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OBSERVATIONS ON THE OCCIPITAL BONE IN A SERIES OF EGYPTIAN SKULLS, WITH ESPECIAL REFERENCE TO THE PERSISTENCE OF THE SYNCHONDROSIS CONDYLO-SQUAMOSA (ZAAIJER; SYNCHONDROSIS INTRAOCCIPITALIS POSTERIOR, B N A.)

BY H. DOROTHY SMITH, B.Sc. Crewdson-Benington Student in Craniology.

WHILE taking a series of measurements at University College, under the supervision of Professor Pearson, on a collection of Egyptian crania of the third dynasty, the writer noted certain peculiarities in the occipital regions of several specimens. In particular a distinct fissure of varying length, extending inwards from the masto-occipital suture (the origin being located generally by the mastoid foramen) towards the foramen magnum, was well shown in several adult specimens. This was identified by Professor Thane with the persistent synchondrosis condylo-squamosa described by T. Zaaijer (*Anatomische Hefte*, herausgegeben von F. Merkel und R. Bonnet, 1. Abteilung, XII. Heft. 1894). An examination of the collection was then undertaken for comparative purposes, and in this the writer is much indebted to both Professor Pearson and Professor Thane for their kind assistance.

The first 1100 specimens examined included the crania of 62 children of whom the youngest was probably about 5 years old. The only instance of persistence of the synchondrosis condylo-squamosa in its whole extent from the masto-occipital suture to the margin of the foramen magnum was found in the cranium of a child of about 7 years of age—and this on the left side only, while on the right the synchondrosis persists to within 8 mm. of the margin of the foramen magnum. In Plate I. will be seen the condition of this cranium with the component parts in situ (A); in figure (C) will be seen the skull with pars basilaris and pars condyloidea removed and these parts are seen separately in Fig. (B).

The best examples among the adult specimens show an irregular fissure (Plates II. and III.), directed transversely inwards from the masto-occipital suture for about 25 mm. Of crania showing a fissure of 15 mm. or more in length,

there are 12 adult and 6 young specimens; and, of these, 8 adult and 6 young specimens show the condition on both sides to a greater or less extent, while the remaining 4 adult (one a young adult) show it on the right side only.

Of the rest, 28 adult and 20 young specimens show a fissure of less than 15 mm. length—in some cases only a slight trace—and of these 29 adult and 11 young show the condition on both sides, and the remaining 19 adult and 9 young, on one side only.

Thus in the 1100 crania examined, the conditions found were:—

Specimens Examined	Synchondrosis Obliterated	Fissure 15 mm. or more	Less than 15 mm.	Percentage showing persistent Synchondrosis Condylo-squamosa
1038 adult	979	12	47	5·7
62 young	36	6	20	41·9

The percentage of adult crania showing indications of the persistence of the synchondrosis condylo-squamosa agrees very closely with the figure (5·3) found by Zaaijer in the collection of adults examined by him, and similarly the condition has been found in a much higher percentage of young crania. The condition also appears to be of more frequent occurrence among the female crania, for of 322 adult female crania 30 show persistence of the synchondrosis condylo-squamosa (9·3 per cent.), while in 457 male crania the condition was found in only 20 specimens (4·3 per cent.).

It was thought worth while to examine the crania of other primates for comparison, and the writer was enabled by the courtesy of Professor Keith to examine the collection at the Royal College of Surgeons. This collection includes a higher percentage of young specimens, and also a number of younger individuals than any in the collection of human crania under observation, and, as was therefore to be expected, a great number of the young specimens show the condition with the synchondrosis extending from the masto-occipital suture to the margin of the foramen magnum, but few adults show more than traces of the developmental condition. Owing to the very great range in size of the crania of the other primates a standard of 15 mm. length of fissure has not been adopted here as in the case of the human crania, and the cases have been grouped according to whether the fissure reaches the margin of the foramen magnum or stops short of it.

Thus 13·3 per cent. of the adult anthropoid ape crania examined show some indication of the synchondrosis condylo-squamosa, while 21·8 per cent. of the young specimens show the whole length of the synchondrosis, and 54·5 per cent. show the condition in varying degrees.

Of the Lemuroidea 17 specimens (14 adult and 3 young) were examined, and a single adult showed a trace of the synchondrosis while two young showed the

whole extent. The remaining specimen—a foetal cheiromys—showed the various components of the occipital entirely separate.

Specimens Examined	Synchondrosis Obliterated	Extending from Masto-occipital suture to Foramen Magnum	Shorter Portions
SIMIIDAE			
12 chimpanzees { 7 adult	7	—	—
12 chimpanzees { 5 young	1	3	1 (acrocephalic)
5 gorillas { 3 adult ...	3	—	—
5 gorillas { 2 young ...	2	—	—
11 orang-outans { 5 adult	3	—	2
11 orang-outans { 6 young	1	4	1
6 gibbons { 2 adult ...	2	—	—
6 gibbons { 4 young ...	1	1	2
CERCOPITHECIDAE			
57 various { 29 adult ...	27	—	2
57 various { 28 young ...	17	3	8
CEBIDAE			
19 various { 10 adult ...	6	—	4
19 various { 9 young ...	2	1	6
HAPALIDAE			
5 various { 4 adult ...	4	—	—
5 various { 1 young ...	1	—	—

Certain specimens among both the Egyptian crania, and the other primates, show the synchondrosis associated with an ossicle at the masto-occipital end (see our Plate IV. B), and the writer finds that the persistent portions of the synchondrosis are invariably at this end. In no case has the condition been found like that cited as typical by A. Rambaud and Ch. Renault (*Origine et Développement des Os*, p. 105 and Fig. 8, Plate VII.), where the ends of the fissure reaching the margin of the foramen magnum remain open after the other portions have been obliterated.

It will be seen in Plate I. A that the line of the synchondrosis condylo-squamosa terminates internally in a small angular projection which forms the lateral boundary of a distinct median bay in the hinder margin of the foramen magnum. This bay, which has a breadth of 9 mm. and a depth of 4 mm., may be termed the *opisthial notch*, as corresponding to the region of the opisthion. Of the 85 crania which show some degree of the persistence of the synchondrosis condylo-squamosa, this notch is more or less marked in 43, whereas of 100 skulls taken at random from the series, which have the synchondrosis completely obliterated, an appreciable notch is present only in 9. A faint indication of the opisthial notch is however frequently to be recognised in the adult skull.

In one somewhat aged male skull, shown in Plate V. A, and presenting only a slight trace of the synchondrosis condylo-squamosa at the end abutting on the masto-occipital suture of the right side, the opisthial notch has a depth of 11 mm. with a maximum breadth of 10 mm. In another adult male skull, with a trace of the synchondrosis on the right side only, the foramen magnum is prolonged backwards as a broad excavation which is not sharply defined laterally, but must be put into the same category. The antero-posterior diameter of the foramen magnum in this case measures 42·5 mm. and the greatest breadth is 30·5 mm.

The opisthial notch evidently results from a defective development of the lower division of the occipital squama, which is formed by the supra-occipital element of the bone, and especially of the part which is derived from the so-called ossicle of Kerckring (*manubrium squamae occipitis*, Virchow). The existence of such a notch in the adult bone is not mentioned in Ledouble's *Traité des Variations des Os du Crâne*, Paris, 1903; but Hamy records a case of a microcephalic foetal skull in which there was a small posterior encephalocele resulting from absence of the ossicle of Kerckring (*Bull. de la Soc. d'Anthropol. de Paris*, 1867, p. 511). Hamy refers to this case again in his paper "Recherches sur les fontanelles anomales du crâne humain," *Journal de l'Anatomie*, T. VII. 1870—71, pp. 591—601, and there uses the name *fontanelle cerebelleuse* for the membranous gap at the base of the occipital squama. This condition would appear to represent a persistence to a greater or less extent of Hannover's *spinoso-occipital membrane*. Ledouble states (*op. cit.* p. 53) that there is a similar defect of ossification in some skulls of normal foetuses in the *Museum d'histoire naturelle* of Paris, as well as in the skull of a hydrocephalic foetus in the Musée Dupuytren.

In only one instance among the Egyptian crania under observation did the writer find any indication of the division of the pars basilaris, which leads to the separation of an anterior segment, first observed by Etienne Geoffroy Saint-Hilaire and named by him *oto-sphenal*, and subsequently designated by Albrecht *basiotique*. This occurs in a young specimen, about 12 years of age (Plate V. B, Cranium E 801), which also shows a slight indication of the synchondrosis condylo-squamosa. Here the basilar portion has on each side a cleft directed transversely inwards from the petro-occipital fissure for a distance of 9 mm. on the right side and 4·5 mm. on the left. The intervening bridge of bone has a breadth on the surface of 15·5 mm. The division is more strongly marked on the endocranial aspect of the bone, where the fissures are only 12 mm. apart, and are connected by a shallow transverse furrow. An incomplete division of the basilar portion was found by Lucy 25 times in 496 skulls (5·04 per cent.). This skull also shows two well marked pressure facets at the hinder margin of the foramen magnum.

Owing to the large number of crania under consideration and difficulties of access it has not yet been possible to examine the whole of this Egyptian series very carefully, but the writer has made a more cursory examination of the

remaining 629 crania in the collection for further examples of the persistence of the synchondrosis condylo-squamosa with the following results:—

Specimens Examined	Synchondrosis Obliterated	Fissure 15 mm. long or more	Less than 15 mm.	Percentage showing Synchondrosis Condylo-squamosa
608 adult	551	8	49	9·3
21 young	6	3	12	71·4

The cranium shown in Plate IV. A is that of a child of about 5 years of age which shows the whole extent of the synchondrosis condylo-squamosa from the masto-occipital suture to the foramen magnum on both sides. This is one of the Wood-Jones collection of Nubian specimens of Roman date obtained from cemetery 14. The condition of the occipital here was noticed by Dr Derry, and the specimen has been kindly lent for the purposes of this paper.

In addition to the Plates already mentioned is included one (Plate VI.) illustrating the development of the human occipital before and after birth, from a series in the Anatomical Museum of University College. The age of these specimens is estimated as follows. (*a*) Foetus of about five months. (*b*) At birth. (*c*) Child of about 1 year. (*d*) About three years. (*e*) About 5 years.

The condition of these specimens would indicate that the obliteration of the synchondrosis condylo-squamosa begins during the third or fourth year of life (Zaaijer, pp. 199—202), but in conclusion it would appear that there is a greater range of variation in the age at which the fusion takes place than is generally stated, and in certain cases the synostosis is not completed even in the adult.

A STUDY OF PYGMY CRANIA, BASED ON SKULLS FOUND IN EGYPT.

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Among the Egyptian crania of the Third Dynasty now being measured under the supervision of Professor Karl Pearson, at University College, London—it was found that nine adult specimens were remarkable for their small size. These crania present marked feminine characters, and have been sexed definitely as female in all cases except one (E 487) which, though exceedingly small, shows certain variations from the more usual form, and is more heavily developed than the others, being thus more open to question as to sex.

In Plates VII—XI will be seen this cranium in various aspects. Plate VII shows the profile, with some general projection of the glabella, beyond which the line rises and slopes somewhat markedly to the bregma. There is some slight post-coronal depression, beyond which there is a further slight rise towards the mid-sagittal area. Beyond this the line curves first gently downwards towards the lambda, where there is an aggregate of ossicles, and then descends almost vertically and finally curves in to the under surface of the skull. The orbits are less finely cut than is usual among these small crania, and the occipital region is somewhat more rugged, while the mastoid processes, though small, are somewhat clumsy, and the left process projects rather further than the right. Besides the ossicles of the lambda, there are two epipteric bones (16 and 14 mm. long) on the left, and one (10 mm. long) on the right, and small ossicles in the lambdoid suture. The sutures are somewhat simple. Both zygomatic arches, and the left orbit, and the nasal bones have been damaged and only three teeth remain in position. These are the first molar on the right side, and the first and second molars on the left side, which show considerable signs of wear, but there is no evidence of wisdom teeth. There is no sign of any ossification of the coronal, sagittal or lambdoid sutures, but the spheno-occipital synchondrosis is obliterated, and the cranium is evidently that of a fully grown individual.

Cranium E 506 (Plates XII—XVI) shows the more usual formation. Here the profile shows an 'infantile' forehead rising vertically from the nasion. The line then curves backwards, and runs almost horizontally, with a slight indication of post-coronal depression, to a point in the mid-sagittal area. Beyond this it curves down to the lambda, and shows some occipital projection before curving in towards the base of the skull. The orbits and zygomatic arches are extremely lightly developed, but the mastoid processes are of fairly large size. The cranium shows metopism with an accompanying breadth of forehead, but there is a tendency towards the obliteration of all sutures. There are some very small ossicles in the lambdoid suture. None of the teeth have been preserved, but the alveolar margin is in good condition, and shows the sockets of the full complement of permanent teeth, including the wisdom teeth. This is the only specimen to which it has been possible to fit a mandible, and the mandible in this case shows a somewhat low and sloping ramus, a shallow sigmoid notch, and slight mental prominence. The three lower molar teeth on each side have been preserved.

Plate XVII shows the above two crania in comparison with a typical adult male and female of normal size in the collection, the male being placed below the female in the plate.

The other crania of small size under consideration show a formation more closely resembling E 506; and, of these, E 21 shows considerable absorption of the hinder portions of the alveolar margin, but appears to have had the wisdom teeth in position. The forehead here is however not vertical, as in the case of E 506, but slopes backwards somewhat from the nasion.

E 420 shows no wisdom teeth, but the other molars in position are considerably ground down, and the sphenoid and occipital bones are united. This cranium is also metopic.

E 579 shows the three upper molar teeth in position on the left side, on the right the second and third molars are wanting, and their sockets are somewhat absorbed. This cranium is rather larger than the others under consideration, and shows somewhat marked supraorbital ridges, and postcoronal depression.

E 666. Here five teeth are present, viz. the two bicuspids and the three molars of the right side; and in association with the wisdom tooth is an accessory molar. The teeth are wanting on the left side.

E 862 shows no wisdom teeth, and the other teeth present show no great signs of wear, but the spheno-occipital synchondrosis is obliterated, and the cranium appears to be that of a young adult. There is an epipteric bone (26 mm. long) on the left side.

E 869 shows the sockets of both upper wisdom teeth. Slight supra-orbital ridges are present, and also a slight metopic crest, and postbregmatic eminence.

E 919 also shows the sockets of both upper wisdom teeth, and though remarkably small has somewhat heavier mastoid processes than is usual among these small specimens.

In addition to the above it has been possible through the kindness of Dr Derry to procure measurements and photographs of two small specimens of other periods.

Plates XVIII and XIX show a specimen of the Wood-Jones collection (*L* 2/63). This is a small female with a wisdom tooth erupted, but not yet quite in position in the mandible. This skull was taken from a cemetery of Christian period, on the Island of Hesa, to the south of the Asswan dam.

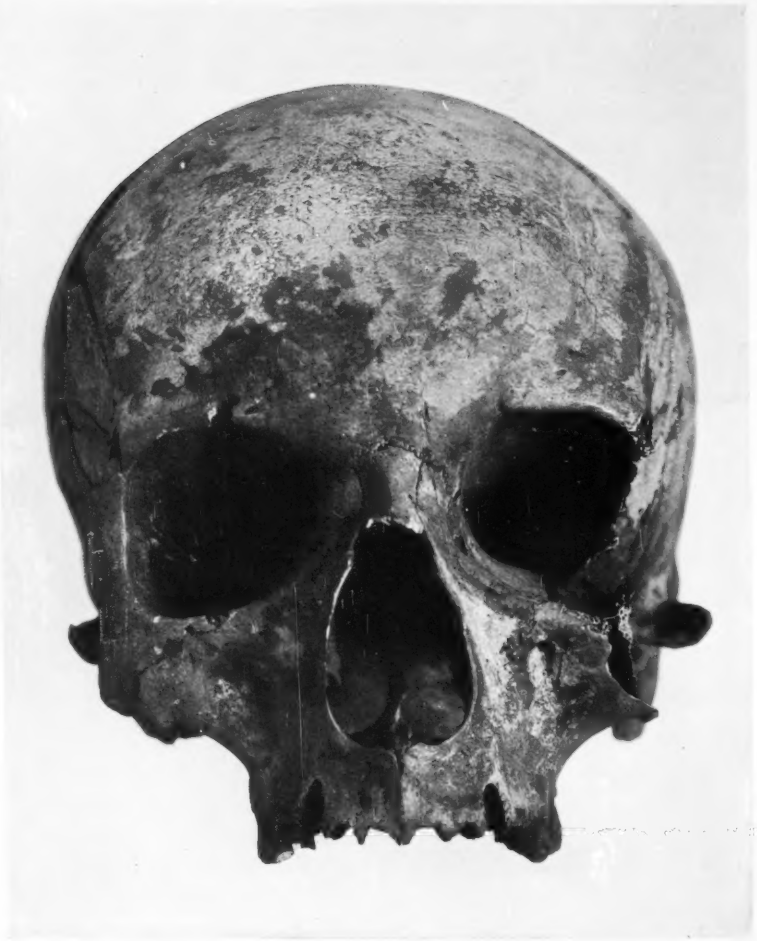
Plates XX and XXI show the skull of a small female (*L* 98/90 *B*) which has had the wisdom teeth in position. This was found by Dr Derry in a grave of Ptolemaic-Roman period at Dakka (70 kilometres south of Asswan on the left bank of the Nile), and, of this specimen, the pelvis has also fortunately been preserved. Plates XXII—XXIV show the pelvic bones in comparison with those of a female of normal size. It will be seen to be very much smaller than the latter, thus indicating a marked degree of correspondence between size of the skull and the probable size of the body.

A table of the series of measurements which it is customary to take in the Biometric Laboratory is given on p. 265, and in this table are included the measurements taken on the above crania, in the order of their notice above, and also, for purposes of comparison, as many of the corresponding measurements on Akka skulls, and Andamanese as were given by the late Sir W. H. Flower in his paper ("Description of Two Skeletons of Akkas, a Pygmy race from Central Africa," *Journal of the Anthropological Institute*, Vol. XVIII. 1888). The measurements of a typical female of normal size from the collection of third dynasty specimens are also given. It was hoped that the measurements taken on living Akkas by Prof. Elliot Smith ("Notes on African Pygmies," *Lancet*, Aug. 12, 1905), might also have been included, but, on investigation, it was found that the methods by which these were taken are hardly comparable, and certain of these measurements as printed appeared to be obviously in error. Unfortunately also the condition of the skulls lent by Dr Derry did not admit of the estimation of their capacity by the method of measurement with mustard seed, and E 919 was also too fragile for this.

It will be seen from the indices that the third dynasty specimens do not show prognathism, being for the most part orthognathous while two specimens are mesognathous. The specimen procured by Dr Derry at Dakka shows a very high degree of prognathism while the Hesa specimen is mesognathous. The Akka specimens have both been estimated as prognathous, but the Andamanese are mesognathous.

Similarly the majority of the third dynasty specimens fall into the mesaticephalic group, only one small, metopic, specimen, and the specimen of normal size being brachycephalic, while both the Dakka and Hesa specimens are brachycephalic. The male Akka is dolichocephalic, and the female is mesaticephalic, while the Andamanese are brachycephalic. With regard to the orbital index there is less

Pygmy Cranium, E 487. About Life Size. *Norma lateralis.*

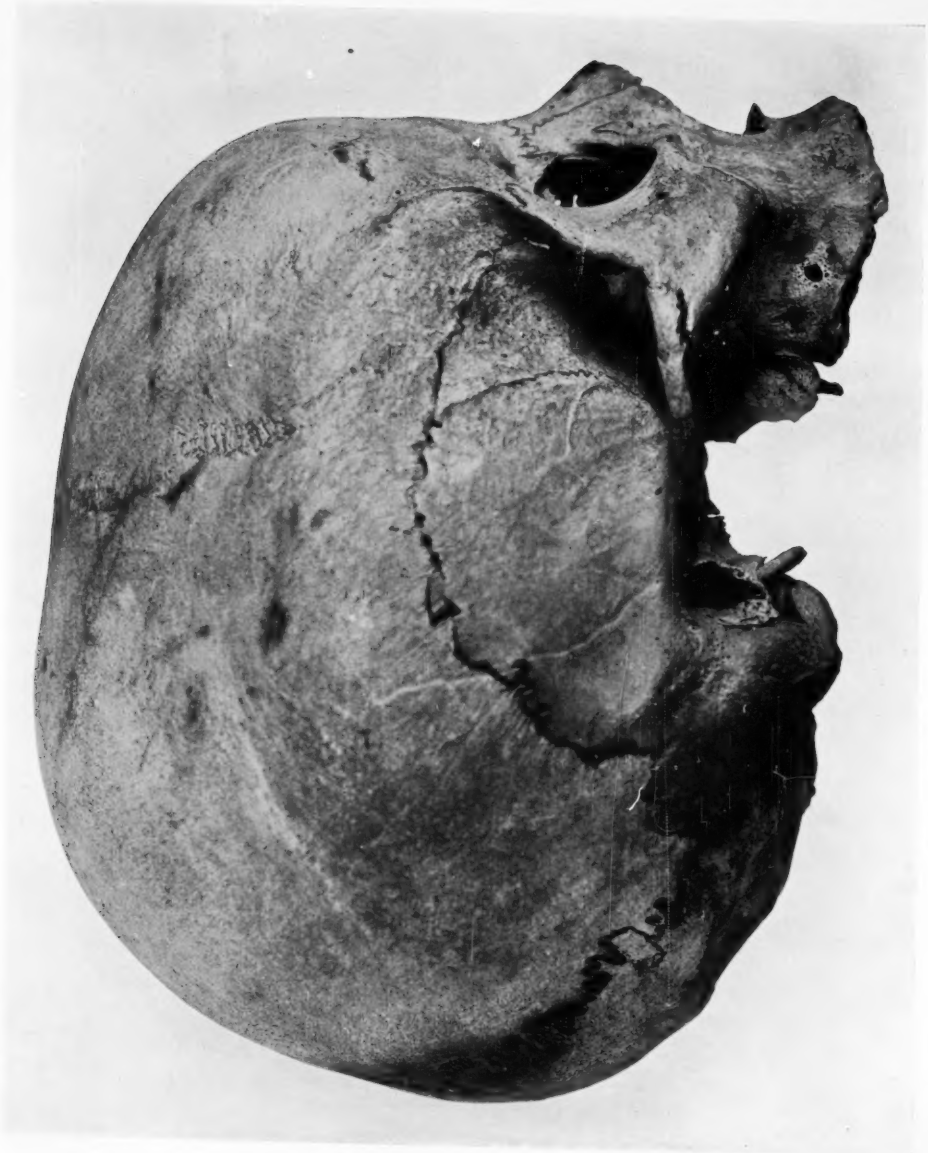


Pygmy Cranium, E 487. About Life Size. *Norma facialis.*

Pygmy Cranium, E 487. About Life Size. *Norma basalis.*

Pygmy Cranium, E 487. About Life Size. *Norma verticalis.*

Pygmy Cranium, E 487. About Life Size. *Norma occipitalis.*



Pygmy Cranium, E 506. About Life Size. *Norma lateralis.*

Pygmy Cranium, E 506. About Life Size. *Norma frontalis.*

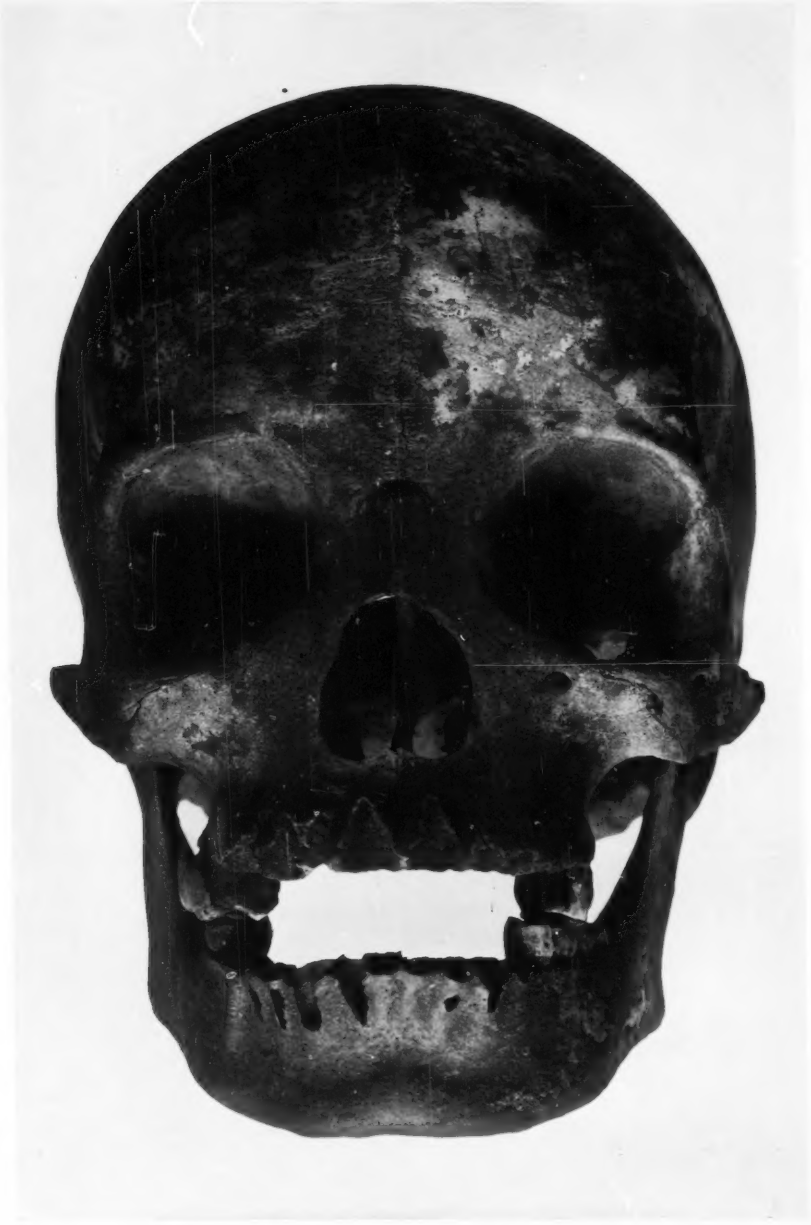
Pygmy Cranium, E 506. About Life Size. *Norma basalis.*

Pygmy Cranium, E 506. About Life Size. *Norma verticalis.*

Pygmy Cranium, E 506. About Life Size. *Norma occipitalis.*

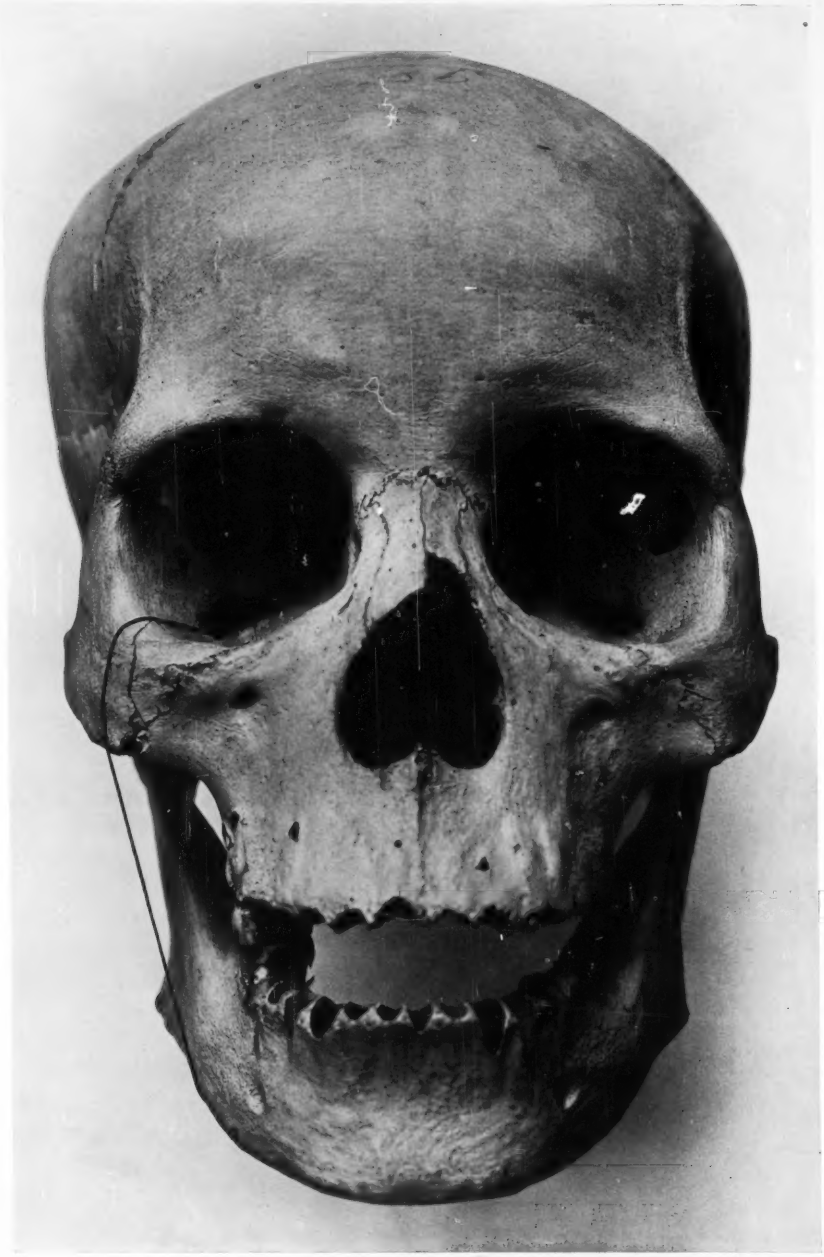
Pygmy Crania, E 506 (above) and E 487 (below), placed longside normal female and male Egyptian Crania. About half Life Size.

Pygmy Cranium, L 2/63, from Hesa. About Life Size. *Norma lateralis.*



Pygmy Cranium, L 2/63, from Hesa. About Life Size. *Norma facialis.*

Pygmy Cranium, L 98/90 B, from Dakka. About Life Size. *Norma lateralis.*



Pygmy Cranium, L 98/90 B, from Dakka. About Life Size. *Norma facialis.*

Pelvis of Pygmy, L 98/90 B (below), from Dakka, compared with that of normal female (above). Bones apart.

Superior View of Pelvis of Pygmy, L 98/90 B (below), from Dakka, and of normal female (above).

Frontal View of Pelvis of Pygmy, L 98/90 B (below), from Dakka, and of normal female (above).

	E 487	E 506	E 21	E 420	E 579	E 666	E 862	E 869	E 919	E 921 Female of normal size	Specimen from Hesa	Specimen from Dakka	Akkas ♂	Akkas ♀	Andamanese ♂	Andamanese ♀
Flowers' ophryo-occipital length ...	166	163·5	165	166	167	165	166	167·5	164	172	160	159	168	163	167·4	160·8
Length (glabella-occipital)	167	163	167	167	169	165	167	168	166	173	160	158·5	—	—	—	—
Length (German horizontal)	166	163	168·2	166·5	168·5	164·5	167	167·5	167	173·5	160	160	—	—	—	—
Greatest horizontal breadth	130	131	129	128	131	126	126·8	127	131	140·5	129	130	125	127	135·8	133·2
Least breadth of forehead	88·8	90	89·1	95	89	92·2	85	87	87·3	87	94	86·7	82	86	—	—
Height of skull (basi-bregmatic) ...	120	123	126	122	131	120	127·5	126·8	125	126	129	119	—	124	—	125·3
Auricular height	106·8	102	106	104	105	100	102	106	110	111	105	107	—	—	—	—
Length of base (basion to nasion) ...	93	92	98	86	100	86·2	92	96	90	95	89·5	92·5	92	92	94·6	90·3
Horizontal circumference	470	469	472·7	476	481	473	468	470	475	499	462	460	468	462	481	462
Sagittal circumference	336	340	333	355	337	344	333	340	348	364	342	328·5	—	—	—	—
Nasion to bregma	117	117	111	120·5	118	117	107	115	119	123	117	119·5	—	—	—	—
Bregma to lambda	99	118	119	125·5	121	125	118	118	116	116	123	109	—	—	—	—
Lambda to opisthion, with steel tape...	120	105	103·5	108	97·5	102	108	107	114	125	101·5	100	—	—	—	—
Lambda to opisthion, callipers ...	99·8	91	87·7	92·5	84·8	89	91·5	91·5	98	105	88·5	87·6	—	—	—	—
Cross circumference of skull... ...	288	282	286·5	283	284	276	277	287	299	312	286	293	—	—	—	—
Upper face height (nasion to alveolar point)	63·5	60	66·3	65	64·2	66·8	64·2	66·2	62·8	67·6	59·2	68	72 ophryon to alveolar	72	— point	—
Face breadth	*d*	85	90·8	96	89·2	91·5	82	85	88·2	89·6	89	90 ?	—	—	—	—
Zygomatic breadth	*d*	116	118·8	116·7	120·3	110·5	112	115·3	114	119·3	119·8	118 ?	—	—	—	—
Nasal height	48·2	44(45·3)	49·2	46·5	49	48	48	47	45	46·3	41·6	42	41	38	—	—
Nasal breadth	23	23	25	22·7	24·6	25	20·5	23	23	21	22	24·5	26	21	—	—
Breadth of Left orbit	*d*	36·5	39	36·5	38·8	*d*	36·3	41	37	38·2	38·2	38	35	35	—	—
Breadth of Right orbit...	37·5	37	39·2	37·5	40·2	*d*	36	39·3	37	38·8	37	37	—	—	—	—
Height of Left orbit	*d*	30	36	29	33·5	34·2	32	33·2	33	33	33·6	32	29	29	—	—
Height of Right orbit	30	29	35·7	29	32·3	34·2	32	32·3	31·5	30·3	30·5	31·5	—	—	—	—
Length of palate	44	43·3	47·8	46·3	44·8	44·5	40	49·6	49·5	45·2	48	53 ?	—	—	—	—
Breadth of palate	33·5	35	35·4	36	40	39·2	36	33·2	33·8	37·5	38·7	37·2	—	—	—	—
Palate length excluding alveolar point	40	40	46	43·5	43·2	42	39	46·2	45	42	45·7	48 ?	—	—	—	—
Profile angle	93°	87°·5	83°	82°	83°	79°	81°	82°	85°	87°	75°	78°	—	—	—	—
Profile length	84	84·5	92·9	88	93·2	85·2	83·7	92·8	88	91	90·9	104·5	100	96	—	—
Length of Foramen Magnum	33·8	33·5	35	33·5	36	32·8	39·3	33·8	32	33·8	31·2	26·5	—	28	—	—
Breadth of Foramen Magnum ...	26	31	30·5	31	29·8	30·3	31·8	29	27	28·8	26·2	22·3	—	—	—	—
Capacity...	1065	1200	1120	1154	1135	1130	1112·6	1072·5	*d*	1299·3	—	—	1102	1072	1244	1128
Length-breadth index	77·8	80·3	77·2	76·6	77·5	76·3	75·9	75·6	78·9	80·9	80·6	81·2	74·4	77·9	81·1	82·8
Gnathic index	90·3	91·8	94·8	102·3	93·2	98·8	90·9	96·6	97·7	95·8	101·5	112·8	108·7	104·3	101·1	101·1
Orbital index	80·0	78·3	92·3	79·4	86·8	*d*	88·1	80·9	89·1	86·3	87·9	84·2	82·9	82·9	90·4	91·4
Nasal index	47·7	52·2	50·8	48·8	50·2	52·0	42·7	49	51·1	45·3	52·9	58·3	63·4	55·3	mesorrhine	
Weight in grams	435·2	308·6	371·3	400·9	458·7	383·75	389·1	416·4	392·2	—	—	—	—	—	—	—

uniformity among the third dynasty crania. The orbits of four of the small specimens are microsemic, those of two small specimens, and the specimen of normal size, are mesosemic; and, of the remaining small specimens, two have megasemic orbits, while the last is defective. Both the Dakka and Hesa specimens have mesosemic, while the Akkas have both microsemic, and the Andamanese have megasemic orbits.

The nasal indices indicate no platyrrhiny in the third dynasty specimens under consideration. Two small specimens and the one of normal size are leptorrhine while the remainder are mesorrhine. The Hesa specimen is mesorrhine, and the Dakka specimen is markedly platyrrhine. Both Akkas also are platyrrhine while the Andamanese are stated to be mesorrhine. There is however in two of the small third dynasty specimens (E 21 and E 579) a marked rounding of the lower margin of the piriform aperture, which gives a somewhat negroid appearance, though for the most part the nasal bones are well formed, and somewhat prominent, and the root of the nose is not markedly broad.

The actual weight of the crania is also not without interest, and at the bottom of the table of measurements will be seen the weights of the small specimens of the third dynasty. In comparison with other crania, however, the weight of individual examples is of small value by reason of variations in preservation and defects; but the mean of the figures given above is 395·13, and this may be compared with the figure 491·15, which is the mean obtained from the weights of 36 adult female crania of normal size, of the same series, and showing the same general condition as regards preservation.

Thus, though closely resembling the Akkas in point of size, the characteristic features of the Negro races are not present in a marked degree in any of these small specimens, except in Dr Derry's specimen from Dakka. This also is the only specimen with which any other portion of the skeleton has been preserved, and, in this case there is marked correspondence between the size of the cranium and that of the pelvis. From what is known of the relation between size of cranium and stature, it is, however, not idle to suppose that the occurrence of adult individuals of small stature was fairly common in Egypt at the period of the third dynasty, although from the evidence afforded by these specimens it is not possible to assert that all such pygmies were of "Negrillo" race. The several forms of dwarfism were quite familiar to the Egyptians, and the possibility that these are cases of dwarfism of the "infantile type" is worthy of consideration. At the same time the fairly perfect character of the teeth rather favours ethnic dwarfism.

NOTES ON THE PIGMENTATION OF THE HUMAN IRIS.

By A. RUDOLF GALLOWAY, M.B., C.M., M.A., Aberdeen.

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In the *Proceedings of the Royal Society* (B), Vol. 80, 1908, p. 85, C. C. Hurst has published a paper "On the Inheritance of Eye-Colour in Man." He therein states that there is distinct discontinuity between:

(1) "The eyes in which two kinds of pigments are present; the one yellow-brown in colour, deposited on the outer or anterior surface of the iris; the other, blue-black in colour, deposited on the inner or posterior surface of the iris." Such eyes he calls duplex.

(2) "The eyes in which the posterior pigment alone is present in the iris, the anterior pigment being absent." Such eyes he calls simplex.

At page 93 he states that "(1) simplex parents mated together give all simplex offspring; (2) duplex parents mated together give either (*a*) all duplex offspring,

or (*b*) duplex and simplex offspring in the proportion of about 3 : 1 ; (3) duplex parents mated with simplex parents give either (*a*) all duplex offspring, or (*b*) duplex and simplex offspring in the proportion of about 1 : 1."

He adds "It is evident therefore that the simplex type, in heredity, behaves as a Mendelian recessive to the duplex type, which is dominant." "The unit characters concerned in the heredity of the duplex and simplex types of eyes are therefore presence (duplex) and absence (simplex) of anterior pigment in the iris, presence being dominant over absence, which is recessive."

From the above statements it follows :

1. That in studying the pigmentation of the human iris, the two surfaces—anterior and posterior—alone are of importance ; the stroma which is about ten times as thick as the posterior layer of retinal pigment epithelium is negligible.

2. That two kinds of pigments may be present in the human iris, the one yellow-brown in colour located on the anterior surface, the other blue-black in colour located on the posterior surface.

3. That Mendelian laws regulate the inheritance of these types.

Since the publication of the paper cited, I have, as occasion allowed, in the out-patient department of the Eye Institution in Aberdeen, where the annual attendance is over 20,000, taken notes of eye-colours and pedigrees which might throw any light on this question. My first difficulty was in finding any eye in which I could state, after careful examination in daylight and by focal illumination with magnification, that there was no anterior pigmentation. This difficulty still continues, and any such case noted in this paper must be taken to mean that no anterior pigmentation was seen at the examination. Several times I have thought that I had discovered a pure blue, or pure grey, or pure blue and grey parentage with no anterior pigmentation, but on careful examination traces of pigment were found in one or other parent.

My next difficulty lay in the immense number of different types of pigmentation. Only very few (each covering many grades) can be given in this paper, and these refer solely to the pedigrees of the families mentioned. This multiplicity of types probably accounts for the following differences from Hurst's observations as noted at pp. 86, 87 of his paper.

1. "Self-coloured duplex" eyes are often "spotted."

2. "Ringed duplex" eyes are often "spotted."

3. The pigment ring of "ringed duplex" eyes is often outside the pupillary zone, occupying an irregular middle zone, and also sometimes, in spots, a peripheral zone of the iris.

4. "Self-coloured duplex" eyes occur with no denser pigment ring round the pupil.

5. Eyes with grades of anterior pigmentation so low that they can be mistaken for "simplices" do occur.

6. "The lighter shades of blue, and coarser greys" are not necessarily "due to the greater density of age" (Hurst, p. 88). In the B family mentioned in this paper, II. 2, aged 9 years, has as densely grey eyes as her mother.

With regard to the first deduction from Hurst's observations concerning the two surfaces of the iris, it is necessary to mention that the two layers of importance developmentally and anatomically are (1) the stroma, consisting mostly of connective tissue, about ten times as thick as (2) the posterior retinal pigment epithelium. The latter is epiblastic, and the former mesoblastic in origin.

For facility of description, the area of the iris may be divided into three zones:

1. Pupillary zone—from the pupil to the lesser circle.

2. Median zone—from the lesser circle to the beginning of the outer or folded part of the ciliary zone (where there is sometimes a greater circle).

3. Peripheral zone, consisting of the outer or folded portion of the ciliary zone, and the periphery of the iris as far as visible.

As the other deductions from Hurst's observations with regard to the two different kinds of pigments, and their Mendelian inheritance, could not be proved or disproved without examining microscopically an iris, showing both, I took occasion to do so in the case of G. C., aged 70, whose right eye I removed on March 30th, 1910, for sarcoma of the choroid in the lower and inner equatorial region. The tumour had not advanced far enough to affect the globe beyond the site of the growth, there being no pain or inflammation, and the sight being only partially obscured. The iris was normal and healthy, with free movement, and was coloured in exactly the same way as that of the left sound eye of the patient. The colour note of the eyes made on March 22nd, 1910, was:

"At a short distance distinctly blue; narrow yellow-brown pupillary zones; median and peripheral zones blue."

"The *right* eye has nine dark brown spots of the apparent size of $\frac{1}{2}$ to $1\frac{1}{2}$ mm. dotted over all three zones, one distinct dark one (up and in) and three more hazy ones occur on the yellow-brown pupillary ring. There is also an irregular peripheral ring, consisting of separate grey spots free of anterior pigment; the median and peripheral zones generally show reticulations well except where the dark brown pigment spots occur.

The *left* eye has seven similar dark brown pigment spots varying from $\frac{1}{4}$ to 2 mm. in apparent size."

The presence of a yellow-brown pigment ring, of dark brown pigment spots, and of blue median and peripheral zones, apparently mostly free from anterior pigment, promised valuable information. The eye, on removal, was taken to the Pathological Department of Aberdeen University, where Professor Dean prepared a large number of serial sections. The globe was divided into an upper and

lower half, the lower containing the tumour. The upper half was used for the sections, which were cut antero-posteriorly from near the horizontal meridian.

Four figures will suffice to demonstrate the important appearances which may be followed all through the series of sections.

Fig. I. (× Zeiss A. unstained.)

Low power unilateral view of a section through the pupil, showing the dark brown posterior pigment epithelium, which increases in thickness near the pupil (behind the pupillary zone); in this region also, the *stroma* is seen to be thickly strewn with pigment cells, many rounded and full of pigment granules; towards the anterior surface these rounded pigment cells are less numerous, some appearing to have ruptured, while at the anterior surface, the pigment consists almost entirely of yellow granules, the cells apparently having distributed their pigment in a granular form, thus causing the pupillary ring. The anterior surface of this pupillary region is seen to be distinctly yellow in colour. In the median and peripheral regions of the iris, beyond the pupillary area, very few pigment cells are seen although in no part are they entirely absent; the anterior surface of these median and peripheral parts, which *to external examination before enucleation appeared blue and free from anterior pigment*, is yellowish and shows the presence of pigment granules, although more faintly than in the pupillary region. It will be noted also that near the pupillary margin, the dark brown posterior pigment epithelium passes suddenly and abruptly into the yellow colour of the anterior pupillary ring, also that the stroma of the pupillary margin has fewer pigment cells than the stroma behind the middle of the pupillary pigment ring. Towards the left limit of the field a dark brown anterior pigment spot is seen, as if some of the pigment cells had apparently passed through the stroma and located themselves on the anterior surface. In the stroma behind this spot, one or two disintegrated pigment cells appear with granular pigment around.

Fig. II. (× Zeiss A. unstained.)

Horizontal antero-posterior section through the pupillary margin.

At the mid-point of the section corresponding to the pupillary margin few stroma pigment cells are seen, while on each side in the parts corresponding to the stroma behind the pupillary pigment ring they are numerous. The yellow anterior surface is most distinct opposite the parts of the stroma which have most pigment cells.

Fig. III. (× Zeiss D. unstained.)

Horizontal antero-posterior section above the level of the pupil through the yellow-brown pupillary zone.

Note the dark pigment epithelium, and the stroma thickly strewn with pigment cells, which tend to disintegrate towards the yellow anterior surface, where pigment mostly in the granular form is seen.

Fig. IV. (× Zeiss D. logwood stain.)

Section through iris, showing the anterior pigment spot seen in Fig. I.

The spot is seen to be due to the presence of dark brown pigment cells of a similar nature to the stroma pigment cells, several of which, whole or disintegrated, appear in the field: the logwood staining shows the structure of the iris tissues.

Professor Dean suggests that these stroma pigment cells are possibly connective tissue cells that have the power of movement. These sections appear to show that they tend to pass from the pigment epithelium layer forward through the stroma, to distribute pigment either in the shape of pigment cells or granules to the anterior surface of the iris. Whether this view be correct or not the sections lend no corroboration to the idea that there are two kinds of pigment, a blue-black behind, and a yellow-brown in front. All the appearances are compatible with the belief that only one pigment is present, which when heaped up in the form of pigment cells, is dark brown in colour, and is yellow when thinly distributed especially in a granular form. Moreover the dark brown pigment cells of the anterior spot are exactly similar in colour to the retinal pigment epithelium of the posterior surface.

It appears probable from the presence of yellow anterior granular pigment in all sections of the median and peripheral zones which were blue and apparently free from pigment, that many so-called pure blue eyes may have this anterior coating of granular pigment, which is revealed only microscopically*. Hence, in addition to the errors concerning two kinds of pigment, etc., another error in the classification of some blue eyes as having no anterior pigment would arise.

Therefore any theory, Mendelian or otherwise, which depends on the supposition that there are two kinds of pigments, with different loci, and that there is no stroma in the human iris, seems based on inadequate examination of the human eye.

Hurst's results so far as they go give approximately the anticipated Mendelian proportions, but as I have endeavoured to indicate in this paper the amount of anterior pigment is a continuous quantity, and nothing short of microscopic post-mortem examination will determine to what extent an eye does or does not possess it. Hurst suggests that half the "clear" or pure blue eyes classified by ordinary observers really possess anterior pigment and that this is the reason why the data collected by Galton do not support the Mendelian theory. If, as I anticipate, many of Hurst's "simplex" irides would show some anterior pigment on microscopic examination, it is evident that he has been singularly fortunate in discovering that the limit of his personal method of examination was that which coincides numerically with Mendelian theory. This coincidence is the more

[* In the case of the eyes of "clinically complete albinos" both in man and in lower forms of life a post-mortem microscopic examination will frequently show some posterior pigmentation, and occasionally some slight anterior pigmentation as well. On p. 85 of his paper Hurst states that in the albino pigment is entirely absent from both surfaces of the iris. EDITOR.]

remarkable in that he quotes the investigations of Davenport as leading to Mendelism in human eye colour, and thus as confirming his own work. Now Davenport collected his data in precisely the same way as Galton did, i.e. by aid of schedules with verbal definitions of eye colour and categories practically identical with Galton's scheme, and therefore, according to Hurst, leading to a wrongful classification of 50 % of the "clear" or pure blue eyes, i.e. eyes without anterior pigment. Yet Davenport—precisely like Hurst—finds his Mendelian quarter. It is thus fairly obvious that if both these authors are right and that if their papers actually confirm each other it must be practically of no importance where the limit is drawn between a "simplex" and a "duplex" eye,—for two different methods of observation which ought to lead to widely different numerical results are appealed to by both observers as confirming each the other's work!

During my search for pure blue parentages, I made the following note on February 22nd, 1910, "Very many blue eyes have a slight yellowish staining, sometimes arranged as a pupillary ring, at other times as a faint wash here and there preferably following the lines of the reticulations. This staining is usually visible only in bright daylight with focal illumination, and not in gaslight." To this, I have now to add the observation that if, after deciding that a blue eye has no anterior pigment, such an eye be rotated upwards, so that an oblique view of the iris surface with foreshortened pupil is obtained, a yellow sheen of anterior pigmentation will often be seen.

These observations account for the fact that I have not, as yet, met with a family from a pure blue or grey, or blue and grey parentage of sufficient size to be of any value to record. Even although many such pure blue or grey-eyed families were obtained, however, I question whether this would indicate any Mendelian inheritance with regard to iris pigmentation: the nearest approach I have been able to make is represented by the following pedigrees, which show some interesting points in inheritance of iris colour. On looking over the notes made when the various individuals of these and other families were examined, I am impressed with the frequency with which presence of the yellow staining of blue and grey eyes is mentioned as occurring in the *upper* parts of the irides. In light brown irides also, the upper parts of the irides are often darker brown (*vide* F. family, II. 3). I would suggest as an explanation of this interesting fact that the upper lid prevents the pigment from being bleached as quickly as in other unprotected parts. This would agree with the microscopic appearances of the sections given in this paper, which show the gradual disintegration of the pigment cells towards the anterior part of the iris.

It is probable that the greatest incidence of light takes place at the most prominent or convex part of the iris surface, at a little distance from the slightly funnel-shaped pupil. Hence the greatest pigment activity is seen there in the sections, and hence the frequency of the ringed pupillary type of pigmentation in light eyes, and the tendency to have darker pupillary rings in dark eyes.

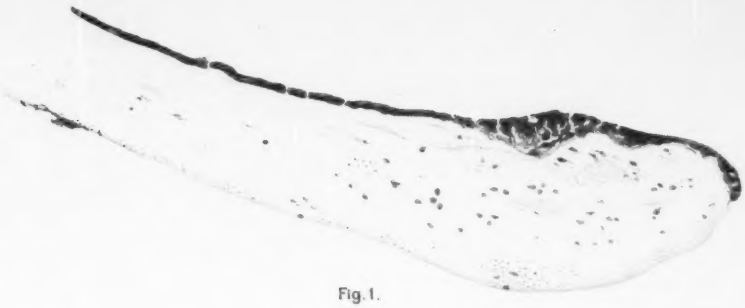


Fig. 1.

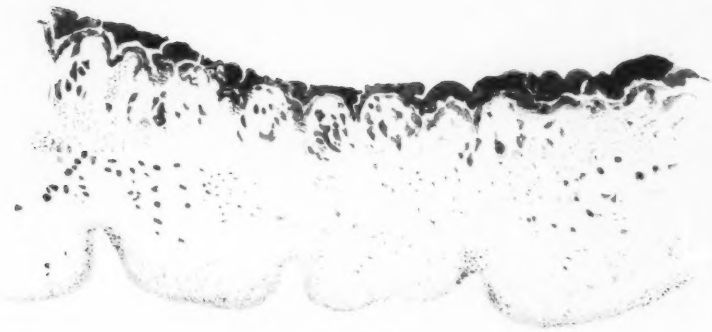


Fig. 2.

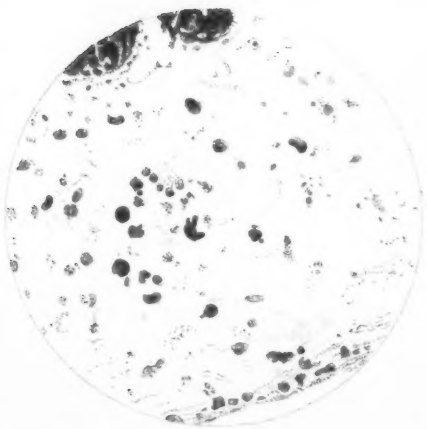


Fig. 3

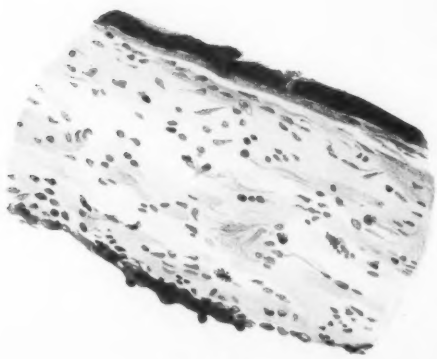


Fig. 4.

Wm Smith Jr del. E. Wilson, Cambridge

SECTIONS OF THE EYE OF G. C. ILLUSTRATING A. RUDOLF GALLOWAY'S MEMOIR

REMARKS ON THE PEDIGREES OF IRIS PIGMENTATION.

A. Family:

This pedigree was constructed prior to April 1910, mainly on account of the large family of sixteen—from two blue-eyed parents II. 3, and II. 4. At that time, following Hurst's classification of simplex and duplex irides, I noted as having no anterior pigment those eyes which appeared to me to have none with focal illumination in good daylight. I have since found that many of these show the yellowish sheen of anterior pigmentation when rotated strongly upwards so that a foreshortened view of the pupil is obtained. This test was not applied either to the parents or children of this interesting family. It will be noted, however, that the mother II. 4 had a faint buff tint on the blue of the upper parts of each iris.

It would serve no useful purpose, therefore, to look for the manifestation of Mendelian phenomena in this family, although the proportion of 14 blues without visible anterior pigment to 2 with tells against this mode of inheritance. I feel that the classification of eyes into "duplex" and "simplex" according to the pigmentation of the iris, is an impossible one, and must lead to inevitable confusion.

It is interesting to note the inheritance, in the daughter III. 9, of the mother's brown spot of anterior pigment, and in the daughter III. 15, as a pupillary ring, of the mother's buff tinting*.

In the eyes of I. 3, faint buff tinting was at first noted, and on a second examination deleted. This yellowish sheen would probably appear on upward rotation in good light in this case also. Three of her sisters, I. 4, 5, 6, had anterior pigmentation well marked.

The mating of III. 3, a pure blue, with III. 4, a uniform rich dark self-brown, is interesting, and has resulted in no iris that is, or can ever become, self-brown. Three of the children have well marked pupillary rings when the parents had none, and IV. 6 is almost an albino. In the mating of II. 5 with II. 6, unfortunately the husband could not be seen, but again from dark brown eyes and almost pure blues there have resulted three with pupillary rings.

F. Family:

The chief points of interest in this pedigree—the mating of two blues which would be classified as pure blues with no anterior pigment by any non-professional examiner—are:

1. The occurrence of a son, II. 3, with brown irides with a complete coating of anterior pigment.

* For examination and report on III. 15 I am indebted to Major F. M. Mangin, R.A.M.C., Specialist in Ophthalmology, Aldershot Command.

2. The buff or yellowish tinting being confined to the upper parts of the irides in two members (I. 2 and II. 1), and pigment being rather more dense in the same regions in II. 3.

3. The appearance of pupillary circles in II. 4, while they are absent in the parents.

B. Family:

This pedigree is interesting:

1. From the mating of blue, almost free from visible anterior pigment, with very rare and peculiar dense grey, almost white, irides free from visible anterior pigment.

2. From the occurrence of the same peculiar white or light grey irides in three of the children, viz. II. 1, 2, 3.

3. From the appearance of pigment circles in four of the five children, the parents being free from them.

4. From the buff staining occurring in the upper parts of the irides of I. 1.

5. From the blue peripheral zones of II. 2. This appearance may partly be due to the fact that in this region the iris is thinner than elsewhere. In a clear yellow yellow-ammer—a *clinically* complete albino with pink eyes—which I have alive at present, this same grey colour occurs as a pupillary ring, where the iris is thickest, the median and peripheral zones being translucent and showing the red fundus reflex through them*.

6. From the absence of photophobia and other signs of albinism.

C. Family:

1. The eyes of I. 1, to any ordinary observer, are blue with no visible anterior pigment.

2. Mated with a blue with no visible anterior pigment, there appears the heterochromic iris of II. 1, showing a large deposit of anterior pigment.

3. Neither of the parents is ringed, but two of the children are, viz. II. 2 and 3.

D. Family:

1. The red pupillary ring of I. 1 corresponds, as is frequently the case, with the original reddish colour of the hair.

2. Red hair is more subject to change of colour, and is more closely connected with albinism than other colours.

3. The two children are verging on albinism and show variation in refraction, II. 1 being hypermetropic, and II. 2 myopic.

* This bird (*Emberiza citrinella* ♀) is now dead, and Dr Usher, who kindly examined the eyes, reports them to be entirely free of pigment, except in the posterior pigment epithelium of the irides, which is very lightly pigmented: see "Canary Breeding," *Biometrika*, Vol. VII. Plate II.

4. The pupillary rings of I. 1 are repeated in both the children.

5. Yellow staining on the blue irides of I. 2 occurs only in the upper and inner regions.

E. Family:

1. The blue eyes and reddish hair of I. 2 are probably connected with the appearance of an incomplete albino II. 2.

2. The examination of this patient II. 2, as detailed, combined with the microscopic examination of G. C.'s eye, has convinced me that it is quite impossible to say when any living person's blue eye is free from anterior iris pigment, and has led me to doubt that this ever truly occurs.

CONCLUSIONS.

1. The human iris cannot be held to consist merely of an anterior and a posterior surface, each, in certain eyes, exhibiting its own kind of pigment.

2. The stroma may contain pigment cells more especially in its deeper parts and granular pigment towards the anterior surface.

3. Pure blue, or so-called "simplex" eyes, which to any outward examination appear free from anterior pigmentation, may show stroma pigment cells, and anterior granular pigmentation when examined microscopically.

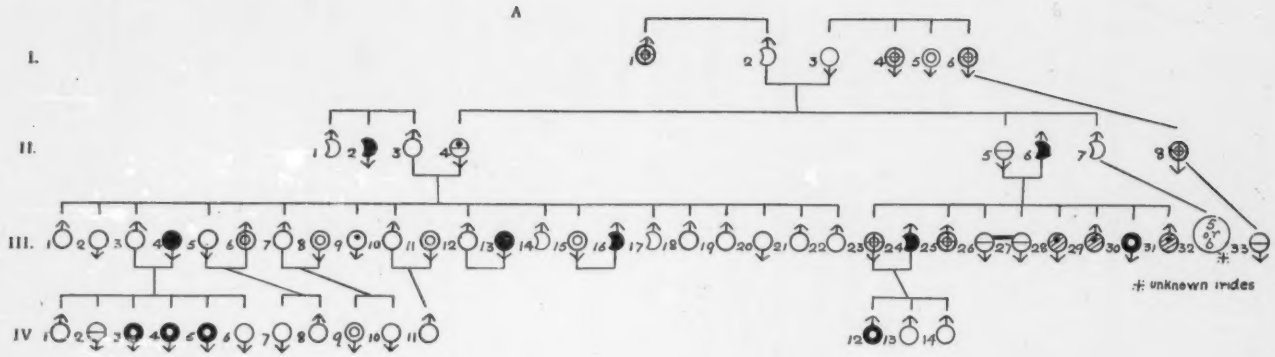
4. Hence the classification of "duplex" and "simplex" eyes is erroneous, and there is no discontinuity or Mendelian inheritance between them.

PEDIGREES OF IRIS PIGMENTATION.

All eyes that have been seen have been examined by myself, except III. 15, Fig. A.

The symbols used have been devised to indicate the main types of iris pigmentation occurring in the following pedigrees, but, as already mentioned, these varieties are so numerous that several grades of colouration must be included under some of the symbols.

The aim of these pedigrees is to indicate how complex is the problem of heredity of eye colour, when we pass from a mere rough appreciation of the extent of pigmentation, as provided by the ordinary verbal descriptions, to an analysis of its distribution throughout the iris.



○ Blue irides.

◎ Blue or grey irides with various coloured circles.

● Dark self-brown irides.

◍ Brown pigment spotting on lighter brown irides.

◓ Heterochromic blue and brown.

Ɔ Blue eyes, not examined.

⊜ Slightly buff tinted blue irides with one or more brown anterior pigment spots.

◗ Brown eyes, not examined.

⊕ Brown or greenish brown irides with pupillary circles, some also with pigment spots.

◴ Grey irides.

⊖ Slightly buff tinted, blue irides.

⊙ Blue irides with one or more brown anterior pigment spots.

◉ Brown irides, with darker-brown pupillary circles.

◌ Greenish blue irides with yellow rayed circles mostly in outer pupillary and middle zones.

⊕ Light brown irides.

APPENDIX, PEDIGREES.

Fig. A. (April, 1910.) I. 1, male, aet. 77. Poorhouse. *Hair*, black. *Eyes*, Right—brown with darker pupillary circles, three or four ¼ mm. darker brown spots in middle and peripheral zones; Left—greenish-brown with darker brown pupillary circles, four or five ¼ mm. brown spots in middle and peripheral zones. I. 2, male, seaman, aet. 55 at death. *Hair*, dark. *Eyes*, pale blue. I. 3, female, aet. 80. *Hair*, light brown. *Eyes*, pale blue, no pigment seen. I. 4, female, aet. 76. *Hair*, brown. *Eyes*, greenish-brown, large yellow-brown rayed pupillary circles, occupying pupillary and middle zones. Peripheral zone, bluish-green. I. 5, female, aet. 72. *Hair*, brown. *Eyes*, pale blue with small buff pupillary circles. I. 6, female, aet. 67. *Hair*, black. *Eyes*, green-brown. Brown pupillary circles, middle and peripheral zones, green. Right has one small (< ¼ mm.) brown tick inner peripheral zone. Left has one small (< ¼ mm.) brown tick outer middle zone. II. 8, female, aet. 34. *Hair*, dark brown. *Eyes*, green-brown. Brown pupillary circles: middle and peripheral zones, green. Right has 3 or 4 small brown dots inner middle zone. Left ditto outer peripheral. III. 33, female, aet. 7 (daughter of II. 8). *Hair*, light brown. *Eyes*, blue; buff mottling arranged in irregular pupillary and peripheral circles. II. 1, male, dead. Blue eyes. II. 2, female, dead. Brown eyes. II. 3, male, aet. 58. *Hair*, blond. *Eyes*, pale blue, no anterior pigment; reticulations well marked. II. 4, female, aet. 57. *Hair*, brown. *Eyes*, blue, each with faint buff tint especially upper part of left iris, which has also one 1½ mm. brown pigment spot in lower pupillary zone. II. 5, female, aet. 55. *Hair*, brown. *Eyes*, pale blue with faint buff mottling on pupillary and peripheral zones. II. 6, male, aet. 51, trawler. *Hair* and *eyes*, dark brown. II. 7, male. *Eyes*, blue. III. 1, male, single, aet. 40, coachman. *Hair*, light brown—lighter on left side when young. *Eyes*, blue, no anterior pigment. III. 2, female, aet. 38. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 3, male, aet. 36. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 4, female, aet. 34. *Hair*, used to be auburn now brown. *Eyes*, rich dark self-brown, uniform colour. III. 5, female, aet. 34. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 6, male, aet. 32. *Hair*, dark brown. *Eyes*, greenish-blue with yellow rayed circles mostly in outer pupillary and middle zones (one in each eye). III. 7, male, aet. 32. *Hair*, black. *Moustache*, light brown. *Eyes*, pale blue, no anterior pigment. III. 8, female, aet. 29. *Hair*, dark brown. *Eyes*, pale blue with pupillary circles of buff. III. 9, female, single, aet. 30. *Hair*, dark brown. *Eyes*, blue—right has one ½ mm. brown spot upper pupillary zone. III. 10, male, aet. 28. *Hair*, brown. *Eyes*, blue with no anterior pigment. III. 11, female, aet.?. *Hair*, brown. *Eyes*, blue with buff pupillary circles. III. 12, male, aet. 27. *Hair*, light brown. *Eyes*, blue, no anterior pigment. III. 13, female. *Hair*, black. *Eyes*, dark self-brown. III. 14, male, single, aet. 25, in Vancouver. *Hair*, brown. *Eyes*, blue. III. 15, female. *Hair*, light brown. *Eyes*, pale blue with buff pupillary circles. III. 16, male, soldier, Aldershot. *Hair* and *eyes*, brown. III. 17, male, died, aet. 9 mos. *Eyes*, blue. III. 18, male, aet. $21\frac{4}{12}$. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 19, male, aet. 19. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 20, female, aet. 17. *Hair*, light brown. *Eyes*, blue, no anterior pigment. III. 21, male, aet. $11\frac{8}{12}$. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 22, male, aet. 10. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 23, female, aet. 32. *Hair*, black. *Eyes*, greenish-brown, large pupillary circles of yellow-brown, and dots of same elsewhere, both zones. III. 24, male, died, aet. 31. *Hair*, dark brown. *Eyes*, dark brown. III. 25, male, aet. 30. *Hair*, dark brown, used to be light brown. *Eyes*, greenish-brown—well-marked pupillary circles of reddish-brown with rays meeting similar spots in peripheral zones—middle and peripheral zones dull green. III. 26, female, aet. 25. *Hair*, light brown. *Eyes*, blue. Right has two or three minute ticks of

yellow in peripheral zone down and out. Left very similar. Twin with III. 27. III. 27, female, aet. 25. Twin with III. 26. *Hair*, brown. *Eyes*, blue, both with very faint spots of buff staining. III. 28, female, aet. 23. *Hair*, dark brown. *Eyes*, self light brown. Right has one 1 mm. dark brown spot in peripheral zone down and out. Left ditto in pupillary zone upwards. III. 29, male, aet 20. *Hair*, once black has now a slight reddish tint. *Eyes*, light brown, uniform yellow-brown pigment layer, no rings. Left has one $\frac{1}{2}$ mm. brown spot peripheral zone up and in. III. 30, female, aet. 16. *Hair*, brown. *Eyes*, light brown with darker pupillary circles. III. 31, male, aet. 14. *Hair*, dark brown. *Eyes*, light self-brown with one or two darker spots. IV. 1, male, aet. 12. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 2, female, aet. 10. *Hair*, light brown. *Eyes*, pale blue, very faint mottling of buff in sinuous lines. IV. 3, female, aet. 8. *Hair*, brown. *Eyes*, light brown with narrow pupillary circles of yellow-brown. IV. 4, female, aet. 5. *Hair*, light brown. *Eyes*, light brown, rather browner round pupils. IV. 5, female, aet. 3. *Hair*, light brown. *Eyes*, light brown, rather browner round pupils. IV. 6, female, aet. 2. *Hair*, light flaxen. Eyebrows and lashes the same—almost an albino. *Eyes*, pale blue, no anterior pigment. IV. 7, female, aet. $4\frac{1}{2}$. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 8, male, aet. 9 mos. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 9, female, aet. 4. *Hair*, brown. *Eyes*, pale blue with indistinct buff pupillary circles and other buff dots here and there. IV. 10, female, aet. 2. *Hair*, brown. *Eyes*, blue, no anterior pigment. IV. 11, male, aet. 2. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 12, male, aet. 6. *Hair*, brown. *Eyes*, brown with darker pupillary circles. IV. 13, male, aet. 5. *Hair*, brown. *Eyes*, blue, no anterior pigment, a greyish white ring in peripheral zone. IV. 14, male, aet. 2. *Hair*, light brown. *Eyes*, blue, no anterior pigment.

Fig. B. (March, 1910.) I. 1, male, aet. 33. *Hair*, brown. *Eyes*, blue, very faint stain of buff on upper part of each iris. I. 2, female, aet. 31. *Hair*, brown, used to be lighter. *Eyes*, very pale grey, almost white, no visible anterior pigment. By magnification the greyish-white colour of the iris is seen to be due to closely packed sinuous fibrous lines. II. 1, male, aet. 11. *Hair*, brown. *Eyes*, pale grey like the mother's, but with pupillary circles of yellow-brown, and one or two other ticks of the same colour here and there. II. 2, female, aet. 9. *Hair*, light brown. *Eyes*, dense pale grey all over except peripheral zones of blue, no visible anterior pigment. The grey irides very like the mother's, and quite as grey, verging on white. II. 3, male, aet. 8. *Hair*, brown. *Eyes*, the same pale grey, almost white irides but with median circles of yellowish-brown. II. 4, male, aet. 6. *Hair*, light brown. *Eyes*, pale blue, with faint pupillary circles of buff. II. 5, male, aet. 2. *Hair*, light flaxen. *Eyes*, blue, pupillary circles of *grey*, and other lines and spots of same, no visible anterior pigment.

Fig. C. (Feb. 1911.) I. 1, male, aet. 43. *Hair*, medium brown. *Eyes*, blue with rather darker pupillary circles. Touched with one or two minute ticks of yellow-brown ; one brown pigment spot in *Right* periphery down and out of ·25 mm. size, a smaller one up and out in median zone. I. 2, female, aet. 45. *Hair*, dark brown. *Eyes*, blue, no visible anterior pigment. II. 1, female, aet. 13. *Hair*, medium brown, rather reddish at tips. *Eyes*, blue. *Right* has a transverse heterochromic band formed by two distinctly red-brown patches of anterior pigment, one at each side of pupil, the outer one $4\frac{1}{2} \times 5\frac{1}{2}$ mm., widest at periphery, the inner one 3×4 mm., widest also at periphery. II. 2, female, aet. 11. *Hair*, red. *Eyes*, light blue, median irregular circles of reddish-yellow anterior pigment. II. 3, female, aet. 9. *Hair*, pale brown with reddish-golden tints. *Eyes*, pale blue with faintest possible staining of pale yellow in pupillary zones.

Fig. D. (March, 1911.) I. 1, male, aet. 37. *Hair*, used to be very fair, gradually got darker, now dark brown, almost black. Eyebrows and moustache *light* reddish brown, they have not altered. *Eyes*, bluish-grey with pale green tint in median and peripheral zones. *Red* complete pupillary rings, with rays stretching across median and peripheral zones. *Refraction*, $\frac{0}{0}$ with manifest hypermetropia of ·5 D. : probable total hypermetropia of 1·5 D. I. 2, female, aet. 38. *Hair*, medium brown, used to be very *fair*. *Eyes*, blue, yellow staining in patches, one in each eye, at upper and inner parts only. Greyish hazy spots form irregular peripheral circles.

Refraction, hypermetropic astigmatism in each eye of about +2 D. cylinder, axis down and in. II. 1, female, aet. 8. *Hair*, yellowish flaxen. *Eyes*, pale blue, narrow yellow pupillary rings. *Refraction*, hypermetropia of 1 D. II. 2, female, aet. 5. *Hair*, pale yellowish-brown, a shade darker than II. 1. *Eyes*, pale blue, lighter than her sister's—faintest possible yellowish staining in irregular pupillary circles. *Refraction*, myopia −6 D.

Fig. E. (April, 1911.) I. 1, male. Unexamined. *Hair*, light brown, darker than that of I. 2. *Eyes*, blue, darker than those of I. 2. I. 2, female, aet. 35. *Hair*, light reddish-yellow, used to be lighter. *Eyes*, blue-grey with irregular pupillary rings of yellow-brown, and one or two small separate ticks of brown in median and peripheral zones of each eye. *Refraction*, +1 D. II. 1. Two older and one younger children (than no. 2) with considerably darker hair and eyes. *Eyes*, blue. II. 2, male, aet. 10. Incomplete albino. Brought by his mother (I. 2) on account of sensitive eyes—photophobic in bright light—and difficulty with lessons. *Hair*, almost white, white eyebrows, lashes a shade darker—pale yellowish. *Eyes*, light blue, no pigment visible anteriorly, and would certainly pass all ordinary tests for absence of anterior iris pigment; but when the eyes are turned strongly upwards so that a foreshortened view of the pupils is obtained, a faint yellowish sheen appears over both irides, and this is more visible round the pupils than elsewhere. Both eyes have peripheral grey dots in irregular circles, no nystagmus, nor pink pupils. *Fundi*, distinctly albinotic, the choroidal interspaces being much paler than the choroidal vessels. *Refraction*, +2 D.

Fig. F. (April, 1910.) I. 1, male, aet. 58. *Hair*, very light flaxen—"white" in youth. *Eyes*, pale blue, very faint buff mottling here and there. He has four brothers and two sisters with blue eyes and one sister with dark eyes. I. 2, female, aet. 57. *Hair*, light brown. *Eyes*, slight buff or yellowish staining in upper part of each iris, seen only by focal illumination in bright daylight. II. 1, male, aet. 37. *Hair*, light brown. *Eyes*, blue, faint touch of buff upper parts of each iris. II. 2, male, aet. 35. *Hair*, dark brown. *Eyes*, pale blue, no visible anterior pigment. II. 3, male, aet. 33. *Hair*, brown. *Eyes*, light brown, left rather darker—diffuse brown anterior pigment—most marked at upper part of each iris, especially *left*. II. 4, female, aet. 31. *Hair*, light brown. *Eyes*, blue with pupillary circles of pale yellow-brown. II. 5, male, aet. 28. *Hair*, brown. *Eyes*, blue. Abroad, not examined. II. 6, male, aet. 20. *Hair*, brown. *Eyes*, blue, no visible anterior pigment.

THE INCREASE IN THE NUMBER OF ERYTHROCYTES WITH ALTITUDE.

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THE subject has previously been investigated by numerous observers and their work has invariably shown that there is a definite increase in the number of erythrocytes in the peripheral blood with a change of altitude. Most of the investigations have however, been conducted either on animals alone, or on a small number of human beings. We on our part have carried out these investigations on biometric lines in order to determine the undermentioned points:—

(1) The normal variations in the number of erythrocytes in apparently healthy individuals.

(2) The effect that a high altitude has on this number.

(3) The effect, if any, that a change of altitude has on anaemic individuals.

The opportunities that were afforded to us to investigate these points were unique in many ways, as we had access to a considerable number of more or less healthy individuals (Natives of India) and furthermore they remained on the spot some 18—20 days whilst undergoing anti-rabic treatment (Högyes dilution method).

This treatment may possibly affect the leucocyte content of the blood but there is no evidence to show that it has any influence on the erythrocyte content, and therefore should not lead to any alteration in our findings on that account.

Kasauli is a hill station in the Punjab in India situated in the Sub-Himalayan range, Lat. 30° 58′ 4″, Long. 77° 2′ 3″ and 6335 feet (about 1920 metres) above sea-level. The patients examined numbered 127 all told; they were taken in the order they came for treatment and were all adult men. Cases severely bitten and those suffering from inflammatory conditions, as the result of the bite, were excluded. We may therefore affirm that the men represented a varied sample of the population. Our patients came from all parts of Burmah, Assam, Central

India and Upper India. If any selection was brought into play, it would be to this extent that more Indians of the lower classes were examined than those of the upper. Amongst our cases some had recently suffered from malarial fever and others probably were suffering from helminthic diseases, if we take eosinophilia to be a characteristic of the latter. Thirty-five were judged to be suffering from anaemia as indicated by the pallor of the mucous membranes; this point was always noted before any blood counts were taken; unfortunately it was not possible at the time to make a haemoglobin estimation as well. Out of the total of 127 individuals 16 came from an altitude of 3000—8000 feet and showed as might have been expected a higher count than the majority who came from the plains of India.

Before giving the results of our analysis, it is necessary to give a brief account of the conditions that may possibly influence an examination of erythrocytes, as well as some of the tests that were applied by us, in order to show how these faults could be excluded.

I. Expansions of glass with diminished atmospheric pressure. As Bürker* has conclusively shown that there is no appreciable effect produced by the expansion of the glass in a Thoma-Zeiss instrument as the result of the decreased atmospheric pressure which occurs at altitudes, this point does not concern us.

II. Errors of technique. When commencing this investigation a number of trial counts were first made on ourselves and the laboratory attendants in order to acquire a greater technical skill and the following points were carefully attended to in order to avoid obtaining false results:—

(*a*) The diluted blood was thoroughly mixed both at the time it was drawn and before blowing it out.

(*b*) The glass cover slip was applied immediately before the corpuscles had time to settle down. Even a few seconds' delay has been shown to materially affect the count*.

(*c*) An exact measurement of the drop was made so that when the cover-slip was applied the fluid extended right to the periphery and none ran over into the trough. No slide was examined unless Newtonian rings were seen.

The blood was diluted 1—200 times with Toisson's fluid and 50 squares of a Thoma-Zeiss slide were counted, which meant that some 350—500 erythrocytes were counted in each individual case.

III. Variability of observations:—

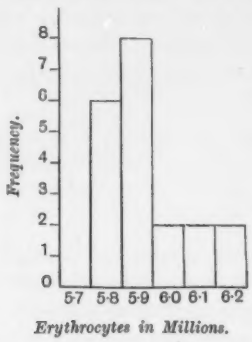
(*a*) To test the degree of error of one's own observations, ten counts were carried out on two consecutive days on the blood of one of our laboratory attendants (see Fig. 1 and Table I).

* R. Bürker, "Die Wirkungen des Höhenklimas auf das Blut," *Münchener medizinische Wochenschrift*, Jahrgang LII., p. 249, 1905, Feb. 7th.

This table shows that the mean or average of these twenty observations was 5,930,000, with a probable error of ± 19,000 and a standard deviation of 127,000.

The probable error of the mean is small compared with the mean, showing that the error of technique was also small. The counts were registered only to the nearest 100,000.

FIG. 1. Results of 20 Observations on one Individual.



(*b*) If a theoretical expression of the probable distribution of erythrocytes per unit of volume can be obtained this would form, when compared with the actual state of affairs, a valuable test of observational and technical accuracy. We have utilized for our purpose a computation given in an article written under the name of "Student" in *Biometrika* (Vol. v. p. 351). The computation is expressed in the formula

$$e^{-m}\left\{1+m+\frac{m^2}{2!}+\ldots\frac{m^r}{r!}+\ldots\right\}^*,$$

this being a very good approximation to the point binomial which would still more closely describe the distribution.

Table I gives the results of a count of 400 squares of a Thoma-Zeiss haemocytometer. The blood in this case was for convenience diluted 500 times.

TABLE I.

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Erythrocytes per square
30	72	105	104	52	27	8	1	1	Number of squares, observed
33	82	103	86	52	26	11	3	2	Number of squares, calculated

It seems evident from the above table that the observed and calculated figures do not differ greatly. And when we apply the necessary statistical tests (*vide*

* m = mean of frequency distribution.

table for testing the goodness of fit of theory to observations, Elderton, *Biometrika* Vol. I. p. 155) we find that $\lambda^2 = 7{\cdot}987$ and $P = {\cdot}43$, which means that the two distributions cannot be considered significantly different (Fig. 2 shows graphically the two distributions). From the foregoing we may assume that our obtained results conform more or less to theoretical requirements.

We will now leave the subject of errors of technique and random sampling and proceed to consider the observations themselves. If an analysis is made of the total first counts (Table II and Fig. 3), it will be seen that the distribution of our cases is a long drawn out one starting at 2,700,000 erythrocytes per c.mm. and

Fig. 2. Firm Line. Actual Observations.
Broken Line. Calculated Values.

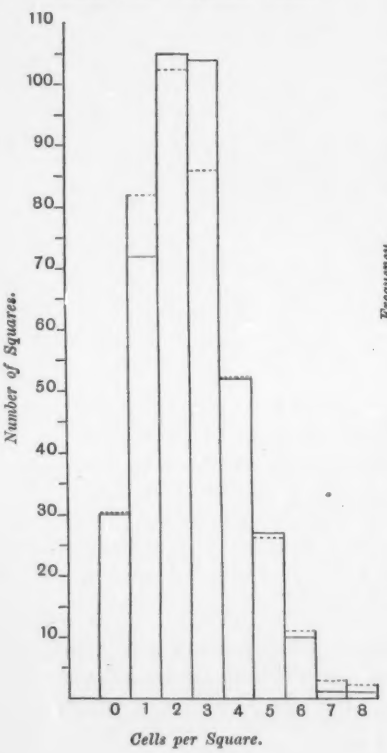
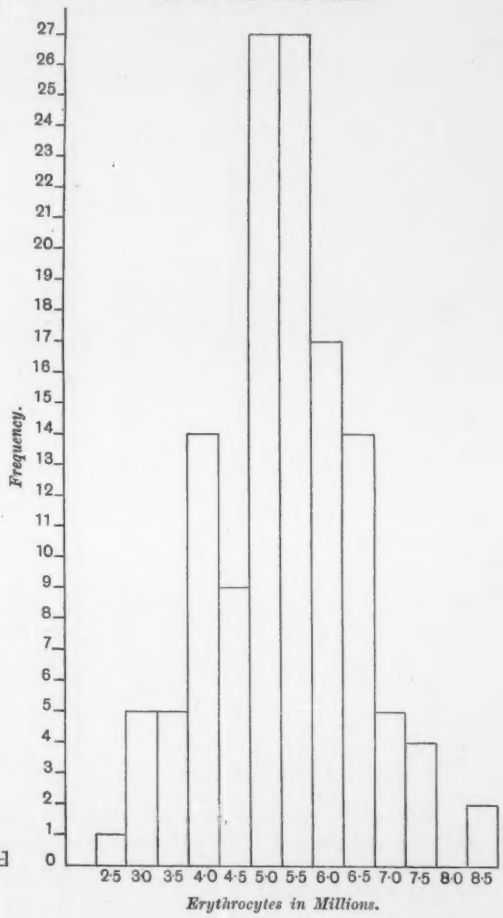


Fig. 3. Total First Counts.



extending to 8,500,000 per c.mm. The mean of the distribution is 5,300,000 erythrocytes per c.mm. with a probable error of ± 67,000 and a standard deviation of 1,127,000. These counts include all individuals, whether suffering from anaemia, enlarged spleens or helminthic diseases and also persons coming from high altitudes as well as those in whom none of these characters are present. They were in fact a random sample of the Pasteur Institute population. In every case, in order to avoid being influenced by the count itself, these points (anaemia, etc.) were carefully noted beforehand and recorded.

TABLE II.

Showing the Frequency of the Total First Counts.

Number of Erythrocytes in 100,000's	Frequency	Number of Erythrocytes in 100,000's	Frequency
27	1	*59*	2
28	0	*60*	5
29	1	*61*	4
30	2	*62*	1
31	0	*63*	4
32	2	*64*	3
33	1	*65*	2
34	2	*66*	2
35	1	*67*	3
36	0	*68*	1
37	1	*69*	2
38	3	*70*	1
39	3	*71*	1
40	2	*72*	0
41	4	*73*	1
42	2	*74*	0
43	1	*75*	1
44	0	*76*	1
45	2	*77*	1
46	6	*78*	0
47	0	*79*	0
48	4	*80*	0
49	3	*81*	0
50	6	*82*	0
51	6	*83*	0
52	8	*84*	1
53	4	*85*	1
54	12		
55	3	Total	130
56	2		
57	6	Mean	53 ± ·67
58	5		
		S. D.	11·3

Now if we eliminate from the total the persons suffering from anaemia and enlarged spleens and those who have come from high altitudes, we are left with the distribution shown in Table III and Fig. 4. The total number now consists of only 75 individuals.

The mean is now increased to 5,620,000 erythrocytes per c.mm. and the increase is chiefly due to the removal of a rather large number of anaemic persons contained in our original total. The probable error is now only ± 43,000 and the standard deviation only 556,000, that is to say the exclusion of the above individuals causes a considerable decrease in the range and variability of our distribution. *The above figures so contracted probably represent fairly accurately the mean and variability for an apparently healthy adult Indian male population living in the plains of India.* The effect of altitude on the number of erythrocytes is well shown in

TABLE III.

Showing the Frequency Distribution of the Total First Counts with the omission of Persons suffering from Anaemia, or Enlarged Spleens, and of Persons from High Altitudes.

Number of Erythrocytes in 100,000's	Frequency	Number of Erythrocytes in 100,000's	Frequency
42	1	*61*	4
43	0	*62*	1
44	0	*63*	3
45	0	*64*	1
46	1	*65*	0
47	0	*66*	0
48	1	*67*	2
49	2	*68*	0
50	5	*69*	2
51	6	*70*	0
52	7	*71*	0
53	4	*72*	0
54	11	*73*	1
55	3		
56	2	Total	75
57	6		
58	5	Mean	56 ± ·43
59	2		
60	5	S. D.	5·56

Table IV and Fig. 5 (*a*) and (*b*). These exhibit the frequency distribution of 100 first counts and the corresponding second counts. It was not necessary in this investigation to consider the exclusion of abnormal individuals (anaemics, etc). The second count was made at an interval of 18 days after the first, that is to say after an 18 days' residence at Kasauli. The counts were limited to cases free from any development of malarial fever during their course of anti-rabic treatment. The means of the first and second counts are respectively 5,328,000 ± 69,600 and 6,492,000 ± 66,500. The difference between the means is 1,164,000, and the probable error of the difference = ± 65,000, from which we may conclude that these two means are significantly different from one another, and that the second count represents an increase on the first. We might have applied our tests to the distributions themselves instead of to their means.

It is of further interest to determine the degree of relationship between the first and second counts and from this obtain an abbreviated expression giving the mean value of a second count as determined from the first for a given altitude. This we believe should have besides a purely scientific interest, one which is of therapeutic value; for it may enable us to determine how far the increase of erythrocytes is likely to occur in anaemic individuals, and furthermore whether the consequent improvement in their condition is or is not counterbalanced by the additional requirements and strain of high altitudes.

FIG. 4. Total First Counts with the omission of Persons suffering from Anaemia, or Enlarged Spleens, and Persons from High Altitudes.

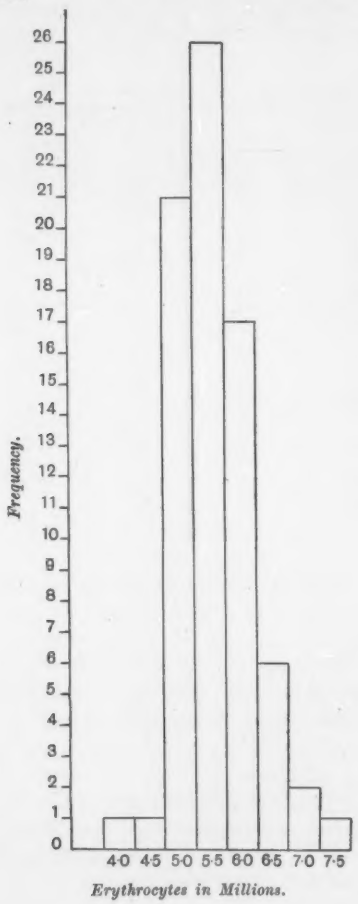


FIG. 5 (*a*). 100 First Counts.

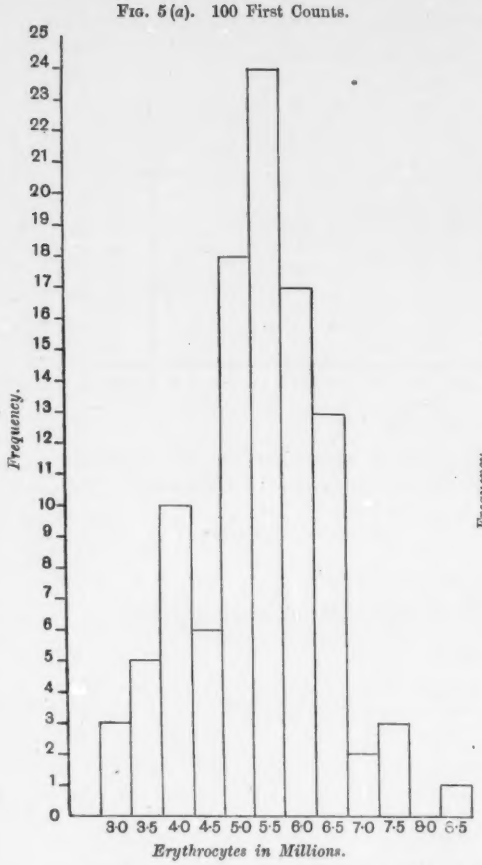


FIG. 5 (*b*). 100 Corresponding Second Counts.

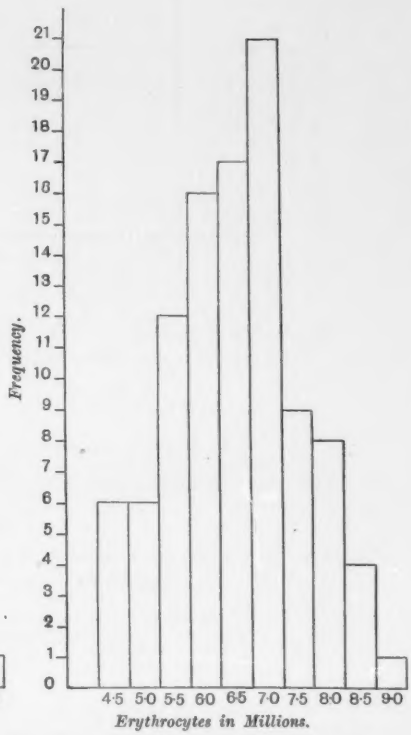


Table IV is a correlation table of the relationship we are considering. In this table the frequency distribution has been somewhat compressed; instead of a unit of 10,000, a unit of 50,000 has been taken. The correlation coefficient works out at $r = 0{\cdot}65 \pm 0{\cdot}04$ which as we might have anticipated is a high one. The relationship between the first and second values obtained from this table is given by the equation

$$C_2 = \bar{C}_2 + r\frac{\sigma_1}{\sigma_2}(C_1 - \bar{C}_1).$$

Where C_2 = the second count, $\bar{C}_2$ = the mean of the second count, which in our correlation table is 6,500,000: σ_1, σ_2 = the standard deviations of the first and second distribution respectively. $\bar{C}_1$ = mean of the first counts, i.e. 5,325,000. An example of the use of this formula will make matters clear.

TABLE IV.

Erythrocytes in 100,000's. First Count.

Erythrocytes in 100,000's. Second Count.	26—30	31—35	36—40	41—45	46—50	51—55	56—60	61—65	66—70	71—75	76—80	81—85	Totals
46—50	1	5	1	—	—	—	—	—	—	—	—	—	7
51—55	—	3	3	3	4	—	—	—	—	—	—	—	13
56—60	—	2	—	4	6	1	—	—	—	—	—	—	13
61—65	—	2	—	1	6	7	1	—	—	—	—	—	17
66—70	—	—	2	3	4	5	6	4	—	—	—	—	24
71—75	—	1	1	—	1	2	3	1	1	—	—	—	10
76—80	—	—	1	—	2	3	2	1	1	—	—	—	10
81—85	—	—	—	1	1	—	—	—	1	1	—	—	4
86—90	—	—	—	1	—	1	—	—	—	—	—	—	2
	1	6	9	5	13	24	19	12	7	1	2	1	100

$r = 0{\cdot}65 \pm 0{\cdot}04$.

For a first count of 5,000,000 erythrocytes, what should be the expectation for a given second count taken 18 days later, at an altitude of 6000 feet?

$$C_2 = 6{,}500{,}000 + 0{\cdot}65 \times \frac{1{\cdot}95}{2{\cdot}16}(5{,}000{,}000 - 5{,}325{,}000),$$

$$= 6{,}325{,}000 \text{ erythrocytes per c.mm.}$$

The following table (Table V) gives the calculated and observed values.

TABLE V.

First Count	Mean of Observed Second Counts	Calculated Mean Second Count
2,800,000	4,800,000	5,027,000
3,300,000	5,220,000	5,322,000
3,800,000	5,800,000	5,617,000
4,300,000	5,900,000	5,912,000
4,800,000	6,220,000	6,207,000
5,300,000	6,500,000	6,502,000
5,800,000	7,090,000	7,092,000
6,300,000	7,160,000	7,387,000
6,800,000	7,800,000	7,682,000
7,300,000	8,050,000	7,977,000
7,800,000	8,300,000	8,282,000

And $\lambda^2 = {\cdot}311$.

$P = {\cdot}998$.

Table VI shows the frequency distribution of 52 observations made on permanent Kasauli residents. They are few in number and widely varying. An average under such circumstances describes but poorly the character of the distribution. Still we may draw this conclusion from our inspection of the figures that the increase which we noted in the erythrocytes of individuals remaining at

an elevation of 6000 feet for only 18 days would seem to be one of a permanent character, as judged from our observations on residents.

TABLE VI.

Showing the Frequency Distribution of Number of Erythrocytes in 52 Kasauli Residents.

Number of Erythrocytes in 100,000's	Frequency	Number of Erythrocytes in 100,000's	Frequency
52	2	*70*	4
53	2	*71*	1
54	1	*72*	0
55	1	*73*	0
56	1	*74*	0
57	4	*75*	1
58	4	*76*	1
59	3	*77*	1
60	3	*78*	1
61	3	*79*	1
62	1	*80*	1
63	0	*81*	1
64	2	*82*	0
65	4	*83*	0
66	3	*84*	0
67	0	*85*	1
68	1		
69	4	Total	52

During the stay at Kasauli 9 out of the 127 cases developed typical malarial fever and in nearly all of these cases there was definite evidence of blood destruction, especially when the blood count was taken a few days after the fever. As the point is an interesting one in relation to malarial fever, we give the figures in Table VII.

TABLE VII.

First Count	Date of development after arrival, in days	Parasites	Second Count	Loss or gain on First Count
6,640,000	1	B. T.*	6,688,000	+ 48,000
3,952,000	1	B. T.	5,939,000	+1,987,000
5,440,000	8	B. T.	5,200,000	− 240,000
5,172,000	6	B. T.	5,104,000	− 68,000
5,232,000	16	B. T.	2,784,000	−2,448,000
3,984,000	5	M. T.†	4,960,000	+ 976,000
5,952,000	17	B. T.	3,216,000	−2,736,000
5,024,000	10	B. T.	4,160,000	− 864,000
5,392,000	6	Nil	5,280,000	− 112,000

* B. T. = Benign Tertian. † M. T. = Malignant Tertian.

We note that the defect due to destruction in malarial fevers would seem to be rapidly recovered from. Those cases which developed fever early after their arrival had at least recovered their original number of erythrocytes by the end of the anti-rabic treatment.

We extracted from our original distribution all anaemias as well as cases of malarial fever. This was done in order to determine the variability of the number of erythrocytes in apparently healthy individuals. Table VIII gives the first and second counts of these extracted anaemias. Most of these, although not all, have reached at the time of their second count the total which we found on the average to be necessary for apparently healthy individuals living at a height of 6000 feet above sea-level. Those who failed to do so, would, we should expect, not benefit markedly by residence at a high altitude, unless carefully treated and kept absolutely at rest in bed. This is in accordance with clinical experience.

TABLE VIII.

Anaemias.

First Counts	Second Counts	Total Gain
2,920,000	4,040,000	1,120,000
3,024,000	3,835,000	811,000
3,040,000	4,241,000	1,201,000
3,168,000	4,672,000	1,504,000
3,168,000	4,188,000	1,020,000
3,376,000	4,632,000	1,256,000
3,440,000	4,688,000	1,248,000
3,536,000	7,328,000	3,792,000
3,696,000	5,104,000	1,508,000
3,792,000	6,496,000	2,704,000
3,840,000	5,208,000	1,368,000
3,840,000	5,080,000	1,240,000
3,928,000	4,696,000	768,000
3,952,000	6,208,000	2,256,000
4,048,000	7,280,000	3,232,000
4,112,000	5,152,000	1,040,000
4,128,000	5,632,000	1,504,000
4,272,000	5,478,000	1,206,000
4,608,000	6,096,000	1,488,000
4,752,000	5,760,000	1,008,000
4,848,000	5,568,000	720,000
5,216,000	6,768,000	1,552,000
5,360,000	5,376,000	16,000
5,424,000	8,704,000	3,280,000
5,792,000	6,308,000	516,000

Concluding Remarks.

1. The mean of our observations is higher than that usually given for an erythrocyte count in a healthy individual, namely, 5,600,000 as against 5,000,000.

2. The deviations from the mean may be considerable and yet consonant with apparent health. The range of our observations extended from 4,200,000 to 7,300,000 for a random sample of a varied population.

3. The effect of altitude is to cause a rapid increase in the erythrocyte content of the blood—quite apparent within a few weeks.

4. Permanent residents at a high altitude exhibit a higher erythrocyte content than those dwelling at lower altitudes.

5. Anaemic individuals do respond to the stimulus of altitude with production of additional erythrocytes but in very different degrees and may not do so in a degree sufficient to meet the requirements of a high altitude.

A STUDY OF THE NEGRO SKULL WITH SPECIAL REFERENCE TO THE CONGO AND GABOON CRANIA.

By the late R. CREWDSON BENINGTON, M.D.

Prepared for Press by KARL PEARSON, F.R.S.

(1) *Introductory.* Attention has already been drawn in *Biometrika* (Vol. VIII. p. 123) to the fact that the late Dr Crewdson Benington during his two years of work in the Biometric Laboratory had spent many months measuring the Congo crania at the Royal College of Surgeons and the crania brought by Du Chaillu from the Gaboon in 1864 and 1880, which latter series are both in the British Museum (Natural History). At the time of Dr Benington's death there was a great deal done, but each of these series was far from completed, and the delay in publication of this paper has arisen from the need of finishing the actual measurements, and reducing statistically not only Dr Benington's series, but other series for comparative purposes. The measurements on the Congo crania at the Royal College were completed by Miss E. Y. Thomson of the staff of the Biometric Laboratory; the Gaboon crania at the British Museum were re-examined especially with regard to their anomalies, by Miss H. Dorothy Smith, Crewdson-Benington Student in craniology. Figs. (i)—(iii) in the text are due to her and the majority of the observations in the "Remarks" on the sheets of individual measurements are either due to or have been revised by her. The whole work of numerical reduction has been carried out by Miss Julia Bell of the Biometric Laboratory. In all anatomical points we have consulted Professor G. D. Thane, who with his wonted kindness has given us aid in a great variety of difficulties. The conclusions which Dr Benington would have drawn from his material of course fail us, for not even the mean characters had been calculated; he had devoted the whole of his reduction work on these skulls to deducing the type contours discussed in the last issue of this Journal. I have made as limited observations on the material as seemed consistent with the general scheme of craniometric work in my Laboratory, leaving the comparative measurements to speak largely for themselves. My own impression after examining the numbers on which many of

the means obtained by both Dr Benington and Dr Shrubsall are based is that they are far too few to base at present any *very* dogmatic statements of racial differentiation upon them. Dr Shrubsall most courteously placed at our disposal a number of unpublished measurements of negro crania, but the frequency of each racial group seemed too small to render them of much service from the standpoint of statistical investigation. Not till at least 50, better 100 crania of each sex and local race are available will it be possible to draw valid conclusions biometrically as to the finer racial differences among the African negroid races. Meanwhile a comparison of the statistical constants* recorded in this paper may be of value as suggesting *possible* but *not proven* differences.

No one appreciates better than the present writer the real difficulty of getting long series of crania, and further how often such long series run the risk of being heterogeneous. But the deduction of local or racial differences by craniometric argument from series of 10 to 20 crania is a perfectly idle task. It cannot be too often asserted that the publication of measurements on small series of crania is of value only because, if they have been made on a standardised system, they can later be pooled with other material. That is the immense advantage that would arise from an international standardised catalogue. The anatomist may from inspection and by appreciation be able to draw conclusions as to racial peculiarities from small cranial series—the biometrician does not feel called upon either to dispute the fact, or to accept such conclusions. He is working by other methods; but he does feel justified in asserting that such conclusions do not receive adequate confirmation by appeal to the slight craniometric differences discoverable in small series of allied human races. Too often we find craniometric distinctions made between two small series, where no evidence is given that two random samples of the same local race might not have exhibited as large differences. Furthermore even, when probable errors are provided, we often find in small, apparently random, samples of the same race, differences quite beyond those of random sampling. The probable explanation of this is that we are meeting in such cases of small numbers with a marked influence of heredity in clan or even with family peculiarities. A cranial series ought to be sufficiently large to insure that the members of a single family or blood group will not markedly influence the results; this is wholly impossible with any cranial series under 50, and the minimum dealt with ought to be 100 adult crania of one sex. The craniologist may often have to content himself with fifty or fewer, but if he does, it will be only wise, as long as he is arguing from craniometry and not from anatomical appreciation, to replace dogmatic statement by cautious suggestion.

(2) *Material used.* The following series have been dealt with, the first three being measured and reduced, the second three reduced only. The seventh series is one which was reduced by Cicely D. Fawcett and published in *Biometrika*, Vol. I, p. 426.

* The statistical constants of Dr Shrubsall's published material have all been independently recalculated in the Laboratory.

A. *Congo Crania* in the Royal College of Surgeons, measured by Dr Benington at the suggestion of Professor Keith. These crania, belonging to the Batetela tribe, provide 50 males and 27 females and 5 non-adult, although it has not been possible to obtain a complete series of measurements on all of them. There are no mandibles. Professor Keith has kindly given us the following account of their origin:

The Collection of Batetela crania was brought home by Mr E. Torday in 1909 when he had completed his travels and explorations in the eastern central parts of the Congo Free State. He gives the following account of them.

"The Batetela skulls were collected by the natives of one or two villages near Makunji. They may be considered as mostly fresh; some were not quite dry when I received them. I was assured that all had died from sleeping sickness and they were given to me to find a remedy for that disease. Sleeping sickness did not appear in this part until thirty to forty years ago. The district in which they were gathered does not cover more than three or four square miles."

The Batetela occupy a large area between 9° and 6° S. and 24°—26° E. The Sungu Batetela are about 24° 20′ E. and 4° 50′ S. Their village is Mokunji or Mukunji, quite close to the Labefu River, the most eastern tributary of the Sankuru.

Some of the crania show marks of having been cut or notched in the supra-orbital region. Parts of the bone may have been removed for medicinal purposes.

B. *Crania from the Gaboon, Group* I, brought by Du Chaillu from Fernand Vaz in 1864. This and the following series appear never to have been adequately measured, still less properly discussed. There are 50 male and 44 female crania in this series. This and the following series are in the British Museum (Natural History), South Kensington.

C. *Crania from the Gaboon, Group* II, brought by Du Chaillu from Fernand Vaz in 1880. There are only 18 male and 19 female in this series, and the numbers being so small it has not been considered worth while calculating more than the means.

D. *Zulu Crania.* Twenty male crania measured by Dr Shrubsall. The measurements are published in his paper on "A Study of A-Bantu Skulls and Crania," in *Journal of the Anthropological Institute*, Vol. I, p. 55 *et seq.*, London, 1899. They belong to the Eastern group of Southern Bantus, inhabiting country south of Zambesi.

E. *Angoni Crania.* Thirty-eight male crania measured by Dr Shrubsall. The measurements are given in the memoir just cited. The crania were sent by Sir H. H. Johnston to the British Museum (*Natural History*), who wrote about them as follows (Shrubsall, p. 77): "The skulls that I sent you were obtained by me in M'ponda's town at the south end of Lake Nyasa. I was given to understand that they were the skulls of Angoni hostages, who had been slaughtered by M'ponda. The heads were taken by me off the stakes of M'ponda's stockade. If, as I imagine, they are the heads of Angoni, they would be the heads of a slightly mixed negro race mainly belonging to the Anyanja stock with a slight Zulu intermixture. It is possible, however, that one or more of them may be Yaos mixed

with Arab blood." Sir H. H. Johnston further indicates that the Angoni were very unlikely themselves to be at all homogeneous; for he tells us that a horde of Zulus early in the 19th century trekked northwards to avoid the tyranny of Chalea, some founded a Zulu kingdom in Fipa, some went northwards even as far as the Victoria Nyanza, "while others working eastwards became the dominant caste among the Wahehe and Wanzindo. After the disruption of the Zulu kingdom in Fipa country the bands formed the present Angoni and Magwang wara." (*loc. cit.* p. 77.)

With such a history there would be almost certainly a mixture of the migrating Zulus and the original population, and it may be seriously doubted whether any weight at all can be given to such differences as are significant between Shrubsall's Zulus and Angoni. All we should be justified in saying would be that the differences may be due to the presence of crania belonging to a mixed race, or to crania belonging to a mixture of races, but no real light can be thrown on what elements go to build up the heterogeneity of the Angoni series.

F. *Kaffir Crania.* Thirty-eight Kaffir male crania measured by Dr Shrubsall (*loc. cit.* p. 55). These Kaffirs appear to consist chiefly of Arna-mpondo, who inhabit Pondoland along the bank of lower Umzi-mvubu, Aba-temlu from between Umtata and Kei rivers, and Ama-xosa, who formerly were between Kei and Fish rivers, but were driven into the Transkei. Again we must feel on very unsafe ground in drawing any very dogmatic conclusion with regard to Kaffir cranial characters from this mixed and sparse material.

G. *Northern Negro Crania.* This series consists of 39 male crania, the measurements of which were extracted from the German Anthropological Catalogue. They belonged to negroes from the North of Africa (see *Biometrika*, Vol. I. p. 426).

G^{bis}. Broca took measurements of 77 modern negro skulls in the collection at Paris, and copies of these measurements were sent to K. Pearson by M. Manouvrier. The skulls were chiefly from North and West coasts of Africa. The following table is taken from Fawcett's paper (*loc. cit.* p. 428):

Character	Male		Female	
	Number	Mean	Number	Mean
Capacity (*C*)*	54	1430 (? 1347)	23	1256 (? 1175)
Maximum Length (*L*) ...	54	185·04	23	174·52
Maximum Breadth (*B*) ...	54	135·20	23	130·52
Height (*H*)	54	134·77	23	126·91
100 *B*/*L*	54	73·28	23	74·85
100 *H*/*L*	54	72·83	23	72·75

* Broca's measures of capacity are exaggerated. See footnote to Fawcett's paper *loc. cit.* p. 428. The means calculated from the series G and G^{bis} can only be of service when we are comparing negroes in general with other races.

The series involved are too heterogeneous to throw any light on intra-negroid differences. The values found for G and G^{bis}, however, lie well inside the differences found by Shrubsall for small series of local negro races, and it would therefore be impossible to assert on the basis of these results *alone* that the negroes of Northern Africa differ sensibly from those of the South.

(3) *Measurements.* The measurements taken by Dr Crewdson Benington are in the main those of previous biometric craniologists. They were made in the manner described by Fawcett (*loc. cit.* p. 416) and Macdonell, *Biometrika* (Vol. III. p. 200). The measurements are as follows: C = capacity. Taken with mustard seed, packing and measuring glass,—not in the present manner of the Biometric Laboratory with the balance and weighing (see *Biometrika*, Vol. III. p. 203). The latter method was not possible as the skulls were measured away from the Laboratory. Dr Benington, however, practised on the "cranes étalons" and against other workers in the Laboratory until his average difference was under 10 cm.[3] F = Flower's ophryo-occipital length; L' = glabellar horizontal length; L = maximum length from glabella to occiput. B = maximum horizontal breadth; B' = least breadth of forehead from one temporal crest to the other; H = basio-bregmatic height*; OH = auricular height as measured on the craniophor, that is the height perpendicular to the horizontal plane, above the line joining the *upper* margins of the auricular orifices. LB = basi-nasal length, from basion to nasion. P = profile angle measured with the goniometer, the skull being on craniophor. Q = cross or tranverse arc from upper rim of one auricular passage to that of the other over the bregma. Q_1 = the same measurement taken not over the bregma, but perpendicular to the horizontal plane. S = sagittal arc from nasion to opisthion; S_1 = part of S from nasion to bregma; S_2 = part of S from bregma to lambda; S_3 = part of S from lambda to opisthion; S_3' = chord of arc S_3. U = horizontal circumference, measured directly above the superciliary ridges and round the most projecting part of the occiput.

$G'H$ = upper face height, nasion to alveolar point; GB = face breadth, from the lower end of one zygomatico-maxillary suture to that of the other; J = zygomatic breadth, from the outermost point of one zygomatic arch to that of other; NH = nasal height, from nasion to the lowest edge of pyriform aperture†; NB = nasal breadth, greatest breadth of nasal aperture, wherever it may be; O_1 = breadth of orbit, R and L, the greatest breadth from side to side (see *Biometrika*, Vol. III. p. 201); O_2 = height of orbit, R and L, taken perpendicular to O_1; G_1 = length of palate, from the point of the *spina nasalis posterior* to an

* H is usually measured in the Biometric Laboratory from the basion to the pencil line on the skull vertically above the auricular axis, but for comparative purposes Dr Benington followed Dr Macdonell and measured to the bregma and thus his measurements are comparable with those of Shrubsall and Broca. In the case of the Congo crania he had the craniophor at the Royal College, and I think his H is the pencil line measurement, and that this accounts for the great difference between the Congo and Gaboon cranial heights.

† *Theoretically* we say from nasion to the point in the median plane which is determined by a tangent to the lower edges of the pyriform aperture. In practice, however, this gives a value sensibly the same as the above.

imaginary surface tangential to the inner alveolar surfaces of the mesial incisors; G_1' = length from base of spine. The reader should consult *Biometrika*, Vol. I. p. 429 and Vol. III. p. 201, for obscurities in different systems of measuring the palate length*:—G_2 = breadth of palate between the inner alveolar walls at the second molars; B_2 = breadth of palate, from end to end of palato-maxillary suture; GL = profile or basi-alveolar length, from basion to alveolar point. fml = length of foramen magnum; fmb = greatest breadth of foramen magnum. In order to determine the flattening of the bridge of the nose, Dr Benington at my suggestion started to measure the transverse distance from dacryon to dacryon with the callipers and also the corresponding minimum arc with the tape. The ratio of the excess of the mesodacryal arc over the mesodacryal chord to the mesodacryal chord, i.e. $DA - DC$ to DC forms a measure of the flattening of the bridge of the nose. Possibly a better measure than this ratio would be the ratio of the subtense of this arc to the chord and a means of determining this is discussed later. These two ratios may be called the Mesodacryal or more shortly the Dacryal Indices. It might be anticipated that they would show very considerable differences between European and negro races.

Pt = Bi-pteric breadth, measured from the hinder extremity of one sphenoparietal suture to the other. St = Bi-stephanic breadth.

The indices calculated are: the cephalic indices $100\,B/L$, $100\,H/L$ and $100\,B/H$; the upper facial index $100\,G'H/GB$; the nasal index $100\,NB/NH$; the right and left orbital indices $100\,O_2/O_1$; the palate index $100\,G_2/G_1$; the occipital index† $Oc.I = 100\dfrac{S_2}{S_3'}\sqrt{\dfrac{S_3}{24(S_3 - S_3')}}$; the foraminal index $100\dfrac{fmb}{fml}$; and the dacryal indices, α and β, as defined above.

On the mandibles, where available, were measured: W_1 = condylar width, or greatest width from outside one condyle to outside of second; W_2 = greatest width at angles from outside one angle to outside of other; h_1 = greatest height, from lowest median projection to top of alveolar margin between mesial incisors; and f = distance between foramina mentalia.

* The following comparative results are of interest:

Length of spina nasalis posterior.

Naqada Skulls	Male 5·05, Female 4·00.	Fawcett
English Skulls, Whitechapel ...	Male 3·61, Female 3·60.	Macdonell
Negroes, Congo	Male 3·09, Female 2·93.	Benington
Negroes, Gaboon, 1864 ...	Male 3·73, Female 3·37.	Benington

It will be noticed that the spine is sensibly longer in the Egyptian than in the English or Negro series. I presume Shrubsall has measured G_1, but in many cases craniologists do not precisely define what they understand by palate length.

† This index, which I now term the Occipital Index, was suggested by me to Dr Macdonell and misnamed by me at that date as the Cerebellar Index.

TABLE I. *Males.*

Character	Congo Series				Gaboon, 1864 Series			
	No.	Mean	Standard Deviation	Coefficient of Variation	No.	Mean	Standard Deviation	Coefficient of Variation
C	47	1343·91 ± 12·45	126·57 ± 8·81	9·42 ± ·66	49	1380·51 ± 10·38	107·69 ± 7·34	7·80 ± ·53
F	50	176·34 ± ·62	6·55 ± ·44	3·71 ± ·25	50	177·62 ± ·52	5·46 ± ·37	3·08 ± ·21
L'	47	177·78 ± ·65	6·80 ± ·46	3·82 ± ·26	—	—	—	—
L	50	177·87 ± ·69	7·05 ± ·49	3·96 ± ·28	50	179·48 ± ·53	5·61 ± ·38	3·12 ± ·21
B	50	138·52 ± ·48	5·00 ± ·34	3·61 ± ·24	50	135·48 ± ·32	3·38 ± ·23	2·49 ± ·17
B'	50	97·46 ± ·37	3·85 ± ·26	3·95 ± ·27	50	96·46 ± ·30	3·19 ± ·22	3·31 ± ·22
H	48	133·81 ± ·40	4·13 ± ·28	3·09 ± ·21	50	135·42 ± ·48	5·02 ± ·34	3·60 ± ·24
OH	47	113·85 ± ·40	4·05 ± ·28	3·56 ± ·25	—	—	—	—
LB	48	98·19 ± ·45	4·58 ± ·32	4·66 ± ·32	48	100·29 ± ·37	3·77 ± ·19	3·76 ± ·26
P	47	82°·76 ± ·37	3°·74 ± ·26	4·52 ± ·32	—	—	—	—
Q	47	310·74 ± 1·01	10·29 ± ·72	3·31 ± ·23	50	310·54 ± ·86	9·02 ± ·61	2·90 ± ·20
Q_1	47	309·28 ± 1·02	10·40 ± ·72	3·36 ± ·23	—	—	—	—
S	50	361·80 ± 1·02	10·73 ± ·72	2·97 ± ·20	50	364·26 ± 1·19	12·51 ± ·84	3·43 ± ·23
S_1	50	125·46 ± ·52	5·47 ± ·37	4·36 ± ·29	50	126·32 ± ·62	6·53 ± ·44	5·17 ± ·35
S_2	50	128·30 ± ·71	7·42 ± ·50	5·78 ± ·39	50	128·58 ± 1·12	11·75 ± ·79	9·14 ± ·62
S_3	49	107·67 ± ·60	6·20 ± ·42	5·76 ± ·39	50	109·14 ± ·94	9·83 ± ·66	9·00 ± ·61
S_2'	50	94·98 ± ·45	4·69 ± ·32	4·94 ± ·33	50	95·92 ± ·65	6·77 ± ·46	7·06 ± ·48
U	50	504·34 ± 1·64	17·24 ± 1·16	3·42 ± ·23	50	501·18 ± 1·09	11·44 ± ·77	2·28 ± ·15
G'H ...	49	63·41 ± ·40	4·14 ± ·28	6·53 ± ·45	50	66·44 ± ·57	6·01 ± ·41	9·04 ± ·61
GB	46	94·76 ± ·55	5·57 ± ·39	5·88 ± ·41	48	95·60 ± ·42	4·32 ± ·30	4·52 ± ·31
J	33	126·48 ± ·81	6·90 ± ·57	5·46 ± ·45	45	129·04 ± ·49	4·85 ± ·35	3·76 ± ·27
NH	49	47·22 ± ·27	2·83 ± ·19	5·99 ± ·41	50	48·16 ± ·23	2·98 ± ·20	6·18 ± ·42
NB	50	26·04 ± ·20	2·05 ± ·14	7·87 ± ·53	50	26·56 ± ·21	2·20 ± ·15	8·29 ± ·56
O_1R ...	46	41·14 ± ·16	1·64 ± ·12	3·99 ± ·28	50	41·46 ± ·14	1·46 ± ·10	3·52 ± ·24
O_1L	47	40·40 ± ·14	1·44 ± ·10	3·56 ± ·25				
O_2R ...	46	34·57 ± ·19	1·95 ± ·14	5·64 ± ·40	50	35·04 ± ·20	2·09 ± ·14	5·96 ± ·40
O_2L ...	47	34·62 ± ·22	2·19 ± ·15	6·33 ± ·44				
G_1'	33	47·24 ± ·45	3·83 ± ·32	8·11 ± ·68	45	48·38 ± ·32	3·23 ± ·23	6·67 ± ·48
G_1	30	50·33 ± ·55	4·49 ± ·39	8·92 ± ·78	35	52·11 ± ·41	3·56 ± ·29	6·82 ± ·55
G_2	46	38·87 ± ·39	3·95 ± ·28	10·16 ± ·72	44	39·50 ± ·30	2·93 ± ·21	7·41 ± ·54
B_2	45	27·64 ± ·42	4·13 ± ·29	14·94 ± 1·09	44	30·43 ± ·26	2·60 ± ·19	8·54 ± ·62
GL	47	96·47 ± ·54	5·46 ± ·38	5·66 ± ·40	48	100·50 ± ·45	4·67 ± ·32	4·64 ± ·32
Fml ...	48	35·94 ± ·23	2·39 ± ·16	6·65 ± ·46	50	35·86 ± ·27	2·80 ± ·19	7·81 ± ·53
Fmb ...	48	29·98 ± ·22	2·30 ± ·16	7·67 ± ·53	50	29·96 ± ·27	2·80 ± ·19	9·34 ± ·64
DC	50	24·90 ± ·25	2·67 ± ·18	10·72 ± ·73	50	24·50 ± ·23	2·38 ± ·16	9·70 ± ·66
DA	50	34·52 ± ·44	4·65 ± ·31	13·47 ± ·92	—	—	—	—
PT	50	108·66 ± ·49	5·17 ± ·35	4·76 ± ·32	—	—	—	—
ST	50	115·74 ± ·52	5·50 ± ·37	4·75 ± ·32	50	113·66 ± ·51	5·33 ± ·36	4·69 ± ·32
100 *B/L* ...	50	77·99 ± ·25	2·58 ± ·17	3·31 ± ·22	50	75·48 ± ·26	2·77 ± ·19	3·66 ± ·25
100 *H/L* ...	48	75·36 ± ·28	2·88 ± ·20	3·82 ± ·26	50	75·50 ± ·23	2·43 ± ·16	3·21 ± ·22
100 *B/H* ...	48	103·58 ± ·44	4·55 ± ·31	4·39 ± ·30	50	100·07 ± ·41	4·27 ± ·29	4·26 ± ·29
100 *G'H/GB*	46	66·88 ± ·48	4·82 ± ·34	7·21 ± ·51	48	69·73 ± ·54	5·57 ± ·38	7·99 ± ·55
100 *NB/NH*	49	55·23 ± ·47	4·91 ± ·33	8·89 ± ·61	50	55·31 ± ·47	4·89 ± ·33	8·84 ± ·60
100 O_2/O_1L	47	85·71 ± ·46	4·72 ± ·33	5·51 ± ·38	50	84·56 ± ·52	5·41 ± ·36	6·40 ± ·43
100 O_2/O_1R	45	84·14 ± ·51	5·12 ± ·36	6·09 ± ·43				
100 G_2/G_1 ...	30	76·80 ± ·77	6·27 ± ·55	8·16 ± ·72	33	76·08 ± ·76	6·50 ± ·54	8·55 ± ·72
Oc. I ...	48	68·83 ± ·77	7·91 ± ·54	11·49 ± ·80	49	68·30 ± ·65	6·78 ± ·46	9·92 ± ·68
100 *Fmb/Fml*	48	83·50 ± ·58	5·96 ± ·41	7·14 ± ·49	50	83·86 ± ·79	8·24 ± ·56	9·82 ± ·67
100 *DI, a* ...	50	39·85 ± ·61	6·32 ± ·43	15·85 ± 1·11	50	45·41 ± ·58	6·04 ± ·41	13·30 ± ·91
W_1	—	—	—	—	27	112·63 ± ·70	5·42 ± ·50	4·81 ± ·44
W_2	—	—	—	—	30	94·27 ± ·77	6·23 ± ·54	6·61 ± ·58
h_1	—	—	—	—	29	32·69 ± ·51	4·11 ± ·36	12·57 ± 1·13
f	—	—	—	—	30	45·37 ± ·29	2·36 ± ·21	5·19 ± ·45

TABLE I. *Males* (*continued*).

Character	Gaboon, 1880 Series		Zulu		Angoni		Kaffir		Northern Negro	
	No.	Mean	No.	Mean	No.	Mean	No.	Mean	No.	Mean
C	*16*	1447·44	—	—	—	—	—	—	*39*	1347·7
F	*18*	180·50	—	—	—	—	—	—	—	—
L'	—	—	—	—	—	—	—	—	—	—
L	*18*	182·33	*20*	184·15	*25*	184·34	*38*	190·62	*39*	182·85
B	*18*	138·22	*19*	137·05	*25*	134·58	*36*	137·36	*39*	133·15
B'	*18*	98·39	*20*	99·83	*23*	98·02	*38*	100·86	*32*	95·91
H	*18*	136·67	*20*	138·13	*24*	136·98	*38*	137·37	*39*	135·31
OH	—	—	—	—	—	—	—	—	*27*	115·04
LB	*18*	101·89	*20*	101·85	*25*	102·46	*38*	105·05	*27*	102·96
P	—	—	—	—	—	—	—	—	*11*	82°·00
Q	*15*	311·73	*17*	307·24	*23*	305·17	*37*	307·16	*27*	306·85
Q_1	—	—	—	—	—	—	—	—	—	—
S	*16*	370·56	*19*	372·37	*24*	368·92	*37*	380·78	*32*	367·72
S_1	*16*	129·38	*19*	128·32	*24*	129·63	*36*	129·51	—	—
S_2	*16*	126·75	*19*	132·37	*24*	130·42	*37*	132·12	—	—
S_3	*16*	114·13	*19*	111·68	*24*	108·88	*36*	119·31	—	—
S_3'	*16*	100·25	—	—	—	—	—	—	—	—
U	*15*	509·53	*17*	520·35	*23*	511·43	*37*	526·68	*32*	508·47
G'H ...	*15*	67·27	*19*	69·47	*25*	70·32	*36*	69·14	*31*	68·00
GB	*17*	98·18	*19*	98·47	*17*	95·59	*32*	96·16	*27*	96·63
J	*17*	132·53	*15*	133·07	—	—	*33*	134·14	*32*	129·47
NH	*18*	47·94	*20*	47·20	*25*	48·46	*36*	48·64	*27*	49·52
NB	*18*	27·06	*20*	27·30	*25*	28·02	*36*	27·33	*34*	27·29
O_1R ...	*18*	42·06	*20*	38·73	*20*	38·03	*37*	39·80	*27*	39·56
O_1L										
O_2R	*18*	35·17	*20*	33·20	*22*	33·32	*37*	34·00	*27*	34·19
O_2L										
G_1'	*14*	50·07	—	—	—	—	—	—	—	—
G_1	*7*	57·29	*17*	52·00	*25*	51·94	*36*	52·13	*26*	49·42
G_2	*12*	40·67	*18*	37·94	*25*	35·80	*36*	37·21	*26*	39·73
B_2	*13*	31·38	—	—	—	—	—	—	—	—
GL	*15*	102·73	*18*	101·94	*24*	103·52	*36*	105·14	*27*	105·33
Fml ...	*16*	35·63	*20*	37·30	*24*	36·38	*38*	37·50	—	—
Fmb ...	*16*	30·00	*20*	30·15	*25*	29·56	*38*	30·03	—	—
DC	*17*	25·53	*20*	24·30	*24*	24·25	*37*	24·14	—	—
DA	—	—	—	—	—	—	—	—	—	—
PT	—	—	*20*	107·35	*25*	108·50	*37*	109·49	—	—
ST	*16*	115·63	*19*	115·00	*25*	114·32	*37*	110·78	—	—
100 *B*/*L* ...	*18*	75·86	*19*	74·27	*25*	73·05	*36*	72·13	*39*	72·87
100 *H*/*L* ...	*18*	74·96	*20*	75 04	*24*	74·25	*38*	72·13	*39*	74·15
100 *B*/*H* ...	*18*	101·22	*19*	98·89	*24*	98·21	*36*	99·97	*39*	98·28
100 *G'H*/*GB*	*14*	68·94	*18*	72·15	*17*	74·06	*32*	72·18	*27*	70·37
100 *NB*/*NH*	*18*	56·49	*20*	58·05	*25*	58·03	*36*	56·33	*27*	55·43
100 O_2/O_1L	*18*	83·73	*20*	85·82	*20*	88·09	*37*	85·51	*27*	87·20
100 O_2/O_1R										
100 G_2/G_1 ...	*7*	70·64	*17*	73·07	*25*	69·10	*36*	71·56	*25*	81·52
Oc. I ...	*15*	66·27	—	—	—	—	—	—	—	—
100 *Fmb*/*Fml*	*16*	84·31	*20*	80·8	*24*	81·3	*38*	80·1	—	—
100 *DI*, *a* ...	—	—	—	—	—	—	—	—	—	—
W_1	*7*	120·86	—	—	—	—	*19*	118·53	*16*	117·00
W_2	*7*	99·00	—	—	—	—	*19*	96·05	*16*	97·15
h_1	*7*	31·71	—	—	—	—	—	—	—	—
f	*7*	46·29	—	—	—	—	—	—	—	—

TABLE II. *Females.*

Character	Congo Series				Gaboon, 1864 Series				Gaboon, 1880 Series	
	No.	Mean	Standard Deviation	Coefficient of Variation	No.	Mean	Standard Deviation	Coefficient of Variation	No.	Mean
C	21	1205·88 ± 15·85	107·68 ± 11·21	8·93 ± ·94	43	1231·70 ± 13·03	126·63 ± 9·20	10·28 ± ·76	17	1240·41
F	27	170·83 ± ·77	5·93 ± ·54	3·47 ± ·32	44	170·57 ± ·63	6·16 ± ·44	3·61 ± ·26	19	170·05
L'	26	171·23 ± ·70	5·26 ± ·49	3·07 ± ·29	—	—	—	—	—	—
L	27	171·37 ± ·79	6·06 ± ·56	3·54 ± ·33	44	171·57 ± ·61	6·01 ± ·43	3·51 ± ·25	19	170·95
B	27	130·91 ± ·72	5·53 ± ·51	4·63 ± ·43	44	130·57 ± ·47	4·67 ± ·34	3·58 ± ·26	19	130·47
B'	27	92·56 ± ·49	3·74 ± ·34	4·04 ± ·37	43	92·84 ± ·41	4·00 ± ·29	4·30 ± ·31	19	93·05
H	26	127·50 ± ·60	4·53 ± ·42	3·55 ± ·33	43	130·05 ± ·60	5·80 ± ·42	4·46 ± ·32	18	131·50
OH	26	109·00 ± ·58	4·35 ± ·41	3·99 ± ·37	—	—	—	—	—	—
LB	27	93·31 ± ·44	3·42 ± ·31	3·68 ± ·34	43	95·84 ± ·42	4·05 ± ·29	4·22 ± ·31	18	97·39
P	25	81°·08 ± ·46	3°·38 ± ·32	4·17 ± ·40	—	—	—	—	—	—
Q	26	298·19 ± 1·39	10·53 ± ·98	3·53 ± ·33	44	296·27 ± 1·10	10·79 ± ·78	3·64 ± ·26	19	298·16
Q_1	26	297·31 ± 1·38	10·44 ± ·98	3·51 ± ·33	—	—	—	—	—	—
S	27	349·81 ± 1·50	11·56 ± 1·06	3·30 ± ·30	44	350·39 ± 1·39	13·67 ± ·98	3·90 ± ·28	19	348·21
S_1	27	120·91 ± ·86	6·61 ± ·61	5·47 ± ·50	44	121·59 ± ·68	6·72 ± ·48	5·52 ± ·40	19	121·11
S_2	27	119·80 ± 1·13	8·70 ± ·80	7·26 ± ·67	44	122·77 ± 1·06	10·44 ± ·75	8·50 ± ·62	19	120·74
S_3	27	109·43 ± 1·01	7·78 ± ·71	7·11 ± ·66	44	105·93 ± 1·04	10·18 ± ·73	9·61 ± ·70	19	106·47
S_3'	27	95·81 ± ·74	5·67 ± ·52	5·92 ± ·55	44	93·05 ± ·77	7·56 ± ·54	8·13 ± ·59	19	93·79
U	27	489·30 ± 1·79	13·80 ± 1·27	2·82 ± ·26	44	481·16 ± 1·41	13·86 ± 1·00	2·88 ± 21	19	480·68
G'H ...	26	59·27 ± ·64	4·83 ± ·45	8·15 ± ·77	38	62·74 ± ·53	4·89 ± ·38	7·79 ± ·61	19	62·47
GB	27	92·20 ± ·57	4·36 ± ·40	4·73 ± ·44	41	90·85 ± ·64	6·07 ± ·45	6·68 ± ·50	17	90·65
J	18	122·14 ± ·68	4·25 ± ·48	3·48 ± ·39	40	120·50 ± ·63	5·92 ± ·45	4·91 ± ·37	15	120·33
NH	27	45·02 ± ·38	2·90 ± ·27	6·44 ± ·59	43	45·14 ± ·32	3·08 ± ·22	6·82 ± ·50	18	45·33
NB	27	25·04 ± ·23	1·74 ± ·16	6·95 ± ·64	43	24·74 ± ·20	1·98 ± ·14	8·00 ± ·59	18	25·67
O_1R	26	41·21 ± ·18	1·36 ± ·13	3·30 ± ·31	43	39·56 ± ·18	1·72 ± ·13	4·34 ± ·32	19	40·11
O_1L	26	40·50 ± ·20	1·50 ± ·14	3·70 ± ·35						
O_2R ...	27	33·57 ± ·23	1·81 ± ·17	5·39 ± ·50	43	33·67 ± ·19	1·84 ± ·13	5·47 ± ·40	19	34·11
O_2L	27	33·35 ± ·24	1·83 ± ·17	5·49 ± ·51						
G_1'	21	48·26 ± ·39	2·63 ± ·27	5·45 ± ·57	36	46·72 ± ·44	3·92 ± ·31	8·40 ± ·67	13	46·69
G_1	21	50·19 ± ·43	2·90 ± ·30	5·78 ± ·60	33	50·09 ± ·47	4·02 ± ·33	8·02 ± ·67	11	51·73
G_2	21	35·93 ± ·36	2·43 ± ·25	6·76 ± ·71	35	37·40 ± ·38	3·33 ± ·27	8·90 ± ·72	12	37·33
B_2	24	30·92 ± ·28	2·01 ± ·20	6·50 ± ·64	36	28·94 ± ·26	2·33 ± ·19	8·06 ± ·64	13	29·69
GL	26	93·90 ± ·50	3·75 ± ·35	3·99 ± ·37	38	98·45 ± ·54	4·91 ± 38	4·98 ± ·39	18	98·17
Fml ...	27	35·33 ± ·39	2·99 ± ·27	8·46 ± ·78	41	33·98 ± ·28	2·61 ± ·19	7·68 ± ·58	18	33·72
Fmb ...	27	28·80 ± ·27	2·09 ± ·19	7·26 ± ·67	42	28·14 ± ·27	2·55 ± ·19	9·07 ± ·67	18	28·50
DC	27	24·72 ± ·21	1·58 ± ·15	6·39 ± ·59	43	23·09 ± ·30	2·87 ± ·21	12·43 ± ·92	19	22·68
DA	27	40·22 ± ·44	3·38 ± ·31	8·40 ± ·77	—	—	—	—	—	—
PT	27	106·24 ± ·63	4·83 ± ·44	4·55 ± ·42	—	—	—	—	—	—
ST	27	112·24 ± ·51	3·96 ± ·36	3·53 ± ·32	42	108·48 ± ·61	5·87 ± ·43	5·41 ± ·40	19	105·32
100 *B/L* ...	27	76·46 ± ·32	2·50 ± ·23	3·27 ± ·30	44	76·15 ± ·34	3·38 ± ·24	4·44 ± ·32	19	76·43
100 *H/L* ...	26	74·58 ± ·33	2·51 ± ·23	3·37 ± ·32	43	75·67 ± ·32	3·07 ± ·22	4·06 ± ·30	18	76·78
100 *B/H* ...	26	102·58 ± ·55	4·19 ± ·39	4·08 ± ·38	43	100·72 ± ·54	5·28 ± ·38	5·24 ± ·38	18	99·68
100 *G'H/GB*	26	64·04 ± ·67	5·05 ± ·47	7·89 ± ·74	38	69·28 ± ·54	4·96 ± ·38	7·15 ± ·56	17	68·20
100 *NB/NH*	27	55·81 ± ·62	4·78 ± ·44	8·56 ± ·79	43	54·93 ± ·43	4·20 ± ·31	7·66 ± ·56	18	56·90
100 O_2/O_1L	26	82·54 ± ·50	3·81 ± ·36	4·62 ± ·43	43	85·23 ± ·48	4·65 ± ·34	5·45 ± ·40	19	85·17
100 O_2/O_1R	26	81·23 ± ·47	3·59 ± ·34	4·42 ± ·41						
100 G_2/G_1 ...	17	70·79 ± ·80	4·90 ± ·57	6·92 ± ·80	30	74·80 ± ·75	6·13 ± ·53	8·20 ± ·72	9	71·98
Oc. I ...	27	67·04 ± ·59	4·58 ± ·42	6·83 ± ·63	44	69·23 ± ·87	8·53 ± ·61	12·32 ± ·90	19	69·45
100 *Fmb/Fml*	27	81·80 ± ·75	5·79 ± ·53	17·07 ± ·65	41	81·16 ± ·74	6·99 ± ·52	8·61 ± ·65	18	84·67
100 *DI* ...	27	41·48 ± ·71	5·71 ± ·51	13·75 ± 1·24	38	46·71 ± ·82	7·49 ± ·58	16·03 ± 1·27	—	—
W_1	—	—	—	—	18	110·00 ± ·68	4·25 ± ·48	3·86 ± ·43	9	106·33
W_2	—	—	—	—	21	90·05 ± 1·05	7·10 ± ·74	7·89 ± ·83	9	90 11
h_1	—	—	—	—	20	30·50 ± ·57	3·78 ± ·40	12·38 ± 1·34	9	28·11
f	—	—	—	—	22	44·27 ± ·34	2·38 ± ·24	5·37 ± ·55	9	47·44

(4) *Biometric Constants for Negro Crania.* Tables I and II give the means, standard deviations and coefficients of variation (accompanied where justifiable by their probable errors) of the Congo and Gaboon 1864 series for males and females respectively. The values of the Gaboon 1880 series, for Fawcett's modern negroes and for Shrubsall's Zulu, Angoni and Kaffir series are given for comparison. Let us first compare the Gaboon and Congo crania with regard to a few of the measured characters. First: Capacity. Taking the males there is a greater difference between the 1880 Gaboon crania and the 1864 Gaboon than between the latter and the Congo. The difference of the last two is 37 cm.^{3} and the probable error of that difference about 16 cm.^{3}, or the difference is less than 2·5 times its probable error. It is difficult to lay stress on this. Now turn to the two Gaboon series. The probable error of the capacity of the 1864 males is 10·38, of the 1880 about 18·16*; the difference of their capacities is 67 and the probable error of that difference 21 cm.^{3} Hence the difference is more than three times the probable error and likely to be significant. On the other hand the modern negroes from North Africa are not in capacity significantly different from either the Congo or Gaboon 1864 series! Shall we not conclude that the Gaboon 1880 series has only larger capacity, because it consists of solely 16 crania, and that these have been probably subject to some unknown form of selection? If we turn to the female crania we find this view is entirely confirmed. There is no significant difference between either Gaboon series and the Congo. Shrubsall has given us no capacities, but, I think, we may say provisionally that for the negro skull the capacity is about 1350 for males and 1230 for females, a result very close to Galton's rule of adding one twelfth to the female to get the male equivalent. Clearly the capacity of the negro skull is for males about 140 cm^3. and for females about 100 cm^3. less than that of modern Europeans†. These are significant differences. The capacity is probably slightly less than that of the ancient Egyptians as judged by Naqada or Theban crania, it is sensibly less than that of a short and primitive race like the Aino, and appears only equalled by that of the modern Copt or Hindoo‡.

Diameters: If we take the chief diameters of the head we have:

	Congo		Gaboon, 1864		Gaboon, 1880	
	♂	♀	♂	♀	♂	♀
Length (*L*) ...	177·9	171·4	179·5	171·5	182·3	171·0
Breadth (*B*) ...	138·5	130·9	135·5	130·6	138·2	130·5
Height (*H*) ...	133·8	127·5	135·4	130·1	136·7	131·5

* Assuming the S.D. of the 1880 series would be approximately the same as that of the 1864 series.

† English ♂ 1477, ♀ 1300; Bavarian ♂ 1503, ♀ 1335; Würtemberger ♂ 1494, ♀ 1337; French ♂ 1494. Data reduced by Macdonell, Fawcett and Lee.

‡ For ancient and modern Egyptians; see *Biometrika*, Vol. I. p. 426; the Hindoo skull is judged from material in the Biometric Laboratory not yet published.

The striking similarity in value of the three diameters for the three female series is only marred by the value of the Congo height i.e. 127·5. The males are in less good agreement, but here again the differences where significant are sometimes such that the Congo is nearer to one Gaboon series than the two Gaboon series are to each other. I do not think any differentiation of these series can be made on the basis of differences so wholly unsystematic as those of this table.

Compared with Europeans the negro has both a shorter and a narrower skull; it is shorter but broader than the ancient Egyptians, being in length and breadth very like the modern Copt (♂: $L = 177{\cdot}0$, ♂ $B = 136{\cdot}8$, ♀: $L = 171{\cdot}5$, ♀ $B = 131{\cdot}1$). On the other hand the negro skull is higher than most European skulls, thus we have:

	Height (H)		100 H/L	
	♂	♀	♂	♀
Aino	139·5	135	75·6	76·7
Copt	137	130	77·5	76·1
Gaboon, 1880	137	132	75·0	76·8
Gaboon, 1864	135	130	75·5	75·7
Northern Negro ...	135	—	74·2	—
Congo	134	128	75·4	74·6
Naqada	135	129·5	73·3	73·2
Bavarian	134	128	74·2	73·9
English {Whitechapel ...	132	125	70·0	69·1
English {Moorfields ...	130	124	68·4	67·2
Würtemberger	131	126	73·0	73·2
French	131	—	72·6	—

In the feature therefore of height of head the negro excels the European, and resembles both in height and cephalic height index a primitive race like the Aino, or even the modern Copt ($H = 137$ and 100 $H/L = 77{\cdot}5$ for males!). This excessive height of the negro skull is maintained in all Shrubsall's series, Zulu 138, Angoni 137, Kaffir 137, although their great length reduces the height index to 75, 74 and 72 respectively. Our North African negroes give 135 and 74·2,—all these for males,—while the mixed series G^{bis} gives 135 and 72·8.

It is worth investigating whether this greater height of the negro skull is above or below the auricular passages. Unfortunately the auricular height has not been determined for Shrubsall's series or our Gaboon series, and is probably 5 mm. too great in Ranke's Bavarian series, which were I expect measured from the *centre line* of the auricular orifices (see *Biometrika*, Vol. VIII. p. 137). We have the results given in the table on the following page.

It will be seen that the Congo crania agree very closely with the North African negroes and that both do not exceed in auricular height the European group. The exaggerated height of the negro skull is therefore associated with the portion of the skull between the basion and the auricular axis. This is well brought out by taking the index formed of the ratio of auricular to total height, when it is seen that the Europeans have a greater development above the auricular

	Auricular Height		Index 100 *OH/H*	
	♂	♀	♂	♀
Aino	119	115	85·5	85·1
Copt	116	109	84·6	83·9
Gaboon, 1880	—	—	—	—
Gaboon, 1864	—	—	—	—
Congo	114	109	85·1	85·5
Northern Negro ...	115	—	85·0	—
Naqada	115·5	113	85·5	87·4
Bavarian	115·5 ?*	109 ?*	86·5 (?)*	85·3 (?)*
English {Whitechapel ...	115	109	86·8	87·6
English {Moorfields ...	114	109	87·7	88·5
Würtemberger	115	112	87·8	89·2
French	113	—	86·4	—

axis, while the negroes most closely resemble the primitive Aino, and the Copt alone stands lower than they do in the relative proportion of the upper height. It must not, however, be overlooked that while European crania are in breadth and length notably superior to the negro of the Congo-Gaboon type, and quite significantly superior to the Zulu-Kaffir groups, they are not superior in auricular height. The negro is just as high from auricular passage to the vertex; the largeness of the index 100 *OH/H* is not due to a development of the upper portion of the European's skull; it appears due to a lessening of the basal part of his skull.

Skull-Base: To test this we may compare the basi-nasal length *LB* of the skull in negro and other races. We have

	Basi-nasal length (*LB*)		Ratio *L/LB*	
	♂	♀	♂	♀
Aino	105	100	176·2	176·7
Kaffir	105	—	181·5	—
Northern Negro ...	105	—	177·6	—
Zulu	103	—	180·8	—
Angoni	102	—	179·9	—
Gaboon, 1880	102	97	178·9	175·5
Gaboon, 1864	100	96	179·0	179·0
Copt	101	96	175·4	178·6
French	100	—	180·5	—
Bavarian	100	96	180·0	181·1
English {Moorfields ...	98·5	96	192·0	191·2
English {Whitechapel ...	101·5	95	186·1	189·2
Naqada	99	95	186·4	187·1
Würtemberger	98·5	94	182·0	184·5
Congo	98	93	181·1	183·7

* *OH* from Ranke's measurements is 120·75 for males and 114·17 for females, giving for the indices 90·2 and 89·2. I have deducted 5 mm. from his values, believing that he has measured from centre of auricular passages, and in accordance with my observation on actual sections, see *Biometrika*, Vol. VIII. p. 137.

It is of interest to see that for the base of the skull the Congo crania appear sensibly differentiated from the other negro crania. Although the Kaffir and Zulu crania are long and give a ratio of maximum length to skull basis equal to that of the French or Bavarian, we see that on the whole the European development has been in the direction of a reduced skull basis, and much increased cranial length.

Arcs: If we now turn to the tape measurements we should expect little difference between European and negro in the transverse arc (Q)*, but marked differences in the sagittal arc (S) and the horizontal circumference (U). The following table shows results:

Cranial Arcs.

	Male				Female			
	$Q+S+U$	Q	S	U	$Q+S+U$	Q	S	U
Aino	1224	328	373	523	1180	317	361	502
Bavarian	1219	330	365	524	1173	319	353	501
Kaffir	1215	307	381	527	—	—	—	—
English, Moorfields...	1211	305	379	527	1172	293	366	513
English, Whitechapel	1209	308	377	524	1161	294	363	504
Würtemberger ...	1209	324	367	518	1174	315	358	501
Zulu	1199	307	372	520	—	—	—	—
French	1196	312	366	518	—	—	—	—
Gaboon, 1880 ...	1192	312	371	509	1127	298	348	481
Naqada	1188	304	373	511	1154	296	364	494
Angoni	1185	305	369	511	—	—	—	—
Northern Negroes ...	1183	307	368	508	—	—	—	—
Copts...	1179	311	366	502	1133	296	352	485
Congo	1177	311	362	504	1137	298	350	489
Gaboon, 1864 ...	1175	310	364	501	1127	296	350	481

It will be seen that the Germans (Bavarians and Würtemberger) head the list for the transverse arc (Q), but while the negroes in this arc excel the English and are close to the French, yet with the exception of the Kaffirs and Zulus in the sagittal and horizontal circumferences they are behind the Europeans. The Congo and the longer Gaboon series of 1864 are at the bottom of the list, and are strikingly alike and also similar as in other characters to the modern Copts. The Angonis and the Northern negroes are very close together and fall between the Naqada and the Copts. It is the Kaffir and Zulu who stand out with a markedly bigger skull, and it is the more to be regretted that we have not data

* Because the negro cranium is not deficient in vertical height, but markedly so in breadth and length.

for the capacities of these races*; they may like the Aino have a very considerable capacity, or the bone of the cranium may be thicker than that of Europeans.

Another extremely interesting point is a comparison of the development of the various sections of the sagittal arc. We can unfortunately only compare the Moorfields English with the Congo and Gaboon series, where alone these measurements have been made. We have:

	Male				Female			
	English	Congo	Gaboon, 1864	Gaboon, 1880	English	Congo	Gaboon, 1864	Gaboon, 1880
Nasion to Bregma...	129·5	125·5	126·5	129·5	125·5	121·0	121·5	121·0
Bregma to Lambda...	128·5	128·5	128·5	127·0	123·5	120·0	123·0	120·5
Lambda to Opisthion	120·5	107·5	109·0	114·0	117·0	109·5	106·0	106·5
Chord of latter ...	98·0	95·0	96·0	100·0	96·0	96·0	93·0	94·0
Occipital Index	58·5	68·8	68·3	66·3	59·2	67·0	69·2	69·4

Now the peculiarity of these results is that the English have only slightly more arcual length between nasion and bregma or bregma and lambda than the three negro groups, but the arc from lambda to opisthion is much larger in the English skull; that is to say the occipital part is much more markedly developed. The measure of this is well seen in the occipital index, the ratio of the radius of curvature of the occipital arc to the chord from lambda to opisthion, a *low* occipital index meaning a well-developed occiput. Clearly the occiput is far more developed in the English than in the negro crania—the difference in the occipital indices being very significant.

In the upper face height $G'H$, there is little to differentiate Negro and European, for though the Germans (Würtemberger and Bavarians) stand at the top, the English and French (68 to 70) are almost identical with Zulus, Kaffirs and Angonis. Only the Congo and Gaboon crania come at the bottom, following closely the Naqada†.

* The known capacities (in cm³.) are as follows:

	♂	♀		♂	♀
Bavarian	1504	1336	Naqada	1381	1288
Würtemberger ...	1494	1337	Gaboon, 1880 ...	1447	1240
English, Whitechapel	1477	1300	Gaboon, 1864 ...	1381	1232
English, Moorfields ...	1474	1365	Copts	1356	1214
French	1473	—	Northern Negro ...	1348	—
Aino	1462	1308	Congo	1344	1206

These results place the European 100 cms. above the negro who is comparable with the prehistoric and modern Egyptians. But as I have said we cannot generalise until the Kaffir and Zulu capacities are found.

† The same remarks apply to the bi-zygomatic breadth, J, which appears in no way as an essentially negroid character.

A more striking comparison may be made between the upper face indices which run as follows:

Upper Face Index, 100 *G'H/GB*.

	♂	♀		♂	♀
Würtemberger ...	78·1	74·1	Angoni	72·2	—
English, Whitechapel	76·5	77·9	Copts	72·1	68·5
Bavarian	74·5	74·4	Naqada	70·6	70·4
French	74·2	—	Gaboon, 1864	69·7	69·3
Kaffir	74·1	—	Gaboon, 1880	68·9	68·2
English, Moorfields ...	72·8	73·6	Aino	68·4	67·7
Theban Mummies ...	72·3	72·3	Northern Negro ...	68·0	—
Zulu	72·2	—	Congo	66·0	64·0

It appears from this list that no modern European race occurs in the second half, but that Zulu and Kaffir appear in the first half *. It is clear that on the whole Europeans have a relatively longer upper face than the negro races.

Another marked feature of the face, its upper prognathism, is measured by the profile angle *P*. Unluckily we have but few records of this for the negro races, but they suffice to show how markedly it differentiates their crania from the modern European, and even from the prehistoric as well as modern Egyptian. We have:

Profile Angle, P.

	♂	♀		♂	♀
Bavarian	89°·1	88°·8	Naqada	84°·4	84°·5
English, Whitechapel	86°·1	87°·1	Copts	84°·2	?
French	86°·5	—	Long Barrow	83°·0	?
Würtemberger ...	86°·4	84°·9	Congo	82°·8	81°·1
Theban Mummies ...	85°·5	85°·6	Aino	82°·0	81°·0
English, Moorfields ...	84°·5	84°·5	Northern Negroes ..	82°·0	—

Thus the prognathous character of the negro's profile is only equalled or excelled by that of the primitive Aino.

It would seem unusual to pass over the discussion of the cephalic indices and yet it is doubtful how far they are really indicative of important racial differences. Thus the Gaboon 1864 males and the Zulus differ but slightly in the height-length index, but the value in one case is produced by the head being unusually short and in the other case by its being abnormally high, relatively, to the majority of human races.

Now it would puzzle any one to take much of profit out of this list as it stands! It represents no clear order in the scale of civilisation nor in that of

* The position of the Kaffirs and Zulus may be due to personal equation in determining the alveolar point. Dr Shrubsall appears to get larger values than we do for *G'H*.

Cephalic Indices.

	Male		Female	
	$100\,B/L$	$100\,H/L$	$100\,B/L$	$100\,H/L$
Bavarians	83·2	74·2 (+9·0)	83·1	73·9 (+9·2)
Würtemberger ...	82·5	73·0 (+9·5)	82·8	73·2 (+9·6)
French	79·8	72·6 (+7·2)	—	— —
Congo	78·0	75·4 (+2·6)	76·5	74·6 (−1·9)
Copts	77·3	77·5 (−0·2)	76·6	76·1 (+0·5)
Aino	76·5	75·6 (−0·1)	77·6	76·7 (+0·9)
Gaboon, 1880 ...	75·9	75·0 (+0·9)	76·4	76·8 (−0·4)
Gaboon, 1864 ...	75·5	75·5 (0·0)	76·2	75·7 (+0·5)
English, Moorfields ...	75·5	68·4 (+7·1)	75·1	67·2 (+7·9)
Theban Mummies ...	75·1	74·7 (+0·4)	76·5	74·7 (+1·8)
English, Whitechapel	74·3	70·1 (+4·2)	74·7	69·1 (+5·6)
Zulus	74·3	75·0 (−0·7)	—	— —
Angonis	73·1	74·3 (−1·2)	—	— —
Naqadas	73·0	73·3 (−0·3)	74·2	73·2 (+1·0)
Northern Negroes ...	72·9	74·2 (−1·3)	—	— —
Kaffirs...	72·0	72·0 (0·0)	—	— —

racial relationship. But quite a different light is thrown on the whole matter, if we take as our character the index $100\,(B-H)/L$, i.e. the difference of the two cephalic indices as shown by the numbers in brackets; we then have the following arrangement:

Würtemberg Women	+9·6	Gaboon Men, 1880	+0·9
Würtemberg Men	+9·5	Coptic Women...	+0·5
Bavarian Women	+9·2	Gaboon Women, 1864 ...	+0·5
Bavarian Men	+9·0	Theban Egyptian Men ...	+0·4
English (Moorfields) Women ...	+7·9	Gaboon Men, 1864	+0·0
French Men	+7·2	Kaffir Men	+0·0
English (Moorfields) Men ...	+7·1	Aino Men	−0·1
English (Whitechapel) Women	+5·6	Coptic Men	−0·2
English (Whitechapel) Men ...	+4·2	Naqada Men	−0·3
Congo Men	+2·6	Gaboon Women, 1880 ...	−0·4
Long Barrow Women... ...	+2·3	Zulu Men	−0·7
Long Barrow Men	+2·2	Angoni Men	−1·2
Theban Egyptian Women ...	+1·8	Northern Negro Men ...	−1·3
Naqada Women	+1·0	Congo Women	−1·9
Aino Women	+0·9		

It is at once seen that this list is far more significant than the previous one; the modern European races stand at one end of the scale, the negroes at the other and the prehistoric and primitive races in the centre. The women as a rule stand above the men; probably, not because the numerator $B-H$ is for women relatively greater, but because L owing to the development of the superciliary ridges is relatively greater in man. On this account when this index has been calculated for a larger number of races, it may be desirable to keep the two

sexes in separate lists. That there is something very anomalous in the position of the Congo males is clear; not only is this one of the two exceptions to the rule that the female stands above the male in the list, but the highest negro male series is associated with a female series at the very bottom of the list. It should however be noted that the other case of interchanged sex order occurs in the 1864 Gaboon series, a series very closely allied to the Congo crania. The affinity already noted between the prehistoric, historic and modern Egyptians and the Gaboon Congo crania is also emphasised in this series. Modern advanced races of man tend to a considerable excess of cranial breadth over height, primitive and prehistoric races like the Naqada and Aino to an equality of these diameters; the negroid races tend to fall below equality and exhibit an excess of cranial height over breadth. The index $100\,(B-H)/L$ may be used to measure this characteristic, and if it has not yet been given a name—and I am not aware that it has—might be called the *acroplatic* index.

Of the measures of the mandible we can say but little, because so few comparative measures exist. Macdonell gives measures of the Whitechapel mandibles, but was unable to sex them satisfactorily, so that they are at present almost useless for comparative purposes*. Shrubsall gives condylar and angle widths for 19 Kaffir mandibles. Benington provides four characters for the two Gaboon series; there were no mandibles with the Congo crania. All we can do is to put against these negro mandible measurements those of Fawcett on the prehistoric Egyptians and Koganei on the Aino. We have the following results:

	W_1		W_2		h		f	
	♂	♀	♂	♀	♂	♀	♂	♀
Aino	102·0	95·5	—	—	34·0	32·5	—	—
Naqada	110·5	106·5	93·5	87·5	33·0	31·5	44·5	43·0
Gaboon, 1864... ...	112·5	110·0	94·0	90·0	32·5	30·5	44·5	45·5
Gaboon, 1880... ...	121·0	106·0	99·0	90·0	31·5	28·0	47·5	46·5
Northern Negro ...	117·0	—	97·0	—	—	—	—	—
Kaffir	118·5	—	96·0	—	—	—	—	—
English, Whitechapel	113·0		95·5		31·0		43·5	

This table shows that in every respect (W_1, W_2 and f) the negro mandible is broader than that of either Aino or prehistoric Egyptian, but that he has a less depth (h) of mandible, i.e., crudely described, less chin.

Another interesting index in the case of the negro is the alveolar index, or the ratio of the profile length to the base of the skull; it is, like the profile angle, a partial measure of the prognathism.

* See *Biometrika*, Vol. VIII. p. 213.

Alveolar (or Gnathic) Index, 100 GL/LB.*

	♂	♀		♂	♀
English (Whitechapel)	94·4	94·8	Aino	99·5	99·8
Scottish†	94·9	94·9	Kaffir	100·1	—
Theban Mummies ...	95·2	96·5	Zulu	100·1	—
Naqada	95·4	95·9	Gaboon, 1864... ...	100·2	102·7
English (Moorfields)...	96·4	96·1	Gaboon, 1880... ...	100·8	100·8
Copts	96·6	98·9	Angoni	101·0	—
Congo	98·2	100·6	Northern Negro ...	102·3	—
Long Barrow... ...	99·4	99·9			

It is regretable that we have not the measurements of any but British crania among the European races. The table brings out, however, several points: the female sex tends to have a higher alveolar index than the male. The Egyptians as represented by prehistoric, historic and modern (Coptic) crania are very sensibly differentiated from the bulk of the negro; the one exception is the Congo male series, the anomalous character of which is again emphasised by the marked difference of male and female indices.

There are few determinations of the *foraminal index*. Macdonell gives it as 84·2 for English males (Moorfields) and 84·5 for English females. Schuster's values are 77·6 for Long Barrow males and 87·5 for females, the numbers being only 11 and 6 crania respectively‡. The Congo crania give: males 83·5, females 81·8; Gaboon, 1864, give males 83·9, females 81·1; Gaboon, 1880, give: males 84·3, females 84·7; Zulu males 80·8, Angoni males 81·3, Kaffir males 80·1. These numbers tend to show that the foramen magnum is somewhat more oval in the southern negroes than in the Congo-Gaboon series, the latter of which approaches close to the English value. But more data are requisite before the matter can be discussed adequately. As far as the material reaches, it is not inconsistent with the Congo-Gaboon crania standing closer to the European than the Zulu-Kaffir groups do. So far as the limited evidence goes, the form of the foramen is not a markedly differential racial characteristic.

Passing to the *palate*, we should, judging from results on the mandible and the negro prognathism, expect to find G_1 and G_2 both greater than in European crania and thus possibly not much differentiation in the palate index. But as the Biometrical School has several times indicated, it is very difficult to be certain that two different measurers of the palate take exactly the same measurements, and there are serious pitfalls likely to arise from personal equation and a lack of standardisation. The following table gives the best results available:

* Determined from means of *GL* and *LB*.

† See *Biometrika*, Vol. III. p. 242.

‡ Mean of 17 crania without regard to sex =81·1.

Measurements of the Palate.

	G_1		G_2		$100G_2/G_1$	
	♂	♀	♂	♀	♂	♀
Congo	50·3	50·2	38·9	35·9	76·8	70·8
Gaboon, 1864... ...	52·1	50·1	39·5	37·4	76·1	74·8
Northern Negroes ...	53·1	—	39·7	—	74·8	—
English (Moorfields)...	53·0	50·0	39·7	37·0	74·9	74·1
Bavarian	44·3?	42·2?	33·2?	32·1?	74·4?	76·0?
Würtemberger ...	51·7	50·2	38·1	37·2	74·2	74·6
French	50·2	—	37·2	—	74·1	—
Zulu	52·0	—	37·9	—	73·1	—
Theban Mummies ...	56·2	52·1	40·8	38·7	72·6	74·3
Aino	53·0	51·4	38·2	37·4	72·1	72·8
Naqada	55·8	53·9	40·3	38·9	71·9	72·3
Kaffir	52·1	—	37·2	—	71·6	—
English (Whitechapel)	53·2	49·1	36·8	35·2	69·2	71·2
Angoni	51·9	—	35·8	—	69·1	—

Some explanation is requisite as to the manner in which this table has been reached. G_1 is defined in the Frankfurt Concordat as the palate length measured from point of the *spina nasalis posterior* to the inner wall of the alveolar rim between the mesial incisors. The inconvenience of this definition is obvious, the alveolar rim is often worn away between the mesial incisors and the spine is often broken off so that the measurement cannot be made. In the series of the above table attempts were made to measure G_1 as thus defined in the Naqada, Congo and Gaboon 1864 series*. The measurements themselves suggest that it was actually used in determining the Würtemberger, Aino, Zulu, Kaffir and Angoni series. Macdonell in his two English series, which were much worn, measured his G_1 from an imaginary surface tangential to the inner alveolar surfaces of the mesial incisors, as suggested by E. Schmidt (see *Biometrika*, Vol. I. p. 429). This for the Naqada crania made a difference of 4·9 and 4·0 mm. for ♂ and ♀ respectively. As the best approximation available these have been added to Macdonell's values of G_1 for the two English series. With a view of using crania with broken spines, Schmidt suggests that the palate shall be measured from the base of the spine. This appears to have been done in the French and Northern Negro series. The average length of the spine in the European male (English) is 3·6 and in the Negro male (Gaboon, 1864) 3·7 mm.; these values have therefore been added respectively to the French and Northern negro series. In the Theban mummies as already indicated (*Biometrika*, Vol. I. p. 430) both terminations of the palate length have been modified in Schmidt's sense and accordingly (*loc. cit.*) 9·9 mm. has to be added to G_1 in the male and 8·0 mm. in the female. Finally the Bavarian measurements seem to have been made *sui generis* and G_2 is clearly

* The Gaboon 1880 series only provided seven male and nine female crania for the palate index and has been disregarded.

not our measurement. As it cannot be corrected, it is idle to correct G_1; and by a lucky chance the index fits well for the male with what we might anticipate.

The table is most hypothetical and its chief service must be to emphasise the need of precise standardisation of the palate measurements. As it stands, it may suffice to show that possibly in palate measurements also the Congo-Gaboon negro is differentiated from the Kaffir-Zulu group. The general want of systematic differentiation between negro and European for palate measurements may either prove that the palate is not a markedly racial character, or indicate that our hypothetical corrections have screened the true values.

Orbits. The orbit is another character which is distinctly of racial importance, but which it is very hard to discuss. There is not the least doubt that in facial appreciation orbital form is a most important factor, and the orbital index ought to be most suggestive in its bearing on racial differences. Unfortunately one component of it, the breadth of the orbit, is almost as vague as the length of the palate. This point has been discussed by both Fawcett and Macdonell*, and a few additional remarks may be made here. It is probable that most writers have taken the dacryon as the start on the nasal side of the greatest breadth. There are great advantages in simplicity of measurement if this be done, and possibly it will be best to standardise the measurement in this manner, but when it is done it must be admitted at once that we are only getting an artificial orbital index. The physiognomical effect of the orbit is far better expressed by the original words of the Frankfurt Concordat—the greatest breadth from side to side of orbit wherever found, measuring from margin to margin—, only as there happens to be no margin on the nasal side, we are thrust back on the alternative of either giving up the definition, or defining what is meant by the 'margin' on the nasal side. Those who give up the margin use the dacryon or, occasionally, a purely arbitrary point leading to endless personal equation in the measurement, and to perfectly idle results, as they can be compared with no other series of measurements. If we retain the idea of margin we must put in an imaginary margin on the nasal side, and measure the greatest diameter from this. Two possibilities occurred to the biometric workers:

(i) To produce with a pencil the margin from the last sensible points of the upper and lower orbital rims as a curve on the nasal wall, starting without breach of continuity with the curvatures indicated by the upper and lower orbital rims, so that these rims meet again without breach of continuity on the nasal wall. The method is not difficult in practice and does not admit of as much personal freedom as the description may suggest to the reader. It was adopted by C. D. Fawcett in her measurement of the Naqada crania. I will term it the curvature method.

(ii) To join with a pencil line the last sensible points on the upper and lower orbital rims, the line to be the curve of shortest length on the nasal wall between

* *Biometrika*, Vol. I. pp. 430, 431 and Vol. III. p. 201.

these two points, i.e. the geodesic. This 'false' margin gives usually abrupt changes of curvature at the terminals of the upper and lower orbital rims. I will term this the 'geodesic' method.

In either (i) or (ii),—which endeavour to maintain the Frankfurt Concordat idea of greatest breadth—the greatest measurement was to be taken, wherever found, an end being on the imaginary margin thus defined. While Fawcett used (i), Macdonell tells us he used (ii)*. He declares that in practice it is identical with following the curvature of the lower orbital rim to its furthest inward point,—but this is limited again by the statement that he does not cross the naso-frontal suture. This really appears to be not (ii) but a third definition, namely to produce the lower orbital rim from its last sensible point *without change of curvature* up to the naso-frontal suture and to measure from some point on this production. I admit that in many cases there is little change of curvature at the terminal of the lower orbital rim when the geodesic is drawn as described in (ii), but I do not think Macdonell's method is absolutely the same as Fawcett's geodesic method especially for male orbits, and thus we have new difficulties in forming racial comparisons.

In Fawcett's paper the differences obtained in the orbital breadth by using (i) and (ii) for one series of crania are given as 1·9 mm. for males and 1·6 mm. for females. What difference, however, is there between the curvature method and the lacrymal point method? The extraordinary uniformity of the breadth measured by the lacrymal point method in prehistoric Egyptians, Theban mummies and modern Copts (see *Biometrika*, Vol. I. pp. 426, 427) shows us that we may safely take for Egyptians, prehistoric to modern, the difference between the curvature method and the lacrymal point method as 4·8 mm. for males and 4·3 mm. for females. Hence to reduce Fawcett and Macdonell's measurements to a lacrymal point measurement we must roughly subtract 4·8 mm. and 4·3 mm. from Fawcett's values and 2·9 mm. and 2·7 mm. from Macdonell's values; the orbital index will then have to be taken as a ratio of means. I believe that this is the best that can be done; I should have preferred to try and get a true orbital index as defined in the Frankfurt Concordat, but there are too many series to modify and too little is known of how they have really been measured.

Dr Benington followed Macdonell in his orbital measurements and accordingly his maximum breadths O_1 have to be reduced by 2·9 mm. and 2·7 mm. respectively for male and female. I have reduced his averages by these and recalculated the orbital indices from the modified averages.

Before we look at the table thus modified we may compare the English (Macdonell) and Negro (Benington) results which were presumably measured by practically the same methods:

* *Biometrika*, Vol. III. p. 201.

	O_1		O_2		100 O_1/O_2	
	♂	♀	♂	♀	♂	♀
English (Moorfields)...	42·0	40·9	32·8	32·7	77·9	80·0
English (Whitechapel)	43·0	41·1	33·4	33·7	77·8	82·1
Congo	40·8	40·9	34·6	33·5	84·9	81·9
Gaboon, 1864... ...	41·5	39·6	35·0	33·7	84·6	85·2
Gaboon, 1880... ...	42·1	40·1	35·2	34·1	83·7	85·2

These results suggest that the Congo-Gaboon type of negro is certainly not broader in the orbit than the European English, but the orbit, especially in the male, appears to be sensibly higher and thus the general appearance of the orbit will be rounder. I think this confirms the impression which arises from photographs of these negroes. Possibly the greater development of the superciliary ridges in the English male will account for some of the difference, which is more marked in the males than in the females.

In the following table the best has been done to correct divergent measurements. The one measurement which is fairly reliable is the height of the orbit and it is extraordinary how little is the racial variation. In the female the racial differences do not cover more than about 2 mm. and in the male about 3 mm. Even the breadth of orbit has really a small range, about 3 mm. for either sex, if we can trust our corrected values. An appreciation of this limited variability will show how essential it is to restrict all personal equation within a fraction of a millimetre, and how hopeless it is to compare measurements which are not exactly standardised.

Measurements on the Orbit. (*Corrected.*)

	O_1		O_2		100 O_1/O_2	
	♂	♀	♂	♀	♂	♀
Congo	37·9	38·2	34·6	33·5	91·3	87·7
Gaboon, 1864 ...	38·6	36·9	35·0	33·7	90·9	91·3
Gaboon, 1880 ...	39·2	37·4	35·2	34·1	89·8	91·9
Angoni	38·0	—	33·3	—	88·1	—
Northern Negro ...	39·6	—	34·2	—	87·2	—
French	38·2	—	33·3	—	87·3	—
Theban Mummies ...	38·1	37·0	33·0	32·3	86·5	89·8
Würtemberger ...	39·8	38·0	34·1	32·2	85·9	85·1
Zulu	38·7	—	33·2	—	85·8	—
Kaffir	39·8	—	34·0	—	85·5	—
Copts	38·2	36·9	32·5	32·5	85·2	86·7
Aino	40·9	39·8	34·9	33·9	85·3	85·2
Naqada	38·1	37·2	32·3	32·0	84·8	86·0
Bavarian	39·9	38·3	33·7	33·2	84·5	86·6
English (Moorfields)...	39·1	38·2	32·8	32·7	83·9	85·6
English (Whitechapel)	40·1	38·4	33·4	33·7	83·3	87·9

The feeling that remains on my mind after study of this list is the sad pity of it,—it is so futile!—one feels quite certain that the French and the Würtemberger are displaced, and that a very little standardisation of the breadth measurement would have provided us with as instructive a table as those for the acroplatic and the gnathic indices. The table represents the measurement of hundreds of orbits and hours and hours of tedious work, and the result for want of careful standardisation is almost worthless. No one is likely to believe that the French and a considerable section of the German population have as round an orbit as Zulu and Kaffir. The table, were it above suspicion, would probably show us the Congo-Gaboon group at one end, the European races at the other, the Kaffir-Zulu group following the Congo-Gaboon negroes and separated from the Europeans by the Egyptian and primitive races. But as it stands, it tells us nothing more than our previous table—that of our few standardised measurements—namely, that the Congo-Gaboon group and the English are at opposite ends of the orbital scale. The table provides one great and important lesson: the urgent need for effective standardisation in craniometric work. If craniologists, instead of trying to do something a little bit different from their colleagues, would only strive to do exactly the same thing, they would double the speed at which their science would develop. At present all we can say is that all work hitherto done on palate and orbit is practically worthless for interracial comparisons!

Lastly, we turn to the nose where we have the height, breadth and nasal index for a number of races and the mesodacryal chord and arc, as well as the mesodacryal indices for a few races.

Nasal Characters. (Table A.)

	Nasal Height		Nasal Breadth		Nasal Index	
	♂	♀	♂	♀	♂	♀
Zulus	47·2	—	27·3	—	58·1	—
Angonis	48·5	—	28·0	—	58·0	—
Kaffirs	48·6	—	28·0	—	56·3	—
Gaboon, 1880... ...	47·9	45·3	27·1	25·7	56·5	56·9
Northern Negroes ...	49·5	—	27·3	—	55·4	—
Gaboon, 1864... ...	48·2	45·1	26·6	24·7	55·3	54·9
Congo	47·2	45·0	26·0	25·0	55·2	55·8
Naqadas	48·9	46·7	25·1	24·3	51·1	52·3
Aino	50·5	47·4	25·6	24·7	50·7	50·5
Theban Mummies ...	50·6	48·4	25·3	24·2	50·1	50·2
Copts	50·7	45·3	24·9	24·6	49·4	54·6
Long Barrow	49·4	47·0	24·1	22·8	49·0	49·1
Bavarian	50·9	48·2	24·8	23·7	48·7	49·2
English (Moorfields)...	50·4	48·0	24·0	23·4	47·6	48·7
English (Whitechapel)	51·2	48·7	24·3	23·2	47·6	47·8
Würtemberger ...	51·5	45·7	23·9	23·0	46·6	50·3
French	51·2	—	23·1	—	45·3	—
Scottish	52·4	48·7	23·2	23·2	44·5	44·1

Nasal Characters. (Table B.)

	Mesodacryal Chord		Mesodacryal Arc		Mesodacryal Index α		Mesodacryal Index β		Simotic Index	
	♂	♀	♂	♀	♂	♀	♀	♂	♀	♂
Gaboon, 1880	25·5	22·7	—	—	—	—	—	—	—	—
Congo	24·4	23·5	32·9	23·2	39·9	41·5	34·5	36·8	25·6	25·2
Gaboon, 1864	24·5	23·1	33·5	32·3	45·4	46·7	42·7	44·8	30·8	28·7
Zulus	24·3	—	—	—	—	—	—	—	—	—
Angoni	24·3	—	—	—	—	—	—	—	—	—
Kaffirs	24·1	—	—	—	—	—	—	—	—	—
Egyptian (Series E) ...	21·5	20·4	34·5	32·4	57·0	55·9	61·3	59·3	44·8	38·6
English (Whitechapel)	22·6	20·9	35·1	31·9	53·9	52·2	56·2	53·4	50·7	47·4

Taking in the first place Table A of nasal characters, we see that the high nasal index of the negro is due to two sources. It occurs chiefly because the breadth of the nose is markedly greater in Negroes than in Europeans, and partly because Europeans have a somewhat longer nose. The Congo-Gaboon group lies closer in breadth to the primitive and prehistoric group, than do the Zulu-Kaffirs. Here again it would seem that the Congo-Gaboon group is the link between the Southern (and Northern ?) negroes and the Egyptian type, just as on the other hand the Egyptians link up the prehistoric Naqadas with the Europeans. The marked gap, however, between the Congo-Gaboon and Egyptian series indicates that we are on different branches of the developmental tree.

The Congo-Gaboon crania are, however, nearer the Egyptian than the "full-blooded" negro crania are, while modern Europeans are more or less continuously linked up through the Nile valley and the Congo-Gaboon group with Kaffir and Zulu. Thus the nasal index appears to emphasise the same form of relationship as the gnathic and the acroplatic indices have already indicated.

Table B is unfortunately very defective. We have only just realised the importance of measurements on the nasal bridge. Unfortunately Dr Benington only measured the mesodacryal arc on the male Congo series, and we have reason to believe that the females were not measured by exactly the same convention. Accordingly Miss K. M. Ryley kindly undertook not only to re-measure the Congo, but to measure also the Gaboon 1864 series, and, for purposes of comparison, fifty crania of each sex of the Egyptian (E) series and the English (Whitechapel) series.

In dealing with the nose the straightforward characters to measure are chords and arcs, e.g. the mesodacryal chord and mesodacryal arc. We have already seen that a dacryal index can be formed = 100 (arc − chord)/chord, but this does not give a readily appreciable idea of the shape of the bridge of the nose. If we want to realise this a better measure is the ratio of the subtense to the chord. We might proceed to find this exactly as we have dealt with the occipital index from chord and arc of opisthion to lambda. But the subtense in that case is small relative to

the chord and an approximate formula suffices. This is not true in the case of the nose, although more nearly true in the case of the Negro than in that of the European nose. For this reason a circular arc is by no means a good approximation to the nasal bridge. In fact in certain noses the subtense is more than half the chord and accordingly to use a circular arc supposes more than a semicircle! After an examination of moulds of the bridge of the nose, it seemed to me that the bridge of the nose could be fairly closely represented in section by the common catenary with axis in the median plane. We might thus pass from arc and chord to subtense and chord by a simple table without much trouble and with sufficient accuracy for practical purposes.

The equation to the common catenary is

$$y = \tfrac{1}{2}c\,(e^{+\frac{x}{c}} + e^{-\frac{x}{c}}),$$

c being the parameter.

Therefore
$$R_1 = \text{arc/chord} = \tfrac{1}{2}(e^{+\frac{x}{c}} - e^{-\frac{x}{c}}) \Big/ \frac{x}{c}$$
$$= \sinh u/u,$$

if u be used to denote x/c.

$$R_2 = \text{subtense/chord} = (y - c)/2x$$
$$= R_1 \times \tfrac{1}{2}\tanh \tfrac{1}{2}u.$$

Hence if we use tables of the hyperbolic sine and cosine we can determine corresponding values of R_1 and R_2. A table of corresponding values is given on pp. 338—9*. $100\,(R_1 - 1)$ is the Dacryal Index β, and $100\ R_2$ the Dacryal Index α, and the table is entered by these indices α and β. In other words we have a means of passing from the measured arc and chord to a *theoretical* subtense. There is little doubt that the ratio of this theoretical subtense to the actual chord gives a fair approximation to the real shape of the bridge of the nose.

It will be seen at once that our mesodacryal indices α and β include more than the nasal bones, for the nasal processes of the superior maxillary bones also contribute. But from the standpoint of the physiognomy of the living it is more than doubtful whether it is the form of the nasal bones only which is concerned in our classification of the nasal bridge—a feature which most certainly plays a large part in racial appreciation. In order to judge fully of the differentiation in this character between Negro and European races we have also measured the nasal bones alone, and obtained the index introduced by Mérejkowsky, and which I venture to term the *Simotic Index*†. In this case the shortest horizontal chord

* Computed by Miss Julia Bell, M.A., from Gudermann's, Glaisher's and Newman's tables of the hyperbolic functions.

† C. de Mérejkowsky: "Sur un nouveau caractère anthropologique," *Bulletins de la Société d'Anthropologie*, T. v. Troisième série, Paris, 1882, pp. 293—304. Mérejkowsky calls this index "indice de la racine du nez."

of the nose (n) is measured from nasal suture to nasal suture, and also the minimum subtense (p), i.e. the line from this chord in the median plane perpendicular to the nasal bridge.. The ratio $100p/n$ is the simotic index and measures the snubbiness or flatness of the nasal bones at the bridge. A special instrument for taking the measurements has been devised by Mérejkowsky himself and a simometer of this type was kindly lent to us by Professor Thane. The instrument does not strike one as mechanically efficient. The distances to be measured are extremely small, one reading in the differential measurement of the subtense being often a fraction of a millimetre, and the bearings for such fine measurements were quite inadequate. These we improved, but the addition of fine micrometer screws and an absolute not a differential determination of the subtense would be requisite for really accurate work. The results are given for what they are worth, and they may be useful in comparing the value of various rhinal indices.

The table on p. 318 brings together all the measurements made by Miss Ryley* and reduced by Miss Bell.

Now some very noteworthy results flow from this table. It will be evident that the Mesodacryal Index α has remarkably little variation within the race compared with either the Mesodacryal Index β or the Simotic Index. It therefore has some advantages in use over the two latter. It measures, however, a quite different character to the Simotic Index and we see at once that the racial order is not the same for simotic and mesodacryal measurements. That is to say an order based purely on the nasal bones may not be the same as one based on the entire bridge of the nose.

Judging first by the simotic index we have the following results:

(a) The Negro races have flatter nasal bones than Egyptian or European.

(b) There is a marked sexual difference, the women being in every case more platyrhinal than the men, the difference however being less marked in the negroid than European and Egyptian races.

(c) From the standpoint of variability the Negroes are far more variable than Egyptian or European.

From this view of the matter we are again confronted by the probability that it is the European who has been more stringently selected from a primitive stock than the Negro, and that in the nasal bones the female has retained rather more of the primitive character—nasal flatness—than the male.

* All the measurements on which the constants of this Table are based for the four races were taken by Miss Ryley. This was done because (i) the measurements are extremely delicate and it is desirable that they should be made by one person for all races investigated in order to avoid errors due to personal equation and (ii) because we feel convinced from our re-measurements that Dr Benington's mesodacryal arc, the "Tape Dacry" of his Tables I and II, has not been measured in the same manner for the Congo male and female crania—an interval of several months elapsed between the two sets of measurements made by Dr Benington and Miss Thomson respectively—and that the "Tape Dacry" column of Table II for the female Congo skulls must be cancelled.

Table of Biometric Constants of Nasal Characters showing their Mean Values and Variability in Four Races.

Character	Congo		Gaboon 1864		Egyptian (Series E)		English (Whitechapel)	
	♂	♀	♂	♀	♂	♀	♂	♀
Mesodacryal Chord: Mean ...	24·43± ·26	23·52± ·26	24·50± ·23	23·09± ·30	21·48± ·19	20·36± ·16	22·57± ·19	20·89± ·22
" " S. D. ...	2·67± ·18	2·09± ·19	2·38± ·16	2·87± ·21	2·01± ·14	1·69± ·11	1·97± ·13	2·29± ·15
" " C. of V....	10·92± ·75	8·89± ·79	9·70± ·66	12·43± ·92	9·35± ·69	8·30± ·61	8·71± ·64	10·96± ·82
Mesodacryal Arc: Mean ...	32·85± ·39	32·16± ·40	33·49± ·28	32·29± ·41	34·52± ·36	32·36± ·24	35·13± ·32	31·90± ·32
" " S. D. ...	4·08± ·28	3·21± ·28	2·91± ·20	3·78± ·29	3·77± ·25	2·53± ·17	3·34± ·23	3·31± ·22
" " C. of V. ...	12·43± ·86	10·00± ·89	8·68± ·59	11·72± ·91	10·91± ·81	7·81± ·57	9·50± ·70	10·38± ·77
Mesodacryal Index α: Mean ...	39·85± ·61	41·48± ·71	45·41± ·58	46·71± ·82	57·00± ·89	55·93± ·56	53·89± ·79	52·15± ·78
" " S. D. ...	6·32± ·43	5·71± ·51	6·04± ·41	7·49± ·58	9·31± ·63	5·91± ·40	8·23± ·56	8·16± ·55
" " C. of V.	15·85±1·11	13·75±1·24	13·30± ·91	16·03±1·27	16·34±1·13	10·56± ·72	15·28±1·05	15·65±1·08
Mesodacryal Index β: Mean ...	34·54± ·87	36·84±1·03	42·70± ·87	44·78±1·27	61·26±1·49	59·29± ·92	56·15±1·27	53·35±1·24
" " S. D. ...	9·03± ·62	8·24± ·73	9·11± ·61	11·58± ·90	15·58±1·05	9·60± ·65	13·29± ·90	13·03± ·88
" " C. of V.	26·13±1·90	22·37±2·08	21·34±1·50	25·87±2·13	25·43±1·82	16·20±1·11	23·67±1·64	24·42±1·70
Simotic Chord: Mean... ...	9·44± ·21	9·46± ·33	9·16± ·26	9·55± ·28	10·58± ·18	10·76± ·19	9·52± ·17	8·98± ·18
" " S. D. ...	2·17± ·15	2·52± ·23	2·78± ·19	2·49± ·20	1·88± ·13	1·99± ·13	1·78± ·12	1·93± ·13
" " C. of V. ...	23·05±1·67	26·64±2·61	30·31±2·22	26·08±2·18	17·77±1·24	18·51±1·29	18·72±1·31	21·53±1·52
Simotic Subtense: Mean ...	2·38± ·08	2·31± ·11	2·87± 11	2·68± ·10	4·67± ·10	4·11± ·09	4·74± ·11	4·14± ·07
" " S. D. ...	·86± ·06	·86± ·08	1·14± ·08	·90± ·07	1·07± ·07	·90± ·06	1·13± ·08	·77± ·05
" " C. of V. ...	36·03±2·78	37·38±3·88	39·73±3·11	33·59±2·92	22·83±1·62	21·94±1·55	23·79±1·69	18·54±1·29
Simotic Index: Mean ...	25·55± ·78	25·19±1·25	30·81± ·89	28·73± ·96	44·77± ·84	38·59± ·73	50·67±1·20	47·36± ·91
" " S. D.	8·04± ·55	9·66± ·89	9·24± ·63	8·70± ·68	8·75± ·59	7·68± ·52	12·52± ·85	9·50± ·64
" " C. of V. ...	31·46±2·37	38·34±4 00	30·00±2·22	30·27±2·58	19·55±1·37	19·91±1·40	24·72±1·77	20·06±1·41

If we turn to the factors on which the simotic index is based, the simotic chord and subtense, we note:

(*d*) That both are far more variable in the Negro than in the European or Egyptian; but

(*e*) That the simotic chord does not differ very widely in English and Negro, both being somewhat less than the Egyptian.

(*f*) That it is the subtense that is widely different in the Negro, its value being a little more than half that of European or Egyptian, while its relative variation is 35—40 °/₀ compared with their 18—24 °/₀—i.e. not quite double. Roughly therefore we may say that the nasal span is the same for all these races and the selection has been that of size of nasal bones covering the span from one nasal process of the superior maxilla to the other. There are indeed among the negro skulls several in which the simotic index could not be measured, because there were practically no nasal bones to measure, e.g. Gaboon 1864, Nos. 10, 11, and 81, and Congo 70 (cf. Remarks on folding sheets of measurements). The significance of this is considerable when we consider the nasal bones of the higher apes*.

Turning to the mesodacryal characters we see:

(*a*) That the mesodacryal chord is larger in the Negroes than in Egyptian or English series, i.e. from the physiognomic standpoint the orbits are farther apart. On the other hand for the mesodacryal arc, while the English and Egyptian males have a somewhat larger value than the Negroes, there is no sensible difference in the females. Thus from the standpoint of the nasal bridge as a whole, it is the interorbital distance, the mesodacryal chord, rather than the arc which is the essentially differentiating feature†. Roughly therefore what the European has gained in nasal bones, he must have lost in superior maxillary processes.

(*b*) The two mesodacryal indices amply suffice to differentiate markedly the Negro from the European or Egyptian crania, but the relatively small difference between European and Egyptian, unlike that of the simotic index, places the English nearer to the Negro than the Egyptian. This point is of considerable interest, because a large negroid admixture has been frequently asserted, without much basis than that of general impression, to exist in these and other series of Egyptian crania.

We see, however, that in both mesodacryal indices, in mesodacryal chord and in simotic chord the English crania stand nearer to the Negro than the Egyptian

* An investigation of the mesodacryal and simotic characters of these apes is now in progress. A female gorilla skull in the Laboratory possesses an almost similar absence of nasal bones at the bridge.

† A familiar illustration would be the suspension of the same lengths of clothes line between two sets of posts at different distances apart, the droop corresponding to the mesodacryal subtense would be greater for the nearer pair of posts.

does; and this inverted order has been observed in some other cranial features*. It is of course far from the rule generally, but it has some bearing on the question of whether the Egyptians really did mix freely with the adjacent negro populations. An examination of the three nasal indices given in this paper will show how the Egyptians and English lie far closer together than Negroes and Egyptians, and this although we are dealing only with the least negroid of the negro races!

(*c*) Another noteworthy feature is that while the simotic index is greater for the males than the females in all races, in the two negro races both mesodacryal indices are greater for the females; while for the English and Egyptians the males have higher indices. In other words the sexual order is *changed*. Taking the bridge of the nose from dacryon to dacryon the negro male is flatter than the female, but if this holds for primitive human types in general, then the male in the advanced races has been more intensely selected than the female, with the result that she is now slightly more flatnosed.

To sum up it appears to me:

(i) That both mesodacryal and simotic indices are markedly differential racial characters.

(ii) That the mesodacryal index α, besides being physically more comprehensible than the mesodacryal index β, has a less percentage range of variation within the individual race. It is easily found by aid of the table now issued.

(iii) That the fuller study of the bridge of the nose promises results of much value for their bearing on selective evolution.

(iv) That notwithstanding the mechanical defects of Mérejkowsky's instrument definite results of much interest can be obtained from it, and it would be quite easy to devise a better instrument†.

(v) That Mérejkowsky has undoubtedly led the way to a series of measurements which if carried out on a long enough series of good sized homogeneous groups would be extremely valuable for racial evolution.

With the nose I conclude our investigation of the differences between the crania of negroid and other races. The reader may question why other absolute lengths or indices have not been considered. The answer to this question is that in a number of cases they have been investigated but they led to no marked differences. In the bulk of these cases I do not believe that the absence of

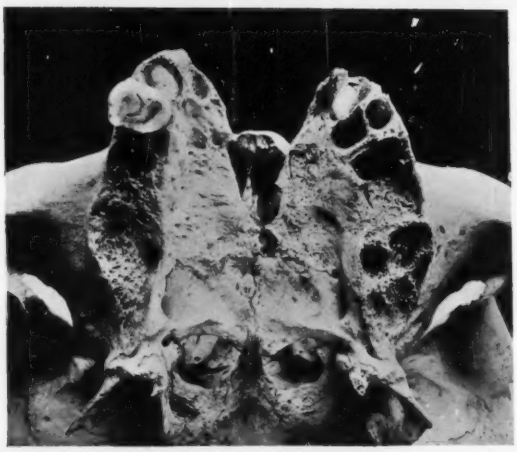
* Cf. *Biometrika*, Vol. VIII. p. 136.

† Taking a rough model of the bridge of the nose in plasticene and constructing from it by aid of a small pair of dividers and a piece of paper the triangle whose base is the lacrymal diameter and sides the distances from the ends of this base to the nearest point on the median plane of the bridge a very good approach to the mesodacryal index α in the living subject can be made. Without any plasticene model at all the reconstruction of the triangle by three measurements on the skull gives quite closely both simotic and mesodacryal indices.

A

B

D



E

C

F

Fernand Vaz Crania.

A, B and C, filed or gnawed orbits. D, filed or gnawed foramen magnum. E and F, Cranium 82, 1864 Series. Cleft of palate and complete absence of praemaxilla.

differentiation is wholly real, indeed I think it may often be traced under the chaos produced by personal equation in measurement. A not uninteresting example of this is the Zygomatic Upper Face Index, or the ratio of the distance of the nasion from the alveolar point to the maximum zygomatic breadth (100 $G'H/J$)—this measurement has been much approved by some craniologists. But its range is very small throughout the races of man, and personal equation both in the matter of the alveolar point, and in placing the callipers on the zygomatic arches brings chaos into the racial table. When the racial range is small even moderate personal equation produces serious effects. The following table will illustrate the point:

Zygomatic Upper Face Index, 100 $G'H/J$ (*found from means of absolute values*).

	♂	♀		♂	♀
Congo	50·1	48·5	Northern Negroes ...	52·5	—
Gaboon, 1880... ...	50·8	51·9	English, Moorfields ...	52·8	52·6
Aino	50·8	50·5	Sweden, Stone Age ...	53·0	—
Gaboon, 1864... ...	51·5	52·1	Theban Mummies ...	53·6	54·4
Kaffirs...	51·5	—	Würtemberger ...	53·6	52·9
Reihengräber... ...	52·0	—	Naqada	53·8	56·3
Zulus	52·2	—	English, Whitechapel	53·8	54·8
Long Barrow ...	52·2	—	Scottish	54·4	55·1
French	52·3	—	Copts	54·6	52·6
Bavarian	52·4	52·9			

The difference between the two English series alone would suffice to carry the North African Negroes to a place fourth or fifth on the list right among their racial fellows. The fact that, only roughly, the female order is the same as the male shows that nothing is final in the list, but generally the appearance of the Negroes at one end of the scale and the Europeans at the other, with the prehistoric races towards the centre, indicates that with longer series and more careful standardisation the zygomatic upper face index might be of value. At present the racial range, under five points for males, is too small to indicate profitable results in the face of small series and large personal equation in measurement.

(5) *On the Variability of the Negro Skull.* It is of interest to consider whether there is less variation in the Negro than in the European skull. The first of the accompanying tables gives the coefficients of variation for the absolute measurements of the Congo and Gaboon 1864 series compared with Naqada, Aino, Theban mummy, English, Bavarian and other European data. The following table gives the standard deviations of the indices. When one examines the gaps in these tables one notices the vast amount of laborious arithmetic still waiting to be done on actually measured series! Such work is too often shirked by the craniologist, but yet it is absolutely needful if we are to determine the relative variability of the various races of man. Without a knowledge of such variability how can we determine the range within which selection can act or has acted? Taking first the table of coefficients of variation for the absolute measurements, we

have only introduced the French* and Scottish data to show what differences may be supposed to arise when the material is very heterogeneous. A careful study of this table shows (i) that the two Negro series are at least as variable as, perhaps on the whole rather more variable, than the prehistoric Naqada, (ii) compared with the English, it is difficult to say which series is decisively more variable. In regard to the second table of the standard deviations of indices, the conclusion must, I think, be that there is no single index in the Negro series of which we can say the variation is markedly less than in either European or primitive races. There is, however, one character in which the Negro differs markedly from the European, the occipital index; in this he appears to be at least three times as variable. No great stress can at present be laid on the point, because the occipital index has at present been ascertained for far too few series, but undoubtedly it is *suggestive*. The indication given would be that the European crania are a stringent selection with regard to occipital development, and that the negro stand closer to more variable and less developed primitive stock. As another marked negroid character we may examine the nasal index. There appears again to be a greater variability in the negroid group; it is, perhaps, less marked, but it is quite significant. Again suggestion, not proven hypothesis, arises if we look at the components of the nasal index, *NH* and *NB* of the table of absolute length variations. We see that the nasal height of the Negro is sensibly more variable than that of the English. There is no very great difference in the variability of nasal breadth. The suggestion is therefore indicated that the nasal differentiation of European and Negro is due to a selection of nasal height in the former, and not to a selection of nasal breadth in the latter. Thus, as in the case of the occipital index, the indications are towards the theory that the European rather than the Negro has been more stringently selected from an original stock†. Such indications, however, cannot reach the plane of probable hypotheses until far larger series of Negro crania have been dealt with and much fuller biometric reductions have been made on European and primitive race crania. At present all we can say is that there is no evidence whatever for the Negro having small variation, or for his being a stringently selected offshoot of a more widely varying primitive type. It is the European who appears to differ from him by the action of more stringent selection.

(6) *Special Crania and Anatomical Peculiarities.* As usual a careful examination was made for anatomical anomalies, and all such are recorded in the "Remarks" column of the folding plates of measurements. It is possible that externally visible anomalies do not occur so frequently in the Negro as in European races; the reason being that deformed infants have in many negro tribes been, until recently, promptly destroyed. However, a collection of photographs

* These are the Paris Catacomb crania and not the French crania of the previous tables, which were those of French prisoners of war dying in Germany.

† Pigmentation investigations—both as to hair, skin and eye-pigment—seem to support the suggestion that the white races could have arisen from a xanthous variant of a dark race, and there is no good evidence at present for any really dark skinned variation having arisen in a white race.

Variability of Absolute Lengths. Coefficients of Variation.

	Congo		Gaboon, 1864		Naqada		Theban Mummies		Aino		English, Whitechapel		English, Moorfields		Bavarian		French*		Scottish†	
Organ ...	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
Capacity	9·4	8·9	7·8	10·3	7·7	6·9	8·1	8·2	6·9	6·8	8·3	8·7	9·0	8·3	7·7	8·2	7·4	7·1	—	—
F ...	3·7	3·5	3·1	3·6	3·3	3·2	—	—	—	—	3·3	3·5	3·2	3·4	—	—	—	—	—	—
L ...	3·8	3·5	3·1	3·5	3·2	3·1	—	—	3·2	3·1	3·3	3·5	3·0	3·3	3·4	3·6	3·8	3·7	4·0	4·0
B ...	3·6	4·6	2·5	3·6	3·3	3·5	—	—	2·8	2·7	3·8	3·5	3·7	3·8	3·9	3·4	4·2	[illegible]	4·1	3·7
B' ...	4·0	4·0	3·3	4·3	5·3	4·5	—	—	—	—	4·3	4·6	4·2	4·2	—	—	—	—	4·6	4·0
H ...	3·1	3·6	3·6	4·5	4·0	3·7	—	—	—	—	4·2	4·0	5·0	3·8	—	—	—	—	—	—
OH ...	3·6	4·0	—	—	3·9	3·5	—	—	3·7	3·2	3·7	4·1	4·1	4·0	4·5	3·9	—	—	—	—
LB ...	4·7	3·7	3·8	4·2	4·9	4·7	—	—	—	—	4·1	4·1	4·6	4·5	—	—	—	—	—	—
Q ...	3·3	3·5	2·9	3·6	3·3	2·7	2·7	2·8	—	—	3·7	4·0	4·1	3·8	—	—	—	—	—	—
S ...	3·0	3·3	3·4	3·9	3·2	3·5	2·7	2·9	—	—	3·6	3·9	3·2	3·7	—	—	—	—	—	—
S_3 ...	5·8	7·1	9·0	9·6	—	—	—	—	—	—	—	—	6·1	6·4	—	—	—	—	—	—
U ...	3·4	2·8	2·3	2·9	2·5	2·3	—	—	—	—	2·9	2·9	2·7	3·3	2·9	3·1	—	—	—	—
G'H ...	6·5	8·2	9·0	7·8	6·1	6·9	—	—	—	—	5·5	7·1	6·0	5·7	—	—	—	—	—	—
GB ...	5·9	4·7	4·5	6·7	5·2	4·8	—	—	—	—	5·6	5·4	4·7	5·8	—	—	—	—	—	—
J ...	5·5	3·5	3·8	4·9	4·2	4·8	—	—	—	—	4·3	4·1	3·6	3·5	—	—	—	—	—	—
NH ...	6·0	6·4	6·2	6·8	6·1	6·8	—	—	—	—	5·1	5·6	5·2	5·7	—	—	—	—	—	—
NB ...	7·9	7·0	8·3	8·0	7·9	7·3	—	—	—	—	8·9	7·1	7·9	8·2	—	—	—	—	6·1	5·9
																			7·6	8·4
O_1 ...	3·8	3·5	3·5	4·3	5·0	5·3	—	—	—	—	4·4	3·8	3·5	4·1	—	—	—	—	4·8	4·3
O_2 ...	6·0	5·4	6·0	5·5	7·2	6·7	—	—	—	—	6·1	4·4	6·5	6·7	—	—	—	—	7·6	7·7
G_1 ...	8·9	5·8	6·8	8·0	6·5	7·4	—	—	—	—	5·7	6·5	8·8	6·2	6·4	6·8	—	—	—	—
G_2 ...	10·2	6·8	7·4	8·9	9·3	8·6	—	—	—	—	7·8	7·7	5·9	7·6	—	—	—	—	—	—
GL ...	5·7	4·0	4·6	5·0	4·8	5·1	—	—	—	—	4·7	5·0	4·3	6·1	—	—	—	—	—	—
W_1 ...	—	—	4·8	3·9	7·5	6·8	—	—	—	—	5·6		—	—	—	—	—	—	—	—
W_2 ...	—	—	6·6	7·9	7·6	7·2	—	—	—	—	8·7		—	—	—	—	—	—	—	—
h ...	—	—	12·6	12·4	9·9	8·5	—	—	—	—	11·7		—	—	—	—	—	—	—	—
f ...	—	—	5·2	5·4	5·7	5·3	—	—	—	—	6·2		—	—	—	—	—	—	—	—

* Crania from the Paris Catacombs, and covering many centuries; very heterogeneous.

† Sir William Turner's series; all periods and all districts combined and therefore very variable.

Variability of Indices (Standard Deviations).

	Congo		Gaboon, 1864		Naqada		Egyptian		Aino		English, Whitechapel		English, Moorfields		Bavarian		French *		Scottish *		Northern Negroes	
Organ ...	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
100 B/L ...	2·6	2·5	2·8	3·4	2·8	3·1	3·4‡	3·4‡	2·4	2·5	3·3	3·0	3·0	2·4	3·5	3·0	4·4	4·2	—	—	2·8	3·5
100 H/L ...	2·9	2·5	2·4	3·1	2·7	3·0	—	—	—	—	3·2	2·8	3·5	2·8	—	—	3·5	3·7	—	—	—	—
100 B/H ...	4·6	4·2	4·3	5·3	4·5	4·7	—	—	—	—	4·6	3·8	4·7	4·0	—	—	4·7	4·3	—	—	—	—
100 $G'H/GB$	4·8	5·1	5·6	5·0	4·5	4·2	—	—	—	—	5·4	6·3	4·8	4·0	3·3	3·3	—	—	—	—	—	—
100 NB/NH	4·9	4·8	4·9	4·2	4·2	4·9	—	—	—	—	4·6	3·9	4·1	4·0	4·4	4·6	—	—	3·8	4·6	—	—
100 O_2/O_1 ...	4·9	3·7	5·4	4·7	5·0	4·7	—	—	—	—	4·2	4·3	4·2	5·7	6·7	5·2	—	—	5·5	6·8	—	—
100 G_2/G_1 ...	6·3	4·9	6·5	6·1	7·4	5·9	—	—	—	—	6·4	6·6	5·6	4·6	—	—	—	—	—	—	—	—
Oc. I ...	7·9	4·6	6·8	8·5	—	—	—	—	—	—	—	—	2·2	2·9	—	—	—	—	—	—	—	—
100 Fmb/Fml	6·0	5·8	8·2	7·0	—	—	—	—	—	—	—	—	6·6	6·1	—	—	—	—	—	—	—	—
Mesodacryal Index a	6·3	5·7	6·0	7·5	—	—	9·3†	5·9†	—	—	8·2	8·2	—	—	—	—	—	—	—	—	—	—
Profile Angle	3°·7	3°·4	—	—	2°·9	3°·7	—	—	—	—	3°·9	2°·8	4°·0	2°·3	2°·8	3°·6	—	—	—	—	—	—

* Both these series are very heterogeneous.

† Egyptian, Series E. ‡ Thebans.

of Nigerian natives recently shown me by Dr Hugh Stannus indicates that most well-known European anatomical deformities occur, if possibly with lessened frequency, even in the adult Negro. A striking example of this occurs in No. 82 of the Gaboon 1864 series reproduced as E and F on our Plate. It is an interesting case of cleft palate with a complete absence of praemaxilla*. But such anatomical defects externally obvious during life, appear to occur rarely in collections of negro crania.

A certain post-mortem injury to these negro crania is referred to by Professor Keith in his note on the Congo series: "Some of the crania show marks of having been cut or notched in the supra-orbital region": see our p. 294. This notching was noticed originally by Dr Benington in the two Gaboon series, and he was surprised to find it again in the Congo skulls at the Royal College. It is not, however, confined to the supra-orbital, or indeed to the orbital regions. Fig. A (Plate XXVI) shows it on the supra-orbital region, Fig. B on the supra- and sub-orbital regions, and Figs. A and C along the external margin. Again Fig. D shows it on the occipital edge of the foramen magnum. It is not easy to understand how the notching came about. Sometimes the markings have almost the appearance of the teeth-gnawings of a rodent; at other times they seem to be due to a rough knife, which might possibly have been used to clear away persistent fragments of flesh. It is noteworthy that they are common to both the Congo and the Fernand Vaz series, and they are not common to the large series of prehistoric and historic Egyptian crania in the Biometric Laboratory. In the latter series a few such markings have been found almost invariably about the foramen magnum, and suggest rough handling by the native finder, who, being paid for the skull, wished to separate it rapidly by aid of his knife from the trunk. In one such case at least it was obvious that the separation of a fused atlas had been the cause of the notching. Whether due to man or to rodent, no doubt some explanation will be forthcoming now that attention is drawn to it and inquiries can be made on the spot.

General Peculiarities. In the Congo series many of the crania show inferior frontal protuberance (i.e. bulging of the temporal area of the frontal); for example No. 78. This condition is associated with a marked Sylvian depression, which lies between the inferior frontal protuberance and the temporal protuberance of the squamous.

There is often a median eminence in the metopic area of the frontal more or less marked and lying between the regions of the frontal eminence. Frontal processes of the squamous: these are varied giving fronto-squamosal sutures of different lengths, e.g. No. 73, where this suture measures 6 mm. on *R*, while on the *L* a slender frontal process of the squamous meets a similar process of the frontal giving the "pterion in *K*."—The *K* may be vertical as in No. 73, or

* Compare Plate G, Fig. E, and Plate I, Figs. 10 and 13 of the *Treasury of Human Inheritance*, Vol. I. pp. 121—133.

horizontal as in No. 83 on the *R.* Epipterics may be associated with the frontal processes of the squamous as in No. 66 on the *L.*

The form of nose varies largely. In many cases, e.g. No. 73, the upper parts of the nasal processes of the superior maxillary bones and the nasal bones themselves are of uniform width. In others the nasal bones are very much reduced and the nasal processes may even meet above them, e.g. No. 70. The form of nose in No. 83 is peculiar. Here the nasal processes of the superior maxillary bones nearly meet, and only a small pointed process of the frontal intervenes. This just meets the small triangular nasal bones (13 mm. long and 10 mm. wide at edges) which are nearly horizontal.

Ossicles and wormians are often of considerable interest; thus in Fig. (i), p. 327, a tracing is provided of a posterior sagittal ossicle occurring in No. 82 and in Fig. (ii), p. 328, we have the large paired ossicles of the lambda (pre-interparietals) occurring in No. 88. Both these crania are female.

Turning to the Gaboon 1864 series, we note several instances of the "metopic crest" or median frontal crest of Schwalbe. This is associated with bregmatic eminence in certain cases, and then the crania have a peculiar formation resembling in transverse section that of the Eskimo* (e.g. No. 86, Table III. B). A good many instances of local metopic eminence also occur (e.g. No. 13, Table III. A).

Fig. (iii) is a tracing of a marked irregularity in the posterior portion of the sagittal suture which seems worthy of graphic reproduction.

A marked feature of both the Congo and Gaboon series is the drawing or filing of the incisors.

Classifying our peculiarities as in previous papers we have the following results:

(i) *Peculiarities of Form.*

Post-coronal depression occurs in: Gaboon 1864 series in three females (10, 54, 74) and in ten males (13, 14, 25, 28, 32, 62, 65, 67, 40 A. and a skull without number marked as (?)); Gaboon, 1880 in one male (3) and one female (37). In the Congo series it occurs in five males (28, 34, 50, 51, 57), seven females (65, 76, 77, 82, 84, 85, 90) and one non-adult (12). It thus does not appear in the Negro as in the European† to be a markedly female character; it occurs in about 12 °/₀ of female and 14 °/₀ of male negro crania. In the case of the English crania (Whitechapel) post-coronal depression or constriction occurred in 20 °/₀ of female and 11 °/₀ of male crania.

Flattening of the Obelion seems very frequent in the negro skull and passes into depression. In Gaboon 1864 it occurred in 13 males (11, 27, 28, 30, 33, 45, 47, 50, 57, 61, 65, 67, 79) and 14 females (4, 10, 12, 17, 21, 22, 24, 38, 39, 46, 48,

* See *Biometrika*, Vol. VIII. p. 181.

† Macdonell, *Biometrika*, Vol. III. p. 218.

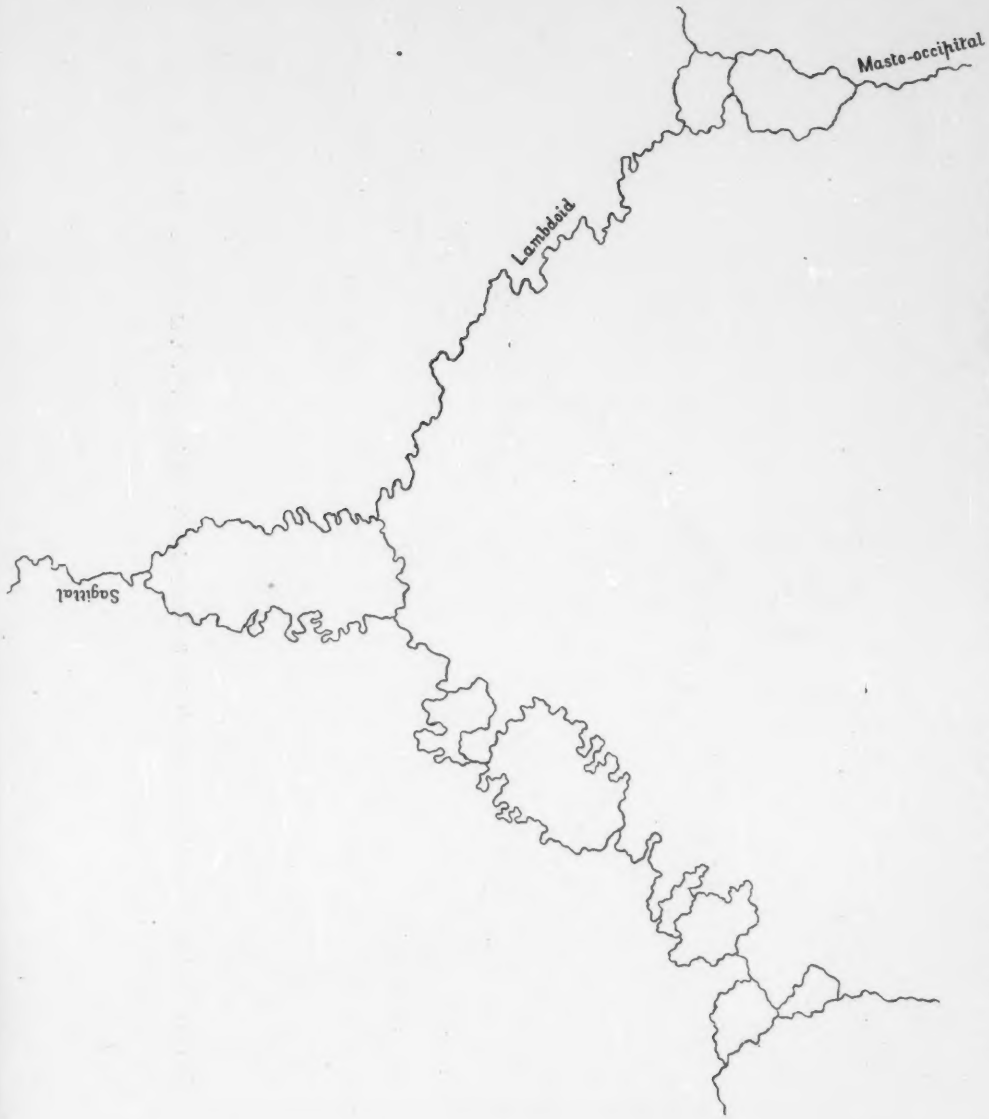


FIG. (i). Tracing of posterior sagittal ossicle. Cranium 82, R. C. of S. Series. Table II, Females.

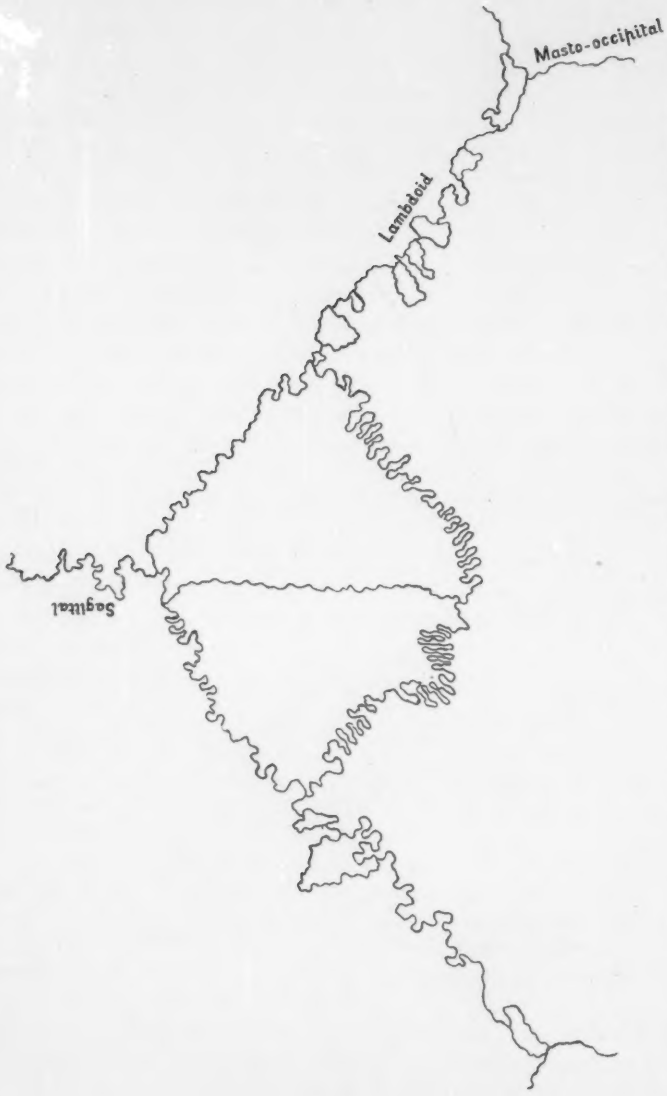


FIG. (ii). Tracing of large paired ossicles of lambda (pre-interparietals). Cranium 88, R. C. of S. Series. Table II, Females.

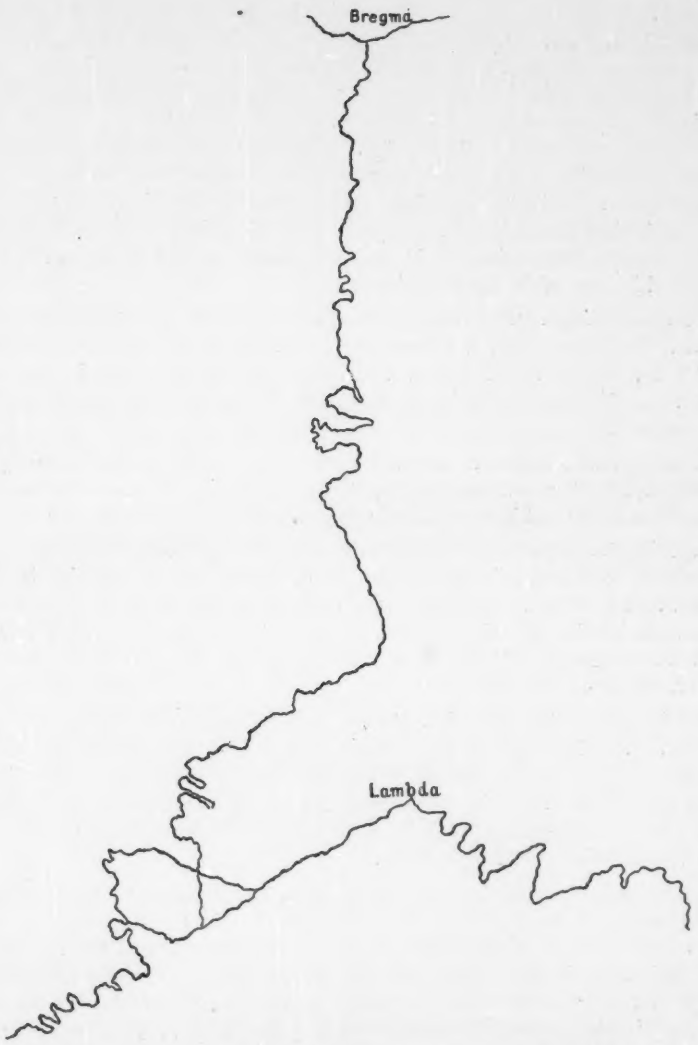


FIG. (iii). Tracing of sagittal and lambdoid sutures. Cranium 75, Table III B.

54, 78, 86), one or two having the obelion depressed (e.g. 38). In Gaboon 1880, the flattening occurred in six males (1, 14, 26, 30, 12[a], 20) and in six females (9, 10, 17, 36, 38, 13). In the Congo series it occurs in 23 males (20, 23, 24, 26, 27, 29, 30, 34, 36, 37, 39, 40, 41, 43, 46, 47, 52, 53 (depression), 54, 56, 57, 58, 59), seven females (66, 68, 74, 84, 86, 87, 91) and two non-adults (11, 12). It has thus been noted in 36 °/₀ of males and 30 °/₀ of females. The Whitechapel English crania showed depression of the obelion in 6 °/₀ of male and 8 °/₀ of female crania, only one case was recorded of flattening. Thus unless the standard was extremely different, flattening of the obelion appears almost a racial character in the Congo-Gaboon group. While bathrocephaly was relatively frequent in the English crania and of a marked type, slight *bathrocephaly* was noticed in only one male skull (59) of the Gaboon 1864 series and in two male skulls (20, 24) of the Gaboon 1880 series and in none of the Congo series.

Temporal Ridges. Pronounced or marked ridges have been held to be a negro feature. In Gaboon 1864 it appears in two females (5, 39) and ten males (8, 27, 36, 55, 61, 65, 71, 76, 77 and (?)). In Gaboon 1880 it occurs in two males (5, 26). In the Congo series in ten males (26, 28, 35, 43, 44, 47, 49, 52, 55, 57), in two females (66, 91) and two non-adults (10 and 13).

Marked *frontal eminences* occurred in the Gaboon 1864 series in five females (22, 38, 46, 54, 64) and five males (14, 18, 61, 65, 85); in the Gaboon 1880 series in one female (15) and in the Congo in six males (23, 29, 30, 41, 46, 48).

Inferior frontal protuberances, often accompanied by Sylvian depressions, occur in Gaboon 1864 series in six females (2, 4, 26, 42, 52, 84) and in six males (13, 34, 71, 80, 85, 89). In Gaboon 1880 series no case is noted, but in the Congo series such inferior frontal protuberances occur in 5 females (65, 72, 78, 80, 86), in 11 males (26, 28, 31, 35, 37, 40, 42, 46, 54, 56, 60) and in 4 non-adults (10, 12, 13, 14).

Sylvian Depression occurs in 7 females (2, 4, 26, 42, 52, 74, 84) and in 7 males (13, 34, 71, 76, 77, 80, 85) of the Gaboon 1864 series. It occurs in two females (10, 39) and three males (20, 23, 24) of the Gaboon 1880 series; while in the Congo series it was noted in two females (78, 88), seven males (17, 23, 26, 35, 56, 60, 62) and two non-adults (12, 13). Thus the Sylvian depression has been noted in 12 °/₀ of females and 14 °/₀ of males.

Marked *metopic eminences* were met with in four females (22, 38, 64, 86) and five males (13, 43, 55, 61, 85) of the Gaboon 1864 series; while they were found in two females (9, 21) of the 1880 Gaboon crania. The Congo series provide the same eminences in four males (15, 28, 29, 43) and four females (12 (young), 66, 68, 87). A 'metopic crest' occurred in a female (5) of the Gaboon 1864 series, and in six males of the same series (27, 44, 63, 65, 77, 79). Such 'crests' were not recorded as occurring in the Congo series.

Marked *bregmatic eminences* were noted in two females (70, 86) of the earlier Gaboon series and a post-bregmatic eminence in the male (32) of the Congo series.

Marked *parietal eminences* occurred in two males (19, 46) of the Congo series. Parietal depression in one male (14) of the 1864 Gaboon series.

Superior occipital bulging was found in one female (10) of the Gaboon 1864 series and in seven males (7, 8, 18, 40 A, 50, 55, 69). It was not recorded as occurring in the Gaboon 1880 series, but in the Congo we find it in one non-adult (12) and in four males (20, 33, 39, 51).

A markedly *retreating forehead* occurs in one male (55) of the 1864 series; in one female (80) and three males (26, 40, 44) of the Congo series. A *prominent glabella* was only recorded in one case a male (55) of the Congo series. A markedly 'infantile' forehead was noted in one female (35) of the 1864 Gaboon series.

Supra-orbital ridges were recorded in two males (45, 59) of the Gaboon 1864 series. *Supra-mastoid ridges* in two males (45, 53) of the same series, and in thirteen males of the Congo series (24, 26, 27, 29, 33, 34, 36, 37, 40, 41, 43, 48, 61).

Subnasal prognathism of a marked character was found in five females (4, 5, 9, 23, 78) and eight males (1, 32, 33, 37, 44, 45, 83, 89) of the Gaboon 1864 series; in two females (12^b, 21) and three males (4, 28, 35) of the 1880 series. It occurred in one female (14) and fourteen males (17, 19, 21, 22, 23, 41, 45, 46, 47, 50, 51, 57, 63, 64) of the Congo series. This negro characteristic therefore is to be found in a marked condition in 21 °/₀ of males but only 9 °/₀ of females. Markedly flat and broad noses were especially noticed in one female (87) and twelve males (26, 28, 29, 33, 37, 38, 39, 42, 43, 48, 49, 59) of the Congo series. The base of nose was depressed in two males (31, 36) of the Gaboon 1864 and in two males (28, 31) of the Gaboon 1880 series, or in 1 °/₀ of females and 14 °/₀ of males we find flattening or depression chiefly of the bridge*. Special peculiarities in the arrangement of the nasal bones (see p. 319) were noted in four females of the 1864 series (21, 29, 39, 81) and in five males (3, 11, 25, 59, 61). In the Congo series nasal peculiarities occur in four females (65, 67, 70, 83) and five males (29, 37, 42, 49, 59). Rounding of the margin of the pyriform aperture, a peculiar negro character, does not occur in a marked manner in many of the Congo-Gaboon series. It was recorded especially in one male (43) of the Gaboon 1864 series and in two females (79, 80) of the Congo series. Many of the peculiarities in the negro nose thus recorded may appear slight, but the general impression formed is that the Negro nose is subject to more variations than the European.

(ii) *Anomalies of the Sutures.* Only a few crania are recorded as *metopic*, e.g. one male (34) and one non-adult (14) of the Congo series and one female (74) of the Gaboon 1864 series. The spheno-occipital synchondrosis was not obliterated in three young females (2, 20, 70) of the 1864 series and a remarkable lateral deviation of the posterior part of the sagittal suture in a female (75) of the same

* This is an individual anatomical confirmation of the difference indicated on p. 318 above, that the negro females have a higher mesodacryal index β than the males.

series (see Fig. (iii), p. 329). Cases of the parieto-squamosal suture being straight and almost horizontal were noted in three Congo males (19, 21, 39).

Frontal processes of the squamous, generally bilateral, were very frequent. They occurred in nine females (10, 12, 60, 70, 75, 78, 82, 86, 88) and seven males (18, 40, 53, 71, 77, 89, (?)) of the Gaboon 1864 series, in two males (5, 32) and two females (19, 37) of the 1880 series; and in two females (66, 73) and three males (32, 50, 57) of the Congo series. Temporal processes of the squamous occurred in one Congo male (90). The pterion* was in '*K*' in three males (8, 31, 73) of the Gaboon 1864 series, in two males (3, 31) and one female (33) of the Gaboon 1880 series; and in one female (83) and one male (45) of the Congo series. There was a marked *sutura notha* in one male (79) of the Gaboon 1864 series, and a *fronto-squamosal suture* occurred in two males (69, 89) of the same series.

(iii) *Interparietals.* A tripartite interparietal, similar to that figured *Biometrika*, Vol. III. Plate XXXVII, occurred in a male (58) of the Gaboon 1864 series. A large simple interparietal occurred also in a female (8) of the Gaboon 1880 series, and another simple interparietal in a male (28) of the Congo series.

Epiterics occur in two females (54, 91) and three males (7, 36, 50) of the Gaboon 1864 and in one male (14) and one female (10) of the 1880 series. In the Congo series they are recorded in seven males (16, 34, 38, 40, 43, 47, 52), four females (66, 69, 70, 74) and one non-adult (14).

(iv) *Ossicles.* Ossicles of the lambda occur in two females (10, 70) and five males (7, 25, 30, 61, (?)) of the Gaboon 1864 series; and in two females (70, 88 (paired), see Fig. (ii), p. 328) and three males (27, 41, 48) of the Congo series. Ossicles of the lambdoid suture are recorded in ten males (3, 8, 14, 15, 25, 34, 47, 50, 57, 61) and seven females (16, 20, 22, 56, 60, 70, 74) of the Gaboon 1864 series; in six males (4, 5, 14, 22, 23, 27) and two females (10, 39) of the Gaboon 1880 series; and in two females (83, 91), thirteen males (16, 17, 20, 21, 22, 23, 25, 27, 28, 34, 38, 47, 56) and one non-adult (14) of the Congo series. Thus ossicles of the lambdoid suture are recorded in 25°/₀ of male and 12°/₀ of female crania. Macdonell found no such sexual difference in his English series, recording them for about 9°/₀ in female and 11°/₀ in male crania.

Posterior sagittal ossicles were recorded in one female (16) of the Gaboon 1864 series, in one female (4) of the Gaboon 1880 series, and in one male (34) and one female (82, see Fig. (i), p. 327) of the Congo series.

Ossicles in the masto-occipital suture occurred in four females (10, 29, 56, 82) and in one male (53) of the Gaboon 1864 series; they occurred in one female (83), two males (22, 23) and one non-adult (14) of the Congo series.

Ossicles in the masto-parietal suture occurred in two females (70, 82) of the Gaboon 1864 series. *Ossicles in the parieto-squamosal suture* in one female (10)

* Flattening of the pterion was only recorded in two crania (30, 31) of the Gaboon 1880 series, both males.

of the 1864 series and one male (7) of the same series. *Ossicles of the sphenomalar suture* in one female (79) and two males (20, 34) of the Congo series, and in one non-adult (14) of the same series ossicles in the squamous suture.

Ossicles at the asterion occur in four females (16, 20, 60, 91) and five males (1, 14, 49, 53, 61) of the 1864 Gaboon crania; in one male (23) and one female (39) of the 1880 Gaboon crania, and in one female (89), two males (34, 38) and one non-adult (14) of the Congo series.

Lastly we may note that a deep *canine fossa* occurred in two males (27, 40) of the Congo series.

There can be no question that every record of anatomical peculiarities varies with the individual observer, who is likely to be more interested in one character than another; nay, the same observer will have his attention concentrated by the occurrence of a very marked anomaly on its appearance in less marked forms in examining further skulls; or again he will seek for certain forms of anomaly because he has read that they are racial features of the group he is studying. There is at present unfortunately a complete want of standardisation in the matter of recording anatomical variations, and no illustrative classification for the guidance of the craniologist. Still admitting that selection plays a large part in all such records, and that the percentages of anomalies of any kind must be very rough, there does seem sufficient justification in the record we have given to show that in the matter of these anomalies and peculiarities, which admit less of the application of metrical methods, there is nothing to indicate that the negro skull of the Congo-Gaboon group is in any way less variable than the skull of any European people.

(7) *Conclusions.* Such conclusions as we draw must be by way of suggestion; one might hope that they may act as incitement to further investigation, above all as some inducement towards the procuring of really ample material and its adequate measurement and reduction. No final conclusions can be drawn until at least 100 crania of each sex for each local race have been obtained and measured by the same individual or by standardised methods. We have endeavoured to indicate the need of this by demonstrating how little profit can be obtained from existing measurements of such important characters as the orbit and palate.

We may summarise the indications of the present paper as follows:

(i) We do not feel justified in differentiating the Congo from the Gaboon crania on the basis of the present material. In some respects the Gaboon 1880 series gives for the males a type which approaches slightly closer to the Angoni and so to the Zulu-Kaffir group than to the Gaboon 1864 series, which is at all points close to the Congo series. This difference is not, however, usually confirmed by examination of the females, and we believe is largely due to the smallness of the 1880 series, which contains some rather large male crania. We

may safely look upon the Gaboon and Congo as forming a fairly representative group*.

(ii) This group differs sensibly from the Kaffir-Zulu group of Shrubsall, but negro crania from the north coast (our Northern negroes) seem to approach these southern negroes more closely than they do the Congo-Gaboon group.

(iii) If we wish to determine characteristic negro features, but few *absolute* measurements are of any service. Thus the usual tape arcs, vertical, sagittal and horizontal, are of small differential value. There are, however, some exceptions to this rule.

(*a*) The Negro skull is higher than the European, when we take the basio-bregmatic height. This excess, however, is not due to the development of the upper part of the cranium, there is sensible equality of the auricular heights. The increased height lies between the auricular passages and the basion. It is in fact due to the more massive character of the base of the negro skull. A good measure of this result is found in the index $100\ OH/H$, which is about 85 for Negroes and 87—88 for Europeans.

(*b*) Notwithstanding that little of an orderly evolutionary scale can be made out from absolute measurements—diameters or arcs—the available data show that both for the Gaboon-Congo and the Northern Negro groups there is a great deficiency of cranial capacity—a difference of 100 to 150 cm.^{3}—between Negro and modern European races.

(*c*) The nature of this difference in cranial capacity appears to be principally occipital in origin. There is some difference in the average of tape measurements from nasion to bregma between English and Negro†, but the chief difference lies in the section from lambda to opisthion. If we measure the occipital prominence by the occipital index, we find the English cranium stands at 58 to 59 and the Negro at 68 to 69, the flatter the cranium from opisthion to lambda the higher is this index. There is, I think, no doubt that relatively to the Negro the occipital portion of the skull in the European is greatly developed‡.

(iv) Certain indices—as distinguished from absolute measurements—seem to be of special value in differentiating European and Negro crania.

(*a*) In the first place the ordinary cephalic indices ($100\ B/L$ and $100\ H/L$) are of small service, but a new index $100\ (B—H)/L$, which I have called the acroplatic index, is not only positive and fairly high for Europeans but is insensible or actually negative for Negroes. In this they agree with the primitive and

* At the same time the mesodacryal and simotic indices show that the two series are by no means wholly alike.

† To some extent indicated in the relative frequency of "markedly retreating foreheads" (see p. 331 above) notwithstanding equality of auricular height.

‡ This has been already indicated in the previous paper on c... contours (*Biometrika*, Vol. VIII. pp. 135, 136) and may be illustrated graphically by superposing Fig. ..V in tissue on Fig. XXVII.

prehistoric peoples; the development of the highly civilised European peoples has been in the direction of increased breadth at the expense of cranial height.

(*b*) The gnathic index gives almost as uniform and suggestive results as the acroplatic index. The Europeans stand at one end of the list, then come the prehistoric, historic and modern Egyptians, then primitive races like the Long Barrow folk and the Aino, then the Negroes, although the full-blooded Negroes are in this respect not clearly differentiated from the Congo-Gaboon group.

(*c*) The Upper Face Index is less clear in its indications, the Kaffir-Zulu group being to some extent displaced. As in the case of the Zygomatic Upper Face Index, this may possibly be due to personal equation. But still the general order—Europeans, Egyptians, Primitive Races, Negroes—appears though with slight confusion.

(*d*) Nasal characters, as we might anticipate, provide a perfectly good scheme of classification. We find full-blooded Negroes, Gaboon-Congo group, primitive and prehistoric peoples and then the European races in excellent sequence. Measurements on the mesodacryal arc and chord are at present too few to be of real service, but they suggest that these characters and the mesodacryal index may ultimately be of much service for differentiation. The simotic index of Mérejkowsky fully justifies its introduction. It places the English at one end of the scale, the Congo at the other with Egyptians and Gaboon crania as intermediate links. The flatness of the Negro nose is seen to be due to defect in simotic subtense, the chord being almost the same as in European crania.

(*e*) We have already referred to the importance of the occipital index, and the failure of orbital and palate indices—in the present absence of standardisation—to be of real value.

(v) The Negro mandible is in all respects broader, where comparison is possible, than that of the primitive races or the European. Relatively to the primitive races and the European, however, it appears (the evidence is still slender) to lack height, or chin development.

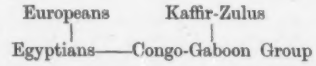
(vi) The profile angle measured by the Ranke goniometer and craniophor—too often neglected by craniologists—justifies its existence even on the small data available. We find the European races head the list, then come the Egyptians, followed by primitive races, Congo Negroes and North Africans. Unfortunately there are no data for the Kaffir-Zulu group, but other characters would suggest that this group will not be found far from our North African Negroes. It is much to be desired that the use of the profile angle should be extended.

(vii) No measures of the variation of the Kaffir-Zulu group have yet been taken, but the variability of the Congo-Gaboon group, whether it be judged by biometric measures or by the appreciation of anatomical peculiarities, appears in no wise less than that of European or of primitive races. Indeed we may even suggest without danger that it is greater. This appears to be markedly the case in

the occipital index and sensibly the case in the nasal index, and in nasal height. Even in 'chin height' the variability of the Gaboon series in either sex exceeds the variability of the English, rendered as it is heterogeneous and therefore exaggeratedly variable by grouping both sexes together. It seems to me therefore that in the characteristic negro features, lack of occipital development, lack of chin height and low nasal index, the Negro is markedly more variable than the European. Accordingly the suggestion can be made that the European and not the Negro has been developed by a more stringent selection in these respects from an original prototype.

(viii) It has been sometimes suggested that the Negro is incapable of development, for he is less variable than the European. This, as far as physical cranial characters are concerned, is not the fact—he is at least as variable as the European, if not more so. The statement must not be interpreted as meaning more than that the Congo-Gaboon group presents as wide a range for the working of natural selection as the European group; it does not signify that a few years of European education or civilised environment could convert the Negro cranium physically or develop it mentally into an instrument equivalent to the European cranium. Physically the European selection has been the work of long centuries, and the thoughtful anthropologist will hardly doubt that mental characters take as long to develop as the physical. But the point is an important one—the Congo-Gaboon group is not one of a stringently selected nature; it appears to have something of the wide variability of a non-differentiated race. It is the European races which suggest stringent selection with regard to these peculiarly negroid characters.

C. D. Fawcett in comparing modern North African Negroes with the primitive Egyptian Naqadas found that in many respects the Naqadas were nearer to European races than to this negro group, which the present investigation shows to be closer to the Kaffir-Zulu than to the Congo-Gaboon group. But now that the latter group has been investigated we find that it stands between the Egyptian group and the full-blooded Negroes. Roughly the links are:



Further we see that the Egyptians and the Congo-Gaboon Negroes are more closely allied than Europeans as a whole or Kaffir-Zulus as a whole to the prehistoric and primitive races—even to such widely separated primitive races as the Long Barrow folk and the Aino. It is not suggested that Europeans have developed under selection from a race of the Egyptian type or Kaffirs and Zulus from a race of the Congo-Gaboon type; but the suggestion is made that the Egyptian and Congo-Gaboon types are possibly lower branches of the respective stems which link European and Negro together through some trunk type in itself closely related to our prehistoric and primitive races. Roughly the scheme suggested is of the kind:

No.	C	F	L′	L	B	B′	H	OH	LB	P	Q	Q_1	S	S_1	S_2	S_3	S_3'	U	G′H	GB	J	NH	NI
15	1208	170	171	172 N.	124	95	133	111	93	88°	295	295	354	127	120	107	96	479	51	90	113	39	24
16	1430	175	175	175	139	98	128	113	92	81°	312	312	360	120	123	117	101	502	61	84	113	44	24
17	1195	168	169	168	132	93	135	112	90	81°	297	296	350	120	120	110	98	475	67	87	117	47	22
18	1175	173	174	174	135	99	126	107	93	89°	296	296	347	122	117	108	91	488	60	85	120	47	24
19	1388	178	d	180	142	89	131	d	95	d	d	d	358	123	120	115	100	509	69	d	d	48	26
20	1415?	170	172	172	139	94	131	113	91	86°	311	311	355	123	127	105	92	489	55	85	116	43	24
21	1235	167	167	169 N.	134	96	134	116	93	81°	312	312	350	125	125	100	89	484	63	85	115	43	25
22	1180	167	167	168 N.	135	97	130	112	90	82°	305	302	355	130	127	95	87	480	59	86	121	44	23
23	1438	173	174·5	176	148	95	132	113	98	78°	318	315	355	133	117	105	93	512	61	91	121	44	24
24	1615	190	193	193	147	100	135	116	104	82°	326	326	386	135	135	116	99	537	66	104	d	52	28
25	d	180	d	180	139	98	d	d	d	d	d	d	362	130	119	d	100	508	61	96	d	50	30
26	1400	179	181	180	144	95	132	115	104	78°	320	319	357	125	122	110	99	515	66	104	136	50	26
27	1315	183	186	185	141	102	130	110	104	88°	305	300	364	117	141	106	94	518	62	107	d	48	28
28	1458	183	183	185	138	101	135	113	102	81°	310	310	370	125	130	115	101	515	72	93	134	51	24
29	1320	178	178	179	142	105	127	116	100	85°	322	320	363	117	141	105	96	516	71	100	134	50	28
30	1088	163	165	165	131	90	132	109	91	82°	300	300	355	125	127	103	90	471	66	90	d	48	27
31	1355	175	d	176	135	95	136	d	97	d	d	d	366	126	131	109	94	498	63	92	d	47	28
32	1190	171	174	173	134	90	132	109	100	79°	294	292	350	125	125	100	86	485	67	97	127	50	26
33	1415	181	183	184	143	97	135	117	97	79°	324	322	360	125	135	100	88	515	63	99	133	48	28
34	1410	177	177	178	140	98	133	110	103	86°	306	306	363	120	143	100	87	510	66	93	124	49	26
35	1232	168	169	170	139	98	133	113	100	81°	315	310	350	118	122	110	96	485	63	100	130	49	26
36	1438	183	183	185	141	98	137	118	101	84°	320	319	370	132	128	110	98	520	67	98	d	51	28
37	1305	173	174	174	137	97	135	114	94	85°	311	309	368	132	126	110	97	495	62	96	123	47	30
38	d	176	177	177	134	96	141	119	100	87°	318	315	370	128	137	105	95	492	d	d	130	d	26
39	1340	180	181·5	182	142	96	131	113	96	80°	309	309	368	127	121	120	100	510	66	96	123	45	27
40	1295	180	180·5	181	141	98	130	109	102	77°	303	299	364	115	133	116	100	512	63	100	134	48	28
41	1200	170	171	171	132	94	134	111	100	80°	299	299	348	115	133	100	90	485	64	89	125	47	24
42	1600	186	189	188	149	106	135	118	101	87°	327	327	380	130	140	110	97	537	60	99	d	49	24
43	1390	185	185	185	130	101	136	114	103	80°	305	302	374	128	132	114	92	520	68	99	135	48	28
44	1585	183	186	187	143	100	145	126	104	84°	334	334	370	130	130	110	101	523	70	98	136	53	29
45	1390	177	179	179	137	97	135	114	103	77°	312	310	365	128	128	99	96	508	63	97	d	40	27
46	1445	178	181	180	146	97	138	114	100	76°·5	317	312	370	125	127	118	102	520	66	95	125	50	29
47	1438	178	180·5	179	140	97	141	123	97	82°	325	325	372	135	137	100	93	510	63	d	d	45	29
48	1255?	173	175	174	135	101	130	110	98	88°	301	297	346	123	133	110	82	494	56	100	d	47	27
49	1345	174	175	174	140	107	131	111	96	82°	307	306	358	122	126	100	95	510	65	102	136	45	24
50	1335	183	185	184	136	95	137	111	105	89°	304	304	371	125	135	111	99	509	61	93	129	47	26
51	1505	182	186·5	184	144	98	136	115	100	84°	314	314	373	125	139	109	98	522	70	96	d	51	23
52	1368	170	175·5	174	139	97	134	119	97	84°	314	311	362	123	134	105	97	495	65	96	133	40	23
53	1215	164	167	166	135	94	134	112	95	80°	303	300	336	120	120	96	89	483	63	99	129	41	26
54	d	176	178	178	140	96	d	115	d	87°	317	314	375	133	125	117	102	512	57	88	124	45	26
55	1430	189	194	194	145	105	144	116	111	83°	317	317	375	132	133	110	91	535	70	—	—	53	26
56	1595	187	187·5	187	147	103	140	123	100	90°	334	333	385	133	138	114	100	541	60	92	—	45	28
57	1150	169	169	171	136	97	132	113	96	79°	310	310	353	120	125	108	96	487	61	98	131	47	28
58	1335	182	182	182	137	97	125	109	93	80°	293	293	363	123	125	105	99	513	60	100	—	47	23
59	1443	182	182	182	142	100	134	120	98	87°	323	320	376	138	131	107	95	522	61	100	—	45	31
60	1300	177	180	178	134	98	138	117	94	86°	310	310	360	125	120	115	97	498	64	93	133	48	23
61	1315	182	183·5	182	134	102	134	113	104	87°	306	305	362	130	130	102	92	510	60	91	—	47	24
62	1125	167	168	167	138	96	128	110	94	78°	308	305	345	125	117	103	90	485	62	93	125	47	28
63	1300	172	174	172	137	93	136	110	99	82°	303	300	351	115	135	101	92	492	63	88	121	47	28
64	1260	170	172	170	134	92	132	109	95	77°	293	293	350	125	110	115	97	486	65	95	128	46	27

CONGO SKULLS.

MALES.

J	NH	NB	O_1 R.	O_1 L.	O_2 R.	O_2 L.	G_1'	G_1	G_2	GL	*fml*	*fmb*	Dacry	Tape Dacry	B_2	PT	St	B/L	H/L	B/H
113	39	24	40	39	33	33	39	41	36	88	36	28	24	30	24	104	112	72·1	77·3	93·2
113	44	24	41	40	35	35	45	48	34	93	37	30	22	30	22	112	118	79·4	73·1	108·6
117	47	22	39	39	33·5	34	41	43	35	89	32	29	19	30	22	97	116	78·6	80·4	97·8
120	47	24	43	42	31	31	41	43	33	90	34	34	22	30	21	110	114	77·6	72·4	107·1
d	48	26	*d*	*d*	*d*	*d*	*d*	*d*	38	92	39	31	25	31	22	111	119	78·9	72·8	108·4
116	43	24	41	40	33	32	39	42	32	87	36	27	24	30	21	109	115	80·8	76·2	106·1
115	43	25	39	38	33	32	44	47	31	95	35	31	25	30	22	114	122	79·3	79·3	100·0
121	44	23	38	38	32·5	31	45	47	36	91	33	28	24	32	25	104	115	80·4	77·4	103·8
121	44	24	41	40	34	35	48	53	40	102	35	27	22	32	28	115	122	84·1	75·0	112·1
d	52	28	45	44	35	35	*d*	*d*	43	102	35	27	26	40	32	112	124	76·2	69·9	108·9
d	50	30	40	40	39	38	*d*	*d*	32	*d*	*d*	*d*	26	35	25	105	110	77·2	—	—
136	50	26	43	41	36	35	53	60	42	107	36	30	25	40	31	106	111	80·0	73·3	109·1
d	48	28	46	45	35	36	47	51	43	100	36	31	26	39	28	108	116	76·2	70·3	108·5
134	51	24	42	41	36	36	48	52	38	105	39	35	25	35	25	117	119	74·6	73·0	102·2
134	50	28	42	41	34	35	47	50	42	104	33	27	32	50	23	115	124	79·3	70·9	111·8
d	48	27	38·5	38	32	33	*d*	*d*	40	90	33	30	23	35	23	110	114	79·4	80·0	99·3
d	47	28	40	40	*d*	33	47	*d*	35	100	37	29	28	38	25	104	108	76·7	77·3	99·3
127	50	26	42	42	33	33	51	55	38	101	37	30	25	34	29	104	106	77·5	76·3	101·5
133	48	28	43	41	37	36	47	*d*	42	89	41?	28	24	30	34	114	117	77·7	73·4	105·9
124	49	26	39	39	35	34	46	50	36	95	37	30	27	35	25	114	120	78·7	74·7	105·3
130	49	26	41	39	32	32	50	53	40	97	33	29	24	30	31	110	116	81·8	78·2	104·5
d	51	28	40	41	35	35	*d*	*d*	40	98	37	30	22	30	24	109	120	76·2	74·1	102·9
123	47	30	41	40	34·5	35	*d*	*d*	*d*	91	33	26	26	30	*d*	109	118	78·7	77·6	101·5
130	*d*	26	42	39	34	33	*d*	*d*	*d*	*d*	34	31	30	45	*d*	100	116	75·7	79·7	95·1
123	45	27	41	40	36	36	48	50	36	100	36	30	28	38	27	100	113	78·0	72·0	108·4
134	48	28	45	43	39	40	51	54	49	102	36	31	26	38	34	116	116	77·9	71·8	108·5
125	47	24	41	40	33	33	53	53	40	99	35	30	19	30	27	105	111	77·2	78·4	98·5
d	49	24	41	40	35	35	49	51	37	96	40	35	30	39	27	123	132	79·3	71·8	110·4
135	48	28	42·5	41	36	37	*d*	*d*	39	103	40	31	29	39	33	105	113	70·3	73·5	95·6
136	53	25	42	42	37	37	51	54	40	105	39	32	22	38	31	113	121	76·5	77·5	98·6
d	46	27	42	42	34	35	55	60	42	107	37	31	23	30	34	114	116	76·5	75·5	101·5
125	50	29	42	40	37	38	*d*	*d*	39	102	36	30	23	31	27	108	114	81·1	76·7	105·8
d	45	25	*d*	*d*	*d*	*d*	*d*	*d*	32	96	34	30	26	40	26	112	115	78·2	78·8	99·3
d	47	27	39·5	40	34	34	*d*	*d*	*d*	91	36	32	28	40	*d*	104	111	77·6	74·7	103·8
136	45	24	41	40	33	33	45	48	40	95	36	28	24	30	30	114	122	80·5	75·3	106·9
129	47	26	40	40	30	30	49	52	34	96	33	26	25	35	25	99	115	73·9	74·5	99·3
d	51	23	*d*	42	37	39	46	51	39	94	34	33	24	35	30	108	111	78·3	73·9	106·0
133	40	25	42	42	36	35	46	50	40	93	37	30	25	35	28	106	116	79·9	77·0	103·7
129	41	26	40	40	35	35	*d*	*d*	41	95	37	32	24	35	30	108	112	81·3	80·7	100·7
124	45	26	40·5	39	33	32	*d*	*d*	39	*d*	*d*	*d*	25	35	30	110	115	78·7	—	—
—	53	26	—	—	—		51	54	43	105	41	30	26	40	30	112	120	74·7	74·2	100·7
—	45	28	43	42	36·5	36	—	—	—	92	37	30	26	35	—	116	130	78·6	74·9	105·0
131	47	28	41	39	34	35	52	46	40	98	31	26	24	30	32	107	116	79·5	77·2	103·0
—	47	23	40	40	36	36	—	—	42	93	39	34	28	40	29	106	108	75·3	68·7	109·6
—	45	31	41	40	35	34	49	52	41	98	37	31	27	35	29	101	107	78·0	73·6	106·0
133	48	23	40	41	37	39	—	—	48	86	37	28	24	35	41	110	117	75·3	77·5	97·1
—	47	24	41	39	32	33	45	48	38	96	38	34	24	35	—	105	112	73·6	73·6	100·0
125	47	26	40	40	32	32	47	52	42	97	31	26	25	32	28	105	115	82·6	76·6	107·8
121	47	28	40	40	34	34	44	—	36	92	35	32	26	30	27	105	115	79·7	79·1	100·7
128	46	27	40·5	40	36	37	—	—	45	97	35	29	18	25	27	108	102	78·8	77·6	101·5

Dacry	Tape Dacry	B_2	PT	St	B/L	H/L	B/H	$G'H/GB$	NB/NH	O_2/O_1 L.	O_2/O_1 R.	G_2/G_1	
24	30	24	104	112	72·1	77·3	93·2	56·7	61·5	84·6	82·5	87·8	About 18 years. Central in
22	30	22	112	118	79·4	73·1	108·6	72·6	54·5	87·5	85·4	70·8	18—20 years. Large epipte the larger one on R. Sr
19	30	22	97	116	78·6	80·4	97·8	77·0	46·8	87·2	85·9	81·4	About 20 years. Sylvian
22	30	21	110	114	77·6	72·4	107·1	70·6	51·1	73·8	72·1	76·7	About 20 years? Central i
25	31	22	111	119	78·9	72·8	108·4	—	54·2	—	—	—	Epipteric 27 mm. long on Slight subnasal prognathi
24	[illegible]	21	109	115	80·8	76·2	106·1	64·7	55·8	80·0	80·5	76·2	About 20 years. Flatteni
25	30	22	114	122	79·3	79·3	100·0	74·1	58·1	84·2	84·6	66·0	Subnasal prognathism. Os Parieto-squamosal suture
24	32	25	104	115	80·4	77·4	103·8	68·6	52·3	81·6	85·5	76·6	Subnasal prognathism. Os
22	32	28	115	122	84·1	75·0	112·1	67·1	54·5	87·5	82·9	75·5	Incisors drawn. Subnasal pression. Ossicles in la
26	40	32	112	124	76·2	69·9	108·9	63·5	53·5	79·6	77·8	—	Incisors drawn. Flattening
26	35	25	105	110	77·2	—	—	63·5	60·0	95·0	97·5	—	Much damaged. Small ossi
25	40	31	106	111	80·0	73·3	109·1	63·5	52·0	85·4	83·7	70·0	Central incisors drawn. Lo inferior frontal protubera
26	39	28	108	116	76·2	70·3	108·5	58·0	58·3	80·0	76·1	84·3	Central incisors drawn. S ossicle of lambda and o
25	35	25	117	119	74·6	73·0	102·2	77·4	47·1	87·8	85·7	73·1	Metopic eminence. Marke rance. Simple interpari suture. Post-coronal dep
32	50	23	115	124	79·3	70·9	111·8	71·0	56·0	85·4	81·0	84·0	Central incisors drawn. mastoid ridge. Flatteni
23	35	23	110	114	79·4	80·0	99·3	73·3	56·3	86·9	83·1	—	Frontal eminences very ma
28	38	25	104	108	76·7	77·3	99·3	68·5	59·6	82·5	—	—	Broad nose (formed as in N
25	34	29	104	106	77·5	76·3	101·5	69·1	52·0	78·6	78·6	69·1	Central incisors drawn. behind the bregma. Sle
24	30	34	114	117	77·7	73·4	105·9	63·6	58·3	87·8	86·1	—	Incisors drawn. Superior
27	35	25	114	120	78·7	74·7	105·3	71·0	53·1	87·2	89·8	72·0	Metopic. Post-coronal dep sagittal ossicles and oth Small epipteric on R.
24	30	31	110	116	81·8	78·2	104·5	63·0	53·1	82·1	78·1	75·5	Temporal ridges marked. temporal protuberance
22	30	24	109	120	76·2	74·1	102·9	68·4	54·9	85·4	87·5	—	Incisors drawn. Top of na
26	30	*d*	109	118	78·7	77·6	101·5	64·6	63·8	87·5	84·2	—	Very flat nose almost entire mastoid ridge. Inferior
30	45	*d*	100	116	75·7	79·7	95·1	—	—	84·6	81·0	—	Flat nose with nasal bones
28	38	27	100	113	78·0	72·0	108·4	68·8	57·8	90·0	87·8	72·0	Incisors drawn. Flat nos bulging. Parieto-squamo
26	38	34	116	116	77·9	71·8	108·5	63·0	58·3	93·0	86·7	90·7	Central incisors drawn. D retreating forehead. Inf Both orbits sliced
19	30	27	105	111	77·2	78·4	98·5	71·9	51·1	82·5	80·5	75·5	Central incisors drawn. F parts of coronal suture Ossicle of lambda 45 × 2
30	39	27	123	132	79·3	71·8	110·4	60·6	49·0	87·5	85·4	72·3	Central incisors drawn. B protuberance. All suture
29	39	33	105	113	70·3	73·5	95·6	68·7	58·3	90·3	84·7	—	Large epipteric (35 mm. lon eminence. Flat nose. Flattening of obelion
22	38	31	113	121	76·5	77·5	98·6	71·4	47·2	88·1	87·7	74·1	Retreating forehead. Mark
23	30	34	114	116	76·5	75·5	101·5	65·0	58·7	83·3	81·0	70·0	Incisors drawn. L. pterion
23	31	27	108	114	81·1	76·7	105·8	69·5	58·0	95·0	88·1	—	Central incisors drawn. B frontal protuberance. P Lower parts of coronal su
26	40	26	112	115	78·2	78·8	99·3	—	55·5	—	—	—	Temporal ridges marked. long on L. Sutures oper
28	40	*d*	104	111	77·6	74·7	103·8	56·0	57·4	85·0	86·1	—	Central incisors drawn. F
24	30	30	114	122	80·5	75·3	106·9	63·7	53·3	82·5	80·4	83·3	Temporal ridges very marke
25	35	25	99	115	73·9	74·5	99·3	65·6	55·3	75·0	75·0	65·4	Incisors drawn. Subnasal much obliterated. Fron mastoid process longer th
24	35	30	108	111	78·3	73·9	106·0	73·0	45·1	92·9	—	76·5	Central incisors drawn.
25	35	28	106	116	79·9	77·0	103·7	67·7	54·3	83·3	85·7	80·0	Temporal ridges marked.
24	35	30	108	112	81·3	80·7	100·7	63·6	63·4	87·5	87·5	—	Central incisors drawn. D
25	35	30	110	115	78·7	—	—	64·8	57·8	82·1	81·5	—	Obliteration of coronal sutu
26	40	30	112	120	74·7	74·2	100·7	—	49·1	—	—	79·6	Central incisors drawn. margin of foramen magn filed or damaged with sor
26	35	—	116	130	78·6	74·9	105·0	65·2	62·2	85·7	84·9	—	Incisors drawn. Obliterati rance. Flattening of obe
24	30	32	107	116	79·5	77·2	103·0	62·3	59·6	89·7	82·9	87·0	Central incisors drawn. Pos on R. and L. Tempor
28	40	29	106	108	75·3	68·7	109·6	60·0	48·9	90·0	90·0	—	Central incisors drawn. N
27	35	29	101	107	78·0	73·6	100·0	61·0	68·9	85·0	85·4	78·9	Incisors drawn. Broad nos Flattening of obelion. O
24	35	41	110	117	75·3	77·5	97·1	68·8	47·9	95·1	92·5	—	Central incisors drawn.
24	35	—	105	112	73·6	73·6	100·0	66·0	51·1	84·6	78·0	79·2	Incisors and canines drawn
25	32	28	105	115	82·6	76·6	107·8	66·7	55·3	80·0	80·0	80·8	Subnasal prognathism. N
26	30	27	105	115	79·7	79·1	100·7	71·6	59·6	85·0	85·0	—	Central incisors drawn. N
18	25	27	108	102	78·8	77·6	101·5	68·4	58·7	92·5	98·9	—	Central incisors drawn. L.

TABLE I.

Remarks
ears. Central incisors drawn. Metopic eminence
rs. Large epipterics 36 mm. long on both sides, and another 12 mm. long inserted behind
er one on R. Small ossicles in lambdoid suture [marked
years. Sylvian depression. Small ossicle in lambdoid suture. Subnasal prognathism
years? Central incisors drawn
27 mm. long on R. Straight and almost horizontal parieto-squamosal suture on L.
ubnasal prognathism. Parietal eminences marked [suture
years. Flattening of obelion. Superior occipital bulging. Small ossicles in lambdoid
prognathism. Ossicles in lambdoid suture and a small one in L. spheno-malar suture.
squamosal suture straight and almost horizontal on L.
prognathism. Ossicles in R. and L. masto-occipital and a small one in lambdoid suture
rawn. Subnasal prognathism. Frontal eminences. Flattening of obelion. Sylvian de-
. Ossicles in lambdoid and masto-occipital sutures
rawn. Flattening of obelion. Supra-mastoid ridge
aged. Small ossicles in lambdoid suture
cisors drawn. Low retreating forehead. Marked temporal ridges. Sylvian depression and
frontal protuberance. Supra-mastoid ridge. Flattening of obelion. Flat nose
cisors drawn. Supra-mastoid ridge. Deep canine fossa. Depression of obelion. Small
of lambda and others in lambdoid suture
minence. Marked temporal ridges and temporo-malar angle. Inferior frontal protube-
Simple interparietal 85 mm. broad and 43 mm. high with small ossicle on L. in lambdoid
Post-coronal depression. Broad nose (formed as in No. 29)
cisors drawn. Broad nose largely composed of nasal processes of Sup. Max. Supra-
ridge. Flattening of obelion. Frontal eminences united in median line (i.e. continuous
minences very marked. Flattening of obelion [with metopic eminence)
e (formed as in No. 29). Marked temporo-malar angle and inferior frontal protuberance
cisors drawn. Nasal bones wider at lower margin than at root. Eminence 20 mm.
the bregma. Slender frontal process of squamous on L.
rawn. Superior occipital bulging. Supra-mastoid ridge. Flat nose. L. orbit sliced
Post-coronal depression and very flat vertex. Flattening of obelion. Two small posterior
ossicles and others in lambdoid and R. and L. spheno-malar sutures and at asterion.
pipteric on R. Supra-mastoid ridge. L. orbit sliced
ridges marked. Sylvian depression between inferior frontal protuberance and slight
l protuberance [of obelion. Supra-mastoid ridge
rawn. Top of nasal bones and nasal processes of Sup. Max. uniform width. Flattening
ose almost entirely formed of nasal processes of Sup. Max. Flattening of obelion. Supra-
ridge. Inferior frontal protuberance [and L. asterion
with nasal bones wide at root. Small epipteric on L. Ossicles in lambdoid suture and R.
rawn. Flat nose. Nasal bones uniform. Flattening of obelion. Superior occipital
Parieto-squamous sutures straight and almost horizontal
cisors drawn. Deep canine fossa. Epipterics 28 mm. long on R., 25 mm. on L. Low
ng forehead. Inferior frontal protuberance. Flattening of obelion. Supra-mastoid ridges.
bits sliced
cisors drawn. Frontal eminences. Flattening of obelion. Supra-mastoid ridge. Lower
coronal suture obliterated. Nose more prominent than usual. Subnasal prognathism.
of lambda 45 × 20 mm.
cisors drawn. Broad flat nose (formed as in No. 29), L. pterion in K. Inferior frontal
rance. All sutures open
pteric (35 mm. long) on L. and almost obliterated on R. Marked temporal ridges. Metopic
e. Flat nose. Nasal bones uniform width. Supra-mastoid ridges. Well marked inion.
ng of obelion [of coronal suture obliterated
forehead. Marked temporal ridges. Heavier mastoid processes than usual. Lower parts
rawn. L. pterion in K. Subnasal prognathism
cisors drawn. Broad low forehead with marked development of frontal eminences. Inferior
protuberance. Parietal eminences marked. Flattening of obelion. Subnasal prognathism.
arts of coronal suture obliterated
ridges marked. Flattening of obelion. Ossicles in lambdoid suture. Epipteric 15 mm.
L. Sutures open. Subnasal prognathism [ossicle of lambda
cisors drawn. Frontal eminences. Broad flat nose. Slight supra-mastoid ridge. Small
ridges very marked. Flat nose (formed as in No. 29)
drawn. Subnasal prognathism. Post-coronal depression. Coronal and sagittal sutures
bliterated. Frontal processes of squamous on R. and L. Nasal bones uniform width. R.
process longer than L. and re-curved [depression. Subnasal prognathism
cisors drawn. Lateral incisors filed. Superior occipital bulging. Lateral post-coronal
ridges marked. Flattening of obelion. Epipteric 20 mm. long on R., defective on L.
cisors drawn. Depression of obelion. Obliteration of coronal suture
on of coronal suture. Flattening of obelion. Slight inferior frontal protuberance
cisors drawn. Prominent glabella and marked temporal ridges. Mastoid processes and
of foramen magnum and both orbits and temporal regions appear to have been gnawed or
damaged with some sharp instrument
rawn. Obliteration of sutures at pterion. Sylvian depression and inferior frontal protube-
Flattening of obelion. Small ossicles in lambdoid suture
cisors drawn. Post-coronal depression. Flattening of obelion. Frontal processes of squamous
and L. Temporal ridges marked. Subnasal prognathism. Obliteration of coronal and
cisors drawn. Nasal bones uniform width. Flattening of obelion [sagittal sutures
rawn. Broad nose with nasal bones and nasal processes of Sup. Max. uniform width at root.
ng of obelion. Obliteration of coronal, sagittal and lambdoid sutures
cisors drawn. Nasal bones prominent. Slight Sylvian depression and inferior frontal
nd canines drawn. Supra-mastoid ridges [protuberance
prognathism. Nasal bones uniform width. Slight Sylvian depression
cisors drawn. Nasal bones uniform width. Subnasal prognathism
cisors drawn. L. mastoid process immature. Orbits sliced or gnawed. Subnasal prognathism

No.	C	F	L′	L	B	B′	H	OH	LB	P	Q	Q_1	S	S_1	S_2	S_3	S_3'	U	G′H	GB	J	NI
65	1327	170	172	172	135	92	136·5	116	97·5	84°	313	312	348	132	118·5	98·5	89	485	61	91	120	45
66	1266	174	175	175	134	93	128	110	94	77°	299	297	356	119	127	110	96	501	54·5	100	128	41
67	1445	182	182·5	182·5	140	88	134	115	95·5	87°	317	316	375	118	132	125	108	512	60	98·5	122	45
68	1285·5	172	172·5	172·5	130	96	130	110	98	87°	302	299	354	127	116	111	98	495	61	92	129	49
69	—	173·5	176	175	136	92	131	120	91	78°	320	320	370	123	139	109	98	495	56	86	119·5	39
70	1123	164	165	163	122	92	121	104·5	88	84°	287	287	341	123	106	112	96	472	49	91	—	43
71	1182	167	167·5	167	137	93	126·5	105	92	86°	295	295	337	122	117	98	87	486	53·5	92	—	43
72	—	164	165·5	165	130	87	123	102	90	—	288	287	331	107	114	111	95	481	—	83	—	44
73	1150	166	167	166	127	92	128	111	91	84°	303	304	344	119·5	110	114·5	98	478	55	87	114	44
74	1068	164	165	165	128	88	116	106	89	80°	292	294	332	115	111	107	91	475	63	87	117	49
75	—	171·5	171·5	172	124	90	129	106	97	79°	287	284	346	130	105	111	96	481	65	96	—	49
76	1393	177	177·5	178	137	91	129	113	94	81°	309	309	362	132	122	108	97	499	67	89	122·5	43
77	1130	167	167·5	167	122	90	120·5	104	94	75°	285	285	343	121	112	111	97	475	57	93	120	43
78	1205	174	175	175	127	90	130	105	98	82°	281	281	343	118	122	103	92	496	59	91	127	43
79	—	175	173	174	129	92	129	110	96	80°	303	300	357	116	126	115	97	491	52·5	90	—	43
80	1135	170	171·5	170	129	86	132	114	92	82°	308	305	354	119	125	111	101	486	56·5	91	115	43
81	948	158	159	158	117	86	122	105	86	79°	276	275	327	102	117	110	97·5	448	58·5	86	—	43
82	—	173·5	176	175·5	137·5	93·5	123	105	96	83°	292	292	352	117	138	98	89	505	55	93	—	43
83	1267	169	171	170	134	94·5	129	110	90	80°	302	301	348	117	115	115·5	98·5	494	52·5	88	—	43
84	1242	179·5	180	179	130	93	126	105	92	82°	292	292	361	124	122	115	96	502	65	99	123·5	43
85	—	185	—	185·5	135	100	—	—	101	—	—	—	358	129	109	120	100	515	66	95	—	5
86	1152	162·5	163	162	130	98	127	109	94	74°	298	297	332	119	123	90	82	477	63	95	121	43
87	1165	171·5	172·5	172	135	98	133	113	93	83°	306	305	357	127	126	104	92	491	65	97·5	123·5	43
88	1230	172·5	173·5	173	135	99	125	106	91	77°	299	296	360	122	117	121	108·5	495	58	97	124·5	43
89	1228	170	170	170	128	95	132	113	90·5	80°	303	303	356	125	113	118	102	483	60	90·5	122	43
90	1257	173	174	174	137	95·5	129	107	96	84°	301	300	352	122	122	108	95·5	505	66	94	129	43
91	1125	167	169	169	129	92·5	126	109·5	93	79°	295	294	349	119	130	100	90	488	62	97	121	43
10	1150	169	—	168	124	90	133	—	—	—	—	—	350	115	135	100	86	470	—	—	—	
11	—	165	162	165	133	88	122	105	90	90°	297	296	328	110	113	105	90	472	54	73	103	
12	1290	165	166	165	133	89	122	107	85	88°	304	303	335	118	117	100	90	479	58	86	109	
13	1390	174	173	174	136	94	130	114·5	95	87°	313	312	350	123	125	102	90	492	60	83	112	
14	1375	172	173	173	132	95	132	115	93	82°·5	305	303	353	120	130	103	91	482	62	86	—	

FEMALES.

GB	J	NH	NB	O_1 R.	O_1 L.	O_2 R.	O_2 L.	G_1'	G_1	G_2'	GL	fml	fmb
91	120	45	24·5	42	42	33	33	50·5	52·5	—	96	37	26
100	128	41·5	27	44	42	33	34	49	52	37	97	38	29
98·5	122	45	27·5	42	41	34	34	48	51	—	91·5	40	32
92	129	49	24	41·5	41·5	36	36	43	46	37	90	34	28
86	119·5	39·5	22	41	39·5	32	32	51	52	35	99	36	30
91	—	42	24	40	39	32	33	—	—	40	87	32	27
92	—	45	26	41	40	33·5	33	45	47	35	85	37·5	30
83	—	44	23	—	39	37	35	—	—	—	—	39	30
87	114	44	24	40	39	32·5	32	47	48	34	90	32	27
87	117	46	25	39	39	32·5	32	47·5	49	32	97	33	25
96	—	46	28	42	—	35	33	—	—	38	98	34	29
89	122·5	47	25	39	38·5	33	32	53·5	56	37	98	38	31
93	120	45·5	28	40	39	32	34	57	53	37	98	30	28
91	127	46	25	42	41	34	36	46·5	48	35	93	38	32·5
90	—	41	23	40	41	33	32	—	—	—	95	34	27
91	115	42	25	40	39	29	29	45·5	50	37	91	32	27
86	—	42	24	41	39	32	31	45	46	34	94	30	28
93	—	46	27	42·5	43	36	34	—	—	41	92	38	30
88	—	40	26	40·5	40	35·5	34·5	46	47	31	91	38	32
99	123·5	48	28	41	41	34	34	49	50	37	93	36·5	27
95	—	51	24	44	43	37	37	—	—	37	97	38	28
95	121	49	23	42	42	33	33	50	51·5	37	101	39	30
97·5	123·5	49	26	42	43	35	35	47	47	37	92	35	26
97	124·5	44	25	41	40·5	31	31	52	56	34·5	96	32	30
90·5	122	44	26	39	38	33·5	33	47	49	32	92	34	28
94	129	48	24	43	42	35	36	49	50	—	92	38	33
97	121	46	22	42	41	33	33	51	53	—	96	31	27

Not Adult.

—	—	—	—	—	—	—	—	—	—	—	—	35	25
73	103	42	20	36	35	32	30	39	41	30	82	38	34
86	109	42	24	37	37	35	35	40	—	37	82	38	28
83	112	43	24	37	36	34	33	45	50	37	93	41	30
86	—	47	21	36	36	33	33	41	49	34	93	39	30

	fml	*fmb*	Dacry	Tape Dacry	B_1	*PT*	*ST*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH*	O_2/O_1 *L.*	O_2/O_1 *R.*	G_2/G_1	
	37	26	23	37	31	109	116	78·5	79·4	98·9	67·0	54·4	78·6	78·6	—	Cent pr en
	38	29	27	43	32	107	113·5	76·6	73·1	104·7	54·5	65·1	81·0	75·0	71·2	Cent Os (F no
·5	40	32	26	43	33·5	108·5	112	76·7	73·4	104·5	60·9	61·1	82·9	81·0	—	Cent ho
	34	28	28	50	31	109·5	117	75·4	75·4	100·0	66·3	49·0	86·7	86·7	80·4	Meto co
	36	30	25	39	30	110	113	77·7	74·9	103·8	65·1	55·7	81·0	78·0	67·3	Inci 24
	32	27	25	40	35	111	116	74·8	74·2	100·8	53·8	57·1	84·6	80·0	—	Inci Na
	37·5	30	23	44	32	107	113	82·0	75·7	108·3	58·2	57·8	82·5	81·7	74·5	Inci
	39	30	24	40	—	96	105	78·8	74·5	105·7	—	52·3	89·7	—	—	Mari bo
	32	27	24	38	29	106	113·5	76·5	77·1	99·2	63·2	54·5	82·1	81·3	70·8	Inci T
	33	25	25	40	28	105	106	77·6	70·3	110·3	72·4	54·3	82·1	83·3	65·3	Inci as
	34	29	27	40	31	102	108	72·1	75·0	96·2	67·7	60·9	—	83·3	—	Inci
	38	31	23	38	29	103	118	77·0	72·5	106·2	75·3	53·2	83·1	84·6	60·1	Inci
	30	28	26·5	41	31	101·5	105	73·1	72·2	101·2	61·3	61·5	87·2	80·0	69·8	Inci
	38	32·5	25	41	31	103	107	72·6	74·3	97·7	64·8	54·4	87·8	81·0	72·9	Cen pr na
	34	27	22	36	—	106	111	74·1	74·1	100·0	58·3	56·1	78·1	82·5	—	Inci ro
	32	27	22	33	30	101·5	109	75·9	77·6	97·7	62·1	59·5	74·4	72·5	74·0	Inci Sc
	30	28	22	37	29	97	105	74·1	77·2	95·9	68·0	57·1	79·5	78·0	73·9	Inci
	38	30	24	41	—	109	114	78·3	70·1	111·8	59·1	58·7	79·1	84·7	—	Inci (s
	38	32	26	43	27	108	114	78·8	75·9	103·9	59·7	65·0	86·3	87·7	66·0	Inci in ne
	36·5	27	25	40	33	104	110·5	72·6	70·4	103·2	65·7	58·3	82·9	82·9	74·0	Orb In
	38	28	25·5	43	30	116	115	72·8	—	—	69·5	47·5	86·0	84·1	—	Inci
	39	30	23·5	39	32	109·5	116	80·2	78·4	102·4	66·3	46·9	78·6	78·6	71·8	Inci of
	35	26	26	40	32	115	118	78·5	77·3	101·5	66·7	53·7	81·4	83·3	78·7	Left M ni
	32	30	25	41	30·5	108	115	78·0	72·3	108·0	59·8	56·3	76·5	75·6	61·6	Lar di
	34	28	24	34	28	98	112	75·3	77·6	96·9	66·3	59·1	86·8	85·9	65·3	Infe w
	38	33	26	40	35	111	115	78·7	74·1	106·2	70·2	50·6	85·7	81·4	—	Cor o M
	31	27	25	45	32	107	113	76·3	74·6	102·4	64·0	48·2	80·5	78·6	—	Mar o

—	35	25	—	—	—	97	106	73·8	79·2	93·2	—	—	—	—	—	8 t i
82	38	34	16	25	25	97	118	80·6	73·9	109·0	74·0	47·6	85·7	88·9	73·2	10 N
82	38	28	18	22	25	102	114	80·6	73·9	109·0	67·5	57·1	94·6	94·6	—	♂ M f
93	41	30	23	30	30	100	112	78·2	74·7	104·6	72·3	55·8	91·7	91·9	74·0	♂ r
93	39	30	25	32	20	99	111	76·3	76·3	100·0	72·1	44·7	91·7	91·7	69·4	♀ i f F

TABLE II.

G_1	G_2/G_1	Remarks
5	—	Central incisors drawn. Lateral post-coronal depression. Inferior frontal protuberance (see Text). Nasal processes of sup. max. form almost entire nose
0	71·2	Central incisors drawn. Pronounced temporal ridges. Flattening of obelion. Ossification of post. 3rd of sagittal. Frontal processes of squamous. (Frontal-squamosal suture 15 mm. R., 18 mm. L. with epipteric.) Flat nose equally nasal and maxillary. Slight metopic eminence (see Text)
0	—	Central incisors drawn. Parieto-squamous suture straight and almost horizontal. Small nasal bones, arch almost entirely maxillary
7	80·4	Metopic eminence. Flattening of obelion. Obliteration of upper parts of coronal and sagittal sutures. Broad nose chiefly maxillary
0	67·3	Incisors not drawn. Single epipteric 24 mm. long R. Two epipterics 24 mm. and 15 mm. long L.
0	—	Incisors drawn. Epipteric bones R. and L. Ossicle of lambda 27 × 27. Nasal processes of sup. max. meet above and exclude very small nasal
7	74·5	Incisors drawn [bones from frontal
—	—	Marked temporo-malar angle and inferior frontal protuberance. Nasal bones uniform width
3	70·8	Incisors drawn. Frontal processes of squamous R. and L. (see Text). Top of nasal bones and nasal processes of sup. max. uniform width
3	65·3	Incisors not drawn. Broad flat nose with nasal and maxillary portions as above. Small epipteric on L. Flattening of obelion
3	—	Incisors not drawn. Upper part of nose mostly maxillary
6	60·1	Incisors drawn. Nasal bones uniform width. Post-coronal depression
0	69·8	Incisors drawn. Nasal bones uniform width. Post-coronal depression
0	72·9	Central incisors drawn. Sylvian depression. Inferior frontal and temporal protuberances (see Text). Obliteration of sagittal suture. Small nasal bones
5	—	Incisors drawn. Nasal bones large. Lower margin of pyriform aperture rounded. Ossicles in R. spheno-malar suture
5	74·0	Incisors drawn. Retreating forehead. Inferior frontal protuberance. Sagittal suture obliterated. Lower margin of pyriform aperture rounded
0	73·9	Incisors not drawn. Small nasal bones
7	—	Incisors drawn. Post-coronal depression. Posterior sagittal ossicle 40 × 15 (see Figure in text)
7	66·0	Incisors drawn. R. pterion in K (horizontal). R. orbit sliced. Ossicles in lambdoid and masto-occipital sutures. Small triangular nasal bones nearly horizontal. Large nasal processes of sup. max. (see Text)
9	74·0	Orbits scratched. Post-coronal depression and flattening of obelion. Incisors drawn. Nose equally nasal and maxillary
1	—	Incisors drawn. Nose as above. Post-coronal depression
6	71·8	Incisors drawn. Nose as above. Inferior frontal protuberance. Flattening of obelion
3	78·7	Left central incisor filed, other absent (not drawn). Epipteric on L. Metopic eminence. Flattening of obelion. Very flat nose, principally nasal bones which are of uniform width
6	61·6	Large paired ossicles of lambda 97 × 47. Sylvian depression. Incisors drawn. Nasal bones small
9	65·3	Inferior frontal protuberance. Ossicles at asterion. Nasal bones uniform width
4	—	Coronal suture obliterated at pterion. Temporal processes of squamous on both sides (probably pterion in K). Some post-coronal depression. Maxillary portion of nose large. Left orbit filed
6	—	Marked temporal ridges. Nasal bones uniform width. Flattening of obelion. Ossicles in lambdoid suture
—	—	8 to 10 years. Very broken and fragile. Marked temporal ridges and inferior frontal protuberance
9	73·2	10 years? Flattening of obelion. Teeth or saw marks above R. orbit. Nasal bones prominent
6	—	♂. 12—13 years. Post-coronal depression. Superior occipital bulging. Metopic eminence. Flattening of obelion. Sylvian depression. Inferior frontal protuberance
9	74·0	♂. About 12 years. Sylvian depression. R. orbit sliced. Temporal ridges marked. Inferior frontal protuberance
7	69·4	?♂, 14—15 years. Metopic. On R. two epipteric bones, and three ossicles in hind part of squamous; one, now wanting, in masto-occipital suture; four ossicles in lambdoid suture, one (at L. asterion) also wanting. R. inferior frontal protuberance. Subnasal prognathism

No.	*C*	*F*	*L*	*B*	*B'*	*H*	*LB*	*Q*	*S*	S_1	S_2	S_3	S_3'	*U*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1	O_2	G_1'	G_1
1	1233	173	173	133	96	134	98	310	358	128	130	100	87	490	76	97	125	50	26	39	37	53	56
3	—	175	176	145	96	131	97	325	355	130	115	110	97	508	70	96	127	50	21	41	37	50	53
7	1515	181	182	138	97	136	101	312	370	122	138	110	95	505	67	97	131	46	26	41	32	—	—
8	1400	178	178	137	97	134	94	310	365	120	140	105	95	505	69	91	125	47	23	41	37	53	56
11	1468	186	188	131	98	139	101	302	383	138	134	111	99	505	66	96	134	49	33	43	33	51	55
13	1380	177	178	135	92	133	98	295	364	130	130	104	95	498	61	98	128	43	25	43	35	50	54
14	1352	175	177	131	93	139	97	302	370	130	135	105	95	495	66	92	124	45	25	41	34	47	—
15	1342	174	175	131	99	133	104	295	350	125	125	100	86	492	65	92	129	49	27	42	34	50	51
18	1430	176	178	136	95	142	100	312	367	135	130	102	92	504	68	93	118	51	25	41	35	48	51
25	1395	173	173	134	100	131	98	312	357	125	122	110	98	500	68	96	130	55	28	41	39	48	51
27	1415	185	186	138	96	142	101	315	388	135	137	116	102	515	67	100	134	46	27	44	33	47	50
28	1333	178	179	133	93	136	97	310	366	130	128	108	95	496	64	91	123	43	24	41	33	41	46
30	1412	170	172	135	96	136	99	308	348	130	122	96	88	488	51	100	140	46	26	43	38	44	46
31	1268	166	172	137	94	132	100	308	348	129	120	99	90	485	60	86	132	46	29	39	31	53	56
32	1435	179	182	138	98	139	106	315	360	122	136	102	95	508	71	97	—	47	22	40	34	52	55
33	1445	185	186	140	98	140	—	323	385	126	136	123	101	520	68	—	—	48	27	43	36	—	—
34	1420	173	177	137	96	138	100	311	358	125	125	108	95	495	67	100	130	47	30	40	33	48	53
36	1300	173	173	136	98	133	90	310	357	127	115	115	92	495	59	90	125	44	24	42	31	48	51
40	1450	186	187	142	99	140	94	314	392	128	138	126	110	520	68	94	124	46	26	42	34	48	50
43	1500	182	183	145	94	135	105	325	372	139	111	122	105	525	70	105	—	50	27	40	36	53	56
44	1324	170	171	133	91	135	96	303	345	120	125	100	91	484	67	90	124	50	29	40	37	52	54
45	1465	180	183	133	97	132	100	312	370	132	138	100	89	508	68	96	127	47	25	41	33	46	48
47	1375	181	182	132	98	138	105	317	370	125	145	100	92	502	64	97	130	50	25	40	35	48	53
49	1279	170	173	138	99	131	93	308	350	113	129	108	96	490	72	91	130	50	27	41	36	46	50
50	1275	175	176	131	93	132	95	297	362	123	119	120	101	485	65	93	123	49	26	40	34	44	—
53	1478	183	184	130	93	140	102	315	377	130	135	112	98	506	53	95	132	43	24	41	32	45	47
55	1375	176	184	135	100	137	104	310	360	115	133	112	101	492	72	102	138	51	28	42	33	52	56
57	1205	172	173	135	101	131	100	308	359	120	128	105	94	493	64	97	126	45	27	40	34	47	—
59	1434	183	186	134	93	138	105	305	370	118	141	111	98	507	67	94	133	49	23	44	36	52	58
61	1517	182	184	140	97	145	102	327	380	140	125	115	101	512	76	96	128	52	28	42	34	43	—
62	1277	169	170	135	98	125	98	303	340	117	95	128	103	486	64	102	128	47	28	41	34	49	51
63	1730	191	192	141	103	146	104	345	396	135	139	122	106	531	72	97	127	50	29	41	37	50	—
65	1318	172	173	134	94	138	102	303	353	119	123	111	95	494	68	95	128	47	25	41	35	48	—
67	1126	168	171	133	90	126	96	295	340	115	105	120	102	475	53	90	120	40	27	40	34	—	—
69	1230	181	182	133	96	132	101	310	360	130	116	114	97	495	63	90	—	47	27	40	33	47	—
71	1400	181	184	135	99	137	106	315	362	125	130	107	94	505	74	97	134	52	27	45	36	49	—
73	1515	182	183	138	99	134	100	315	372	120	140	112	99	511	70	102	139	48	29	41	34	51	55
76	1520	187	187	137	105	135	106	320	371	135	125	111	95	523	70	99	140	50	25	45	36	48	50
77	1230	176	176	136	96	131	99	310	362	115	128	119	105	500	70	99	129	47	29	42	37	50	—
80	1210	170	170	135	91	127	99	295	340	125	115	100	87	486	66	97	133	49	25	41	34	42	49
87	1505	178	179	133	98	148	104	316	370	122	135	110	99	497	66	92	131	51	27	41	35	47	49
89	1460	176	179	136	94	136	107	310	360	120	130	110	97	500	70	96	134	51	27	44	36	46	53
?	1452	177	180	135	97	135	100	300	364	122	175	67	65	500	68	96	127	47	25	41	33	44	49
11A	1322	179	180	134	102	130	—	315	364	120	127	117	98	503	53	91	128	45	25	43	35	—	—
40A	1412	182	185	134	94	135	103	305	372	125	133	114	100	512	57	—	—	51	27	43	40	47	51
69A	1230	174	175	136	93	130	103	310	360	130	116	114	97	495	67	96	129	48	27	41	38	47	—
37	1345	179	181	140	93	126	99	307	357	125	120	112	98	508	77	99	127	54	31	42	40	56	62
79	1365	184	188	132	100	137	100	312	379	130	132	117	102	510	75	105	133	51	28	43	38	45	48
83	1410	178	188	131	100	145	106	315	360	130	130	100	95	498	73	99	125	53	28	42	34	52	57
85	1368	180	180	133	94	130	99	310	364	137	130	97	89	505	57	85	125	46	29	38	37	—	—

1864 SERIES. CRANIA FROM FERNAND VAZ, GABOON.

MALES.

	G_1'	G_1	G_2	GL	fml	fmb	Dacry	B_2	St	B/L	H/L	B/H	$G'H/GB$	NB/NH	O_2/O_1	G_2/G_1
	53	56	38	104	36	29	25	30	123	76·9	77·5	99·3	78·4	52·0	94·9	67·9
	50	53	38	100	40	30	21	28	123	82·4	74·4	110·7	72·9	42·0	90·2	71·7
	—	—	40	105	35	30	24	29	112	75·8	74·7	101·5	69·1	56·5	78·1	—
	53	56	36	100	33	20	25	29	122	77·0	75·3	102·2	75·8	48·9	90·2	64·3
	51	55	44	100	38	33	22	33	103	69·7	73·9	94·2	68·8	67·4	76·7	80·0
	50	54	40	98	36	29	23	30	112	75·8	74·7	101·5	62·2	58·1	81·4	74·1
	47	—	35	92	35	29	21	25	113	74·0	78·5	94·2	71·7	55·6	82·9	—
	50	51	42	106	33	28	21	28	113	74·9	76·0	98·5	70·7	55·1	81·0	82·4
	48	51	43	102	39	28	26	30	116	76·4	79·8	95·8	73·1	49·0	85·4	84·3
	48	51	38	99	37	30	28	28	121	77·5	75·7	102·3	70·8	50·9	95·1	74·5
	47	50	36	101	32	27	25	32	111	74·2	76·3	97·2	67·0	58·7	75·0	72·0
	41	46	—	—	38	29	22	—	120	74·3	76·0	97·8	70·3	55·8	80·5	—
	44	46	—	94	42	31	23	—	111	78·5	79·1	99·3	51·0	56·5	88·4	—
	53	56	37	101	35	30	25	29	120	79·7	76·7	103·8	69·8	63·0	79·5	66·1
	52	55	41	110	38	29	21	33	115	75·8	75·4	99·3	73·2	46·8	85·0	74·6
	—	—	41	100	34	30	23	32	116	75·3	75·3	100·0	—	56·3	83·7	—
	48	53	40	103	37	32	24	27	117	77·4	78·0	99·3	67·0	63·8	82·5	75·5
	48	51	37	90	38	31	25	32	115	78·6	76·9	102·3	65·6	54·6	73·8	72·6
	48	50	39	98	34	29	23	36	116	75·9	74·9	101·4	72·3	56·5	81·0	78·0
	53	56	40	103	32	31	24	34	110	79·2	73·8	107·4	66·7	54·0	90·0	80·0
	52	54	37	101	39	30	24	30	105	77·8	79·0	98·5	74·4	58·0	92·5	68·5
	46	48	39	100	35	30	24	36	113	72·7	72·1	100·8	70·8	53·2	80·5	81·3
	48	53	41	105	36	28	20	30	114	72·5	75·8	95·7	66·0	50·0	87·5	77·4
	46	50	40	92	34	31	29	29	116	79·8	75·7	105·3	79·1	54·0	87·8	80·0
	44	—	33	95	37	27	23	25	106	74·4	75·0	99·2	69·9	53·1	85·0	—
	45	47	39	96	37	30	24	29	111	70·7	76·1	92·9	55·8	55·8	78·1	83·0
	52	56	45	106	38	35	27	30	110	73·4	74·5	98·5	70·6	54·9	78·6	80·4
	47	—	37	101	30	26	27	28	118	78·0	75·7	103·1	66·0	60·0	85·0	—
	52	58	40	105	40	31	22	33	113	72·0	74·2	97·1	71·3	46·9	81·8	69·0
	43	—	43	102	36	26	28	30	111	76·1	78·8	96·6	79·2	53·9	81·0	—
	49	51	45	95	39	32	25	30	116	79·4	73·5	108·0	62·8	59·6	82·9	88·2
	50	—	42	109	37	32	27	30	124	73·4	76·0	96·6	74·2	58·0	90·2	—
	48	—	38	98	35	30	21	32	107	77·5	79·8	97·1	71·6	53·2	85·4	—
	—	—	—	93	30	30	22	—	108	77·8	73·7	105·6	58·9	67·5	85·0	—
	47	—	39	106	30	28	25	32	103	73·1	72·5	100·8	70·0	57·5	82·5	—
	49	—	37	100	38	29	23	26	120	73·4	74·5	98·5	76·3	51·9	80·0	—
	51	55	42	101	39	29	29	36	117	75·4	73·2	103·0	68·6	60·4	82·9	76·4
	48	50	41	—	39	31	27	32	122	73·3	72·2	101·5	70·7	50·0	80·0	82·0
	50	—	—	98	34	26	31	—	108	77·3	74·4	103·8	70·7	61·7	88·1	—
	42	49	46	104	36	37	25	31	113	79·4	74·7	106·3	68·0	51·0	82·9	93·9
	47	49	39	94	35	36	27	29	112	74·3	82·7	89·9	71·7	52·9	85·4	79·6
	46	53	35	108	35	31	26	31	109	76·0	76·0	100·0	72·9	52·9	81·8	66·0
	44	49	35	100	40	34	24	26	109	75·0	75·0	100·0	70·8	53·2	80·5	71·4
	—	—	—	96	34	27	24	—	117	74·4	72·2	103·1	58·2	55·6	81·4	—
	47	51	37	98	37	34	26	33	117	72·4	73·0	99·3	—	52·9	93·0	72·6
	47	—	39	99	30	28	23	32	103	77·7	77·7	100·0	69·8	56·3	92·7	—
	56	62	44	107	34	28	23	31	112	77·4	69·6	107·9	77·8	57·4	95·2	71·0
	45	48	38	99	36	33	25	31	108	70·2	72·9	96·4	71·4	54·9	88·4	79·2
	52	57	42	108	38	33	28	32	116	69·7	77·1	90·3	73·7	52·8	81·0	73·7
	—	—	—	102	33	31	27	—	116	73·9	72·2	102·3	67·1	63·0	97·4	—

CRANIA FROM FERNAND VAZ, GABOON.

MALES.

fml	*fmb*	Diam	B_2	*St*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH*	O_2/O_1	G_2/G_1	W_1	W_2	h_1	*f*	
36	29	25	30	123	76·9	77·5	99·3	78·4	52·0	94·9	67·9	—	—	—	—	40—50 years. Large su nent. Subnasal prog
40	30	21	28	123	82·4	74·4	110·7	72·9	42·0	90·2	71·7	—	—	—	—	40—50 years. Nasal b Sutures gaping and
35	30	24	29	112	75·8	74·7	101·5	69·1	56·5	78·1	—	—	—	—	—	30—40 years. Superior long behind this, in p inion. Forepart of c
33	20	25	29	122	77·0	75·3	102·2	75·8	48·9	90·2	64·3	—	—	—	—	30—40 years. Well-ma long on L. Ossicles
38	33	22	33	103	69·7	73·9	94·2	68·8	67·4	76·7	80·0	—	—	—	—	Aged. Obliteration of Nasal bones absent as of Sup. Max. between
36	29	23	30	112	75·8	74·7	101·5	62·2	58·1	81·4	74·1	113	98	30	46	Obliteration of sagittal eminence. Sylvian (Fronto-squamosal su
35	29	21	25	113	74·0	78·5	94·2	71·7	55·6	82·9	—	110	90	31	42	Aged. Parietal depres eminences well mark
33	28	21	28	113	74·9	76·0	98·5	70·7	55·1	81·0	82·4	117	96	36	47	30—40 years. Small Extensive caries
39	28	26	30	116	76·4	79·8	95·8	73·1	49·0	85·4	84·3	110	102	37	47	30—40 years. Marked obliterated) R. and L
37	30	28	28	121	77·5	75·7	102·3	70·8	50·9	95·1	74·5	109	83	26	44	Aged. Nasal bones mu
32	27	25	32	111	74·2	76·3	97·2	67·0	58·7	75·0	72·0	113	87	—	46	Aged. Sagittal and co ridges. Forehead ri
38	29	22	—	120	74·3	76·0	97·8	70·3	55·8	80·5	—	—	—	—	—	40—50 years. Lateral
42	31	23	—	111	78·5	79·1	99·3	51·0	56·5	88·4	—	—	—	—	—	Aged. Depression of high (somewhat oblit
35	30	25	29	120	79·7	76·7	103·8	69·8	63·0	79·5	66·1	110	99	29	45	30—40 years. Small r
38	29	21	33	115	75·8	76·4	99·3	73·2	46·8	85·0	74·6	—	—	—	—	30—40 years. Post-co prognathism
34	30	23	32	116	75·3	75·3	100·0	—	56·3	83·7	—	—	—	—	—	30—40 years. Flatten
37	32	24	27	117	77·4	78·0	99·3	67·0	63·8	82·5	75·5	—	—	—	—	30—40 years. R. orb lambdoid suture
38	31	25	32	115	78·6	76·9	102·3	65·6	54·6	73·8	72·6	—	—	—	—	Aged. Root of nose d Epipteric 20 mm. lo
34	29	23	36	116	75·9	74·9	101·4	72·3	56·5	81·0	78·0	114	92	32	45	40—50 years. Fine Obliteration of coron
32	31	24	34	110	79·2	73·8	107·4	66·7	54·0	90·0	80·0	103	101	29	51	40 years. Metopic em
39	30	24	30	105	77·8	79·0	98·5	74·4	58·0	92·5	68·5	105	85	35	45	30—40 years. Subnas
35	30	24	36	113	72·7	72·1	100·8	70·8	53·2	80·5	81·3	113	97	31	43	Epipteric on L. Flatte
36	28	20	30	114	72·5	75·8	95·7	66·0	50·0	87·5	77·4	113	97	41	48	30—40 years. Flatten suture on R. and L.
34	31	29	29	116	79·8	75·7	105·3	79·1	54·0	87·8	80·0	117	90	32	44	30 years. R. incisor f
37	27	23	25	106	74·4	75·0	99·2	69·9	53·1	85·0	—	104	88	28	44	Epipteric 17 mm. lon Flattening of obelio
37	30	24	29	111	70·7	76·1	92·9	55·8	55·8	78·1	83·0	—	—	—	—	40—50 years. Obliter and L. asterion. F
38	35	27	30	110	73·4	74·5	98·5	70·6	54·9	78·6	80·4	109	98	41	52	30—40 years. Retreat occipital bulging
30	26	27	28	118	78·0	75·7	103·1	66·0	60·0	85·0	—	111	89	33	47	30—40 years. Flatter lambdoid sutures an
40	31	22	33	113	72·0	74·2	97·1	71·3	46·9	81·8	69·0	—	—	—	—	40—50 years. Superc
36	26	28	30	111	76·1	78·8	96·6	79·2	53·9	81·0	—	—	—	—	—	40—50 years. Flatte lambda. Others in
39	32	25	30	116	79·4	73·5	108·0	62·8	59·6	82·9	88·2	—	87	26	44	Aged. Lambdoid sutu
37	32	27	30	124	73·4	76·0	96·6	74·2	58·0	90·2	—	123	100	36	41	About 20 years. Teet No. 5
35	30	21	32	107	77·5	79·8	97·1	71·6	53·2	85·4	—	—	—	—	—	40—50 years. Some depression. Tempo
30	30	22	—	108	77·8	73·7	105·6	58·9	67·5	85·0	—	—	—	—	—	Weather worn. Oblit
30	28	25	32	103	73·1	72·5	100·8	70·0	57·5	82·5	—	105	83	35	46	About 40 years. Sligh 9 mm. long on R.
38	29	23	26	120	73·4	74·5	98·5	76·3	51·9	80·0	—	114	100	36	45	30—40 years. Sagittal on R. and L. Sylv
39	29	29	36	117	75·4	73·2	103·0	68·6	60·4	82·9	76·4	—	—	—	—	40—50 years. R. squ
39	31	27	32	122	73·3	72·2	101·5	70·7	50·0	80·0	82·0	121	93	30	46	Temporal ridges mark
34	26	31	—	108	77·3	74·4	103·8	70·7	61·7	88·1	—	—	100	41	46	Temporal ridges mar squamous on L. F
36	37	25	31	113	79·4	74·7	106·3	68·0	51·0	82·9	93·9	—	—	—	—	Aged. Coronal, sagi defective. Sylvian
35	30	27	29	112	74·3	82·7	89·9	71·7	52·9	85·4	79·6	—	100	30	48	Sagittal and lambdoid sphenoid and front
35	31	26	31	109	76·0	76·0	100·0	72·9	52·9	81·8	66·0	121	96	31	45	Subnasal prognathism 20 mm. long on L.
40	34	24	26	109	75·0	75·0	100·0	70·8	53·2	80·5	71·4	117	99	30	44	Aged. Post-coronal Ossicle of lambda
34	27	24	—	117	74·4	72·2	103·1	58·2	55·6	81·4	—	111	95	28	44	Sutures gaping. Tee
37	34	26	33	117	72·4	73·0	99·3	—	52·9	93·0	72·6	121	97	30	41	Post-coronal depressi
30	28	23	32	103	77·7	77·7	100·0	69·8	56·3	92·7	—	105	83	35	46	
34	28	23	31	112	77·4	69·6	107·9	77·8	57·4	95·2	71·0	—	—	—	—	Subnasal prognathism
36	33	25	31	108	70·2	72·9	96·4	71·4	54·9	88·4	79·2	116	97	33	45	Sagittal, coronal and Sloping forehead ri
38	33	28	32	116	69·7	77·1	90·3	73·7	52·8	81·0	73·7	116	106	36	44	40? Subnasal pr gn
33	31	27	—	116	73·9	72·2	102·3	67·1	63·0	97·4	—	—	97	—	37	Coronal suture m ch

TABLE III *A*.

	Remarks
	years. Large superciliary ridges. Marked mastoid processes and inion. Nasal bones promi- . Subnasal prognathism. Ossicles at R. asterion
	years. Nasal bones flat. Ossicles in lambdoid suture (now wanting at R. and L. asterion). ures gaping and fragile
	years. Superior occipital bulging. Epipteric 41 mm. long on L., and another ossicle 32 mm. behind this, in parieto-squamosal suture. Ossicle of lambda 22 × 10 mm. Small mastoids and n. Forepart of central incisors sliced off. Dentition good
	years. Well-marked temporal ridges. Pterion in K on R. Fronto-squamosal suture 15 mm. on L. Ossicles in lambdoid suture. Small inion. Some superior occipital bulging
	Obliteration of coronal and sagittal sutures. Flattening of obelion. Nose peculiarly formed. al bones absent save for small median rudiment, and their place taken by broad nasal processes up. Max. between which frontal sends down a pointed process 7 mm. long
	ration of sagittal suture. R. lateral incisor filed. Post-coronal depression. Marked metopic nence. Sylvian depression and inferior frontal protuberance. Frontal processes of squamous onto-squamosal suture 9 mm. long on R. and 10 mm. on L.)
	Parietal depression on both sides. Ossicles in lambdoid suture and at asterion. Frontal nences well marked. Coronal suture obliterated. Post-coronal depression
	o years. Small ossicles in lambdoid suture. R. lateral incisor filed, other incisors wanting. ensive caries
	o years. Marked frontal eminences. Frontal process of squamous and epipteric (somewhat terated) R. and L. Superior occipital bulging [lambdoid suture
	Nasal bones much reduced. Post-coronal depression. Small ossicle of lambda and others in Sagittal and coronal sutures entirely obliterated. Flattening of obelion. Marked temporal es. Forehead rises to a marked eminence similar to the condition in No. 5 (Female,
	o years. Lateral post-coronal depression. Flattening of obelion [Table III *B*.)
	Depression of obelion and median sagittal area. Ossicle of lambda 55 mm. wide and 35 mm. h (somewhat obliterated)
	o years. Small recurved mastoids. Root of nose depressed. Pterion in K on L.
	o years. Post-coronal depression. Obliteration of posterior half of sagittal suture. Subnasal gnathism [ture sliced
	o years. Flattening of obelion. Subnasal prognathism. R. orbit and margin of piriform aper- o years. R. orbit sliced. Sylvian depression and inferior frontal protuberance. Ossicle in nbdoid suture
	Root of nose depressed. Marked fronto-malar angle and temporal ridges. L. orbit sliced. ipteric 20 mm. long on L.
	0 years. Fine large skull said to have been "prepared" by Owen. Thick cranial walls. literation of coronal and sagittal sutures. Frontal processes of squamous on R. and L. ars. Metopic eminence. Very prognathous. Lower margin of piriform aperture rounded
	40 years. Subnasal prognathism. Form of frontal as in No. 5 teric on L. Flattening of obelion. Subnasal prognathism. Supra-orbital and supra-mastoid ridges
	40 years. Flattening of obelion. Sutures open. Small symmetrically placed ossicles in lambdoid ture on R. and L. ars. R. incisor filed—others wanting. Ossicles at R. and L. asterion
	teric 17 mm. long on R. Symmetrically placed ossicles in lambdoid suture on R. and L. attening of obelion. Superior occipital bulging
	50 years. Obliteration of sagittal and lambdoid sutures. Ossicles in R. masto-occipital suture d L. asterion. Frontal process of squamous on R. Supra-mastoid ridges
	40 years. Retreating forehead. Temporal ridges marked. Metopic eminence. Slight superior ipital bulging
	40 years. Flattening of obelion. Obliteration of lower ends of coronal and parts of sagittal and nbdoid sutures and small ossicles in the last [cephaly. Marked inion
	50 years. Superciliary and supra-orbital ridges marked. Nasal bones prominent. Slight bathro- 50 years. Flattening of obelion. Flat nose. Frontal and metopic eminences. Ossicle of mbda. Others in lambdoid suture, and R. and L. asterion
	l. Lambdoid suture entirely and coronal and sagittal largely obliterated. Post-coronal depression ut 20 years. Teeth good. Third molars uncut. Temporal ridges marked. Form of frontal as in o. 5
	-50 years. Some obliteration of coronal and sagittal sutures. Flattening of obelion. Post-coronal pression. Temporal ridges marked. Frontal eminences and metopic crest [obelion ther worn. Obliteration of coronal and sagittal sutures. Post-coronal depression. Flattening of ut 40 years. Slight superior occipital bulging. Pterion defective on L. Fronto-squamosal suture mm. long on R.
	-40 years. Sagittal suture obliterated and lower ends of coronal. Small frontal processes of squamous n R. and L. Sylvian depression between inferior frontal protuberance and temporal protuberance -50 years. R. squamous broken. L. pterion in K [(most marked on R.)
	nporal ridges marked. Sylvian depression
	nporal ridges marked. Sylvian depression. Obliteration of coronal suture. Frontal process of quamous on L. Form of frontal as in No. 5
	d. Coronal, sagittal and lambdoid sutures obliterated. Teeth much worn and alveolar rim efective. Sylvian depression and inferior frontal protuberance
	ittal and lambdoid sutures much obliterated and antero-inferior angle of parietal synostosed with phenoid and frontal
	nasal prognathism. Sutures open. Frontal processes of squamous (Fronto-squamosal suture o mm. long on L., 12 mm. on R.). Inferior frontal protuberance
	d. Post-coronal depression. Temporal ridges marked. Frontal process of squamous on L. ssicle of lambda 44 × 63 mm.
	ures gaping. Teeth absent and alveolar margin much absorbed t-coronal depression. Slight superior occipital bulging
	nasal prognathism
	ittal, coronal and lambdoid sutures obliterated. Flattening of obelion. Marked "Sutura Notha." loping forehead rising to an eminence as in No. 5
	Subnasal prognathism. Mental prominence strong [and frontal eminences
	onal suture much obliterated. Sylvian depression and inferior frontal protuberance. Metopic

No.	*C*	*F*	*L*	*B*	*B'*	*H*	*LB*	*Q*	*S*	S_1	S_2	S_3	S_3'	*U*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1	O_2
2	—	163	163	123	85	—	—	288	342	120	108	114	96	452	50	71	95	35	19	36	31
4	1039	163	165	125	90	115	93	277	325	105	117	103	92	456	61	89	118	41	25	40	35
5	1277	175	175	130	92	130	98	288	365	125	131	109	99	489	70	102	124	49	24	39	34
9	1198	166	168	126	94	130	94	288	350	120	122	108	95	471	71	90	120	50	24	39	37
10	1165	165	166	131	92	128	90	295	347	120	116	111	99	475	—	98	125	47	29	40	39
12	1292	163	163	143	97	133	93	285	357	125	130	97	84	488	61	84	127	42	23	40	33
16	1553	182	183	141	95	125	90	322	378	135	135	108	99	508	61	93	118	45	22	38	32
17	1134	163	166	129	92	130	98	297	342	125	116	101	92	464	63	87	116	47	25	40	33
19	1142	166	167	131	91	124	94	300	335	110	135	90	81	472	—	—	—	43	25	38	35
20	1260	172	172	126	95	135	98	305	350	120	125	105	92	475	67	87	112	46	21	40	37
21	1335	178	179	135	96	136	100	310	363	125	120	118	101	503	68	99	123	48	26	43	36
22	1295	171	173	122	96	137	99	302	350	127	118	105	90	480	67	95	122	50	27	42	36
23	1300	174	175	132	86	127	94	295	360	115	130	115	95	488	64	90	115	46	26	39	34
24	1270	161	165	135	93	138	98	308	348	118	132	98	92	478	61	81	120	47	24	39	33
26	1276	170	172	133	—	131	89	290	345	117	118	110	97	488	—	—	—	43	25	—	—
29	1220	173	175	130	88	130	93	295	363	125	125	113	104	485	66	93	118	46	23	40	32
35	1283	178	178	131	91	132	102	304	355	130	135	90	85	492	66	87	120	47	25	36	31
38	1137	172	174	123	91	133	96	292	356	122	116	118	105	470	—	86	122	44	24	41	36
39	1060	156	157	129	84	126	92	288	330	110	113	107	86	452	64	89	122	44	27	38	33
41	1093	169	169	131	96	123	93	283	340	120	115	105	91	479	53	94	124	43	22	40	35
42	1025	162	162	125	90	124	95	272	322	114	101	107	90	460	57	86	122	45	27	37	31
46	1192	171	172	135	98	126	96	308	336	122	114	100	91	486	62	92	121	45	26	39	33
48	1266	179	180	125	93	133	101	292	367	130	120	117	99	491	62	86	113	46	26	40	35
51	1220	167	167	132	97	131	95	294	343	120	125	98	89	475	64	93	127	44	27	42	34
52	1587	181	181	137	94	141	100	310	376	137	130	109	99	513	68	96	128	48	25	38	33
54	1205	173	173	128	99	133	102	296	355	122	136	97	87	485	68	93	124	47	27	41	35
56	1290	170	172	128	93	136	106	292	343	125	121	97	92	475	67	96	126	49	23	41	34
58	1215	177	177	127	93	125	99	285	342	112	168	68	63	490	60	94	125	48	26	42	35
60	1262	170	173	130	86	132	94	292	362	122	130	110	97	478	65	96	121	44	27	41	31
64	1290	177	177	127	97	133	98	305	353	127	125	101	88	485	64	97	120	46	25	42	34
66	1236	173	173	129	91	132	99	297	345	120	118	107	95	480	67	92	—	44	24	40	34
68	1175	168	168	136	86	123	92	303	345	122	118	105	95	486	61	87	121	46	26	37	35
70	1300	172	173	135	100	131	96	304	348	123	127	98	88	491	59	89	124	42	24	40	33
72	1200	165	166	128	90	120	87	290	335	125	110	100	85	464	51	77	113	37	21	36	31
74	1220	169	169	130	95	126	91	292	355	120	120	115	89	478	59	87	118	45	22	39	31
75	1035	172	173	130	96	128	92	300	348	122	130	96	89	476	59	92	117	44	26	40	34
78	1295	178	179	130	95	130	96	308	355	125	120	110	97	498	67	98	126	50	26	40	34
81	1257	174	176	138	101	131	96	303	375	130	125	120	101	495	54	100	132	40	24	43	32
82	1262	169	170	135	96	126	95	300	347	114	119	114	98	483	—	91	122	—	—	41	33
84	1398	174	175	137	91	137	100	320	360	120	124	116	103	493	66	—	—	47	24	37	33
86	1033	171	172	132	94	140	101	285	343	120	113	110	96	473	64	90	124	44	24	39	30
88	1305	179	179	130	94	140	99	307	376	129	121	126	107	499	66	91	121	47	28	40	34
91	1430	177	179	131	92	134	98	304	365	130	120	115	96	493	62	99	118	44	26	39	33
90	930	157	158	124	87	117	89	272	320	110	110	100	85	457	67	88	116	46	24	39	34

1864 SERIES. CRANIA FROM FER

FEMALES.

B	O_1	O_2	G_1'	G_1	G_2	GL	*fml*	*fmb*	Dacry	B_2	*St*	B/L
9	36	31	32	35	28	—	—	25	19	23	102	75·5
5	40	35	44	48	35	95	33	26	21	28	103	75·8
4	39	34	50	52	40	104	32	30	22	33	108	74·3
4	39	37	48	52	38	97	33	26	21	28	101	75·0
9	40	39	—	—	—	—	36	30	25	—	109	78·9
3	40	33	46	50	41	95	30	26	22	26	121	87·7
2	38	32	48	50	37	95	33	27	21	33	119	77·1
5	40	33	43	—	38	96	34	29	22	31	109	77·7
5	38	35	—	—	—	—	—	—	23	—	109	78·4
1	40	37	45	47	34	95	37	29	21	30	111	73·3
6	43	36	46	47	36	98	39	30	24	27	—	75·4
7	42	36	49	52	37	100	34	28	24	31	106	70·5
6	39	34	48	51	40	101	33	27	25	29	107	75·4
4	39	33	45	48	30	93	32	29	22	25	105	81·8
5	—	—	—	—	—	—	—	28	—	—	113	77·3
3	40	32	52	55	37	104	31	27	23	28	105	74·3
5	36	31	47	50	36	101	36	28	23	28	113	73·6
4	41	36	—	—	—	—	30	28	22	—	104	70·7
7	38	33	51	54	—	99	31	24	20	29	99	82·2
2	40	35	47	—	37	91	34	30	28	28	110	77·5
7	37	31	42	46	38	97	38	26	24	29	98	77·2
6	39	33	—	—	36	97	34	28	25	25	117	78·5
6	40	35	44	49	38	96	32	27	18	28	105	69·4
7	42	34	49	52	33	94	33	22	26	27	113	79·0
5	38	33	50	54	37	98	36	32	25	28	118	75·7
7	41	35	45	52	39	103	34	28	29	33	105	74·0
3	41	34	54	57	42	109	35	32	24	30	96	74·4
6	42	35	—	—	—	97	38	32	26	—	104	71·8
7	41	31	—	—	38	101	32	31	18	31	108	75·1
5	42	34	46	47	42	100	38	26	25	31	114	71·8
4	40	34	51	54	39	105	33	27	22	30	111	74·6
6	37	35	46	50	—	95	35	29	22	—	106	81·0
4	40	33	49	52	42	101	35	30	27	30	120	78·0
1	36	31	40	44	31	87	36	30	14	26	107	77·1
2	39	31	44	48	37	89	31	27	26	30	113	76·9
6	40	34	45	48	—	92	34	24	26	—	111	75·1
6	40	34	47	52	37	99	39	35	23	31	108	72·6
4	43	32	46	49	41	103	35	27	26	29	113	78·4
—	41	33	—	—	—	—	31	29	24	—	110	79·4
4	37	33	50	53	41	106	36	30	22	30	103	78·3
4	39	30	44	48	34	108	32	29	26	28	111	76·7
8	40	34	49	—	40	97	37	28	26	29	112	72·6
6	39	33	47	50	42	104	34	31	21	33	113	73·2
4	39	34	53	57	38	99	27	23	20	30	99	78·5

FROM FERNAND VAZ, GABOON.

FEMALES.

B_2	St	B/L	H/L	B/H	$G'H/GB$	NB/NH	O_2/O_1	G_2/G_1	W_1	W_2	h_1	f	
23	102	75·5	—	—	70·4	54·3	86·1	80·0	—	—	—	—	Very young. Bas and inferior fro
28	103	75·8	69·7	108·7	68·5	61·0	87·5	72·9	—	—	—	—	Sagittal, coronal and inferior fro
33	108	74·3	74·3	100·0	68·6	49·0	87·2	76·9	—	—	—	—	Dentition good. Marked tempor resembling the
28	101	75·0	77·4	96·9	78·9	48·0	94·9	73·1	—	—	—	—	30—40 years. S
—	109	78·9	77·1	102·3	—	61·7	97·5	—	—	—	—	—	40—50 years. obelion. Lat ossicles of lamb occipital bulgin
26	121	87·7	81·6	107·5	72·6	54·8	82·5	82·0	—	—	—	—	30—40 years. Frontal process
33	119	77·1	68·3	112·8	65·6	48·9	84·2	74·0	—	—	—	—	Young adult. U and small ossic
31	109	77·7	78·3	99·2	72·4	53·2	82·5	—	105	97	30	45	30 years. Mastoi nent. Flatten
—	109	78·4	74·3	105·7	—	58·1	92·1	—	—	—	—	—	40 years. Both o arches removed
30	111	73·3	78·5	93·3	77·0	45·7	92·5	72·3	—	—	—	—	Frontal and me ossicles in lam
27	—	75·4	76·0	99·3	68·7	54·2	83·7	76·6	—	—	—	—	Aged. Flattening
31	106	70·5	79·2	89·1	70·5	54·0	85·7	71·2	—	—	—	—	30 years. Slight lambdoid sutu
29	107	75·4	72·6	103·9	71·1	56·6	87·2	78·4	105	86	—	42	About 30 years.
25	105	81·8	83·6	97·8	75·3	51·1	84·6	62·5	105	90	34	46	40—50 years. S
—	113	77·3	76·2	101·5	—	58·1	—	—	—	—	—	—	Aged. Teeth abs and inferior fro
28	105	74·3	74·3	100·0	71·0	50·0	80·0	67·3	—	—	—	—	Both orbits sliced
28	113	73·6	74·2	99·2	75·9	53·2	86·1	72·0	—	—	—	—	40—50 years. I
—	104	70·7	76·4	92·5	—	54·6	87·8	—	—	82	31	45	Aged. Coronal s Very small ma a bregmatic em
29	99	82·2	80·3	102·4	71·9	61·4	86·8	—	105	81	28	43	Aged? Some abs marked. Pron
28	110	77·5	72·8	106·5	56·4	51·2	87·5	—	113	91	32	47	Aged. Sagittal s
29	98	77·2	76·5	100·8	66·3	60·0	83·8	82·6	—	—	—	—	40 years. Sylvia
25	117	78·5	73·3	107·1	67·4	57·8	84·6	—	—	97	36	47	40—50 years. S Broad flat nas
28	105	69·4	73·9	94·0	72·1	56·5	87·5	77·6	107	89	25	40	Coronal suture m
27	113	79·0	78·4	100·8	68·8	61·4	81·0	63·5	113	91	31	47	Aged
28	118	78·7	77·9	97·2	70·8	52·1	86·8	68·5	—	—	—	—	Sylvian depressi lambdoid sutu
33	105	74·0	76·9	96·2	73·1	57·5	85·4	75·0	115	99	36	45	Metopic. Epipte marked
30	96	74·4	79·1	94·1	69·8	46·9	82·9	73·7	113	96	30	45	About 30 years.
—	104	71·8	70·6	101·6	63·8	54·2	83·3	—	110	89	32	43	Aged. Teeth abs Os pentagonale skull W. 219)
31	108	75·1	76·3	98·5	67·7	61·4	75·6	—	—	—	—	—	Frontal processe Small ossicles
31	114	71·8	75·1	95·5	66·0	54·4	81·0	89·4	105	101	35	48	About 20 years. and R. mastoi
30	111	74·6	76·3	97·7	72·8	54·6	85·0	72·2	110	100	29	47	
—	106	81·0	73·2	110·6	70·1	56·5	94·6	—	—	—	—	—	Aged. Teeth abs
30	120	78·0	75·7	103·1	66·3	57·1	82·5	80·8	—	—	—	—	Ossicle of lambda sutures. Fro Marked bregm
26	107	77·1	72·3	106·7	66·2	56·8	86·1	70·5	—	71	22	43	Young. Basilar
30	113	76·9	74·6	103·2	67·8	48·9	79·5	77·1	—	—	—	—	About 30 years. suture
—	111	75·1	74·0	101·6	64·1	59··	85·0	—	109	84	—	40	Aged. Teeth abs processes of sq suture at a poi
31	108	72·6	72·6	100·0	68·4	52·0	85·0	71·2	115	87	33	47	Young adult. M Frontal proces
29	113	78·4	74·4	105·3	54·0	60·0	74·4	83·7	119	84	30	44	Coronal suture this series. A
—	110	79·4	74·1	107·1	—	—	80·5	—	—	—	—	—	Aged. Frontal p sutures. Cleft canine is the s are united in t
30	103	78·3	78·3	100·0	—	51·1	89·2	77·4	114	90	28	41	Sylvian depressi
28	111	76·7	81·4	94·3	71·1	54·6	76·9	70·8	106	93	32	45	30—40. Good Long pointed rising to a very
29	112	72·6	78·2	92·9	72·5	59·6	85·0	—	—	—	33	41	Frontal processes
33	113	73·2	74·9	97·8	62·6	59·1	84·6	84·0	—	—	—	—	Young adult. T
30	99	78·5	74·1	106·0	76·1	52·2	87·2	66·7	111	93	23	43	About 39—40 yc

TABLE III *B*.

	Remarks
	Very young. Basilar synchondrosis. Permanent incisors deep in alveoli. Slight Sylvian depression and inferior frontal protuberance
	Sagittal, coronal and lambdoid sutures much obliterated. Teeth somewhat worn. Sylvian depression and inferior frontal protuberance. Subnasal prognathism. Flattening of obelion
	Dentition good. Sagittal, coronal and lambdoid sutures much obliterated. Subnasal prognathism. Marked temporal ridges. "Front bombé." Metopic crest rising to a marked eminence somewhat resembling the form in the Esquimaux
	30—40 years. Subnasal prognathism. R. orbit sliced
	40—50 years. Teeth absent and alveolar margin decayed. Small mastoid processes. Flattening of obelion. Lateral post-coronal depression. Frontal processes of squamous R. and L. Two small ossicles of lambda and others in R. and L. parieto-squamosal and masto-occipital sutures. Superior occipital bulging. Great width of base
	30—40 years. Obliteration of sagittal suture. Flattening of obelion. Left lateral incisor filed. Frontal processes of squamous (Fronto-squamosal suture 12 mm. long on L. and 7 mm. on R.)
	Young adult. Upper wisdom teeth in sockets; erupted in mandible. Small posterior sagittal ossicle and small ossicles in lambdoid suture and asterion
	30 years. Mastoid processes somewhat larger than is usual in this series, and nasal bones more prominent. Flattening of obelion
	40 years. Both orbits sliced, also margin of foramen magnum and mastoid processes cut and zygomatic arches removed as if by some sharp instrument
	Frontal and metopic eminences. Wisdom teeth not erupted. Basilar synchrondrosis. Small ossicles in lambdoid suture and asterion
	Aged. Flattening of obelion. Nasal bones wider at lower margin than at root
	30 years. Slight metopic and more marked frontal eminences. Flattening of obelion. Ossicle in lambdoid suture
	About 30 years. Subnasal prognathism
	40—50 years. Some obliteration of sagittal and lambdoid sutures and flattening of obelion
	Aged. Teeth absent and alveolar margin absorbed. Sagittal suture obliterated. Sylvian depression and inferior frontal protuberance
	Both orbits sliced. Ossicles in R. and L. masto-occipital sutures. Small nasal bones
	40—50 years. Infantile forehead. Coronal suture obliterated at lower ends
	Aged. Coronal suture obliterated. Obelion depressed. Teeth absent and alveolar margin absorbed. Very small mastoid processes. Metopic and frontal eminences and a slight metopic crest rising to a bregmatic eminence (resembling No. 5)
	Aged? Some absorption of alveolar margin. Sutures open. Flattening of obelion. Temporal ridges marked. Prominent nasal bones
	Aged. Sagittal suture completely obliterated, and coronal at pterion. Broad flat nose
	40 years. Sylvian depression between temporal protuberance and slight inferior frontal protuberance
	40—50 years. Some obliteration of coronal suture. Molars well ground. Flattening of obelion. Broad flat nasal bones
	Coronal suture much obliterated. Flattening of obelion. Very small mastoid processes
	Aged
	Sylvian depression and inferior frontal protuberance. Some obliteration of coronal, sagittal and lambdoid sutures. Flat nose
	Metopic. Epipteric on L. Post-coronal depression. Flattening of obelion. Frontal eminences marked
	About 30 years. Ossicles in lambdoid and masto-occipital sutures
	Aged. Teeth absent and alveolar margin much absorbed. Sutures distinct. Interparietal tripartite. Os pentagonale and R. os triangulare separate (see Plate XXXVII, *Biometrika*, Vol. III. Part II. special skull W. 219)
	Frontal processes of squamous (Fronto-squamosal sutures 12 mm. long on R. and 7 mm. on L.). Small ossicles in lambdoid suture and R. and L. asterion
	About 20 years. Frontal and metopic eminences. Mastoid processes larger than usual in this series and R. mastoid deeply grooved
	Aged. Teeth absent and alveolar rim much absorbed. Obliteration of coronal suture
	Ossicle of lambda 28 mm. wide and 16 mm. high. Others in lambdoid and R. and L. masto-parietal sutures. Frontal processes of squamous. (Fronto-squamosal sutures 18 mm. long R. and L.) Marked bregmatic eminence
	Young. Basilar synchondrosis. Second molar just appearing
	About 30 years. Metopic. Slight post-coronal depression. Sylvian depression. Ossicles in lambdoid suture
	Aged. Teeth absent. Alveolar margin absorbed. Lower ends of coronal suture obliterated. Frontal processes of squamous R. and L. Posterior portion of sagittal suture irregular and joins lambdoid suture at a point 40 mm. to L. of median line. (See figure in text.)
	Young adult. Male? Flattening of obelion. Mandible relatively prominent. Subnasal prognathism. Frontal process of squamous on L.
	Coronal suture entirely and sagittal largely obliterated. Formation of nose as in No. 11 a male of this series. Antero-inferior angle of parietal synostosed with sphenoid and frontal
	Aged. Frontal processes of squamous R. and L. Small ossicles in masto-parietal and masto-occipital sutures. Cleft of palate with complete absence of praemaxilla, but anterior to the socket of the R. canine is the socket of a rudimentary tooth—not seen on L. The posterior portions of the palate are united in the median line. (See Plate accompanying this paper.)
	Sylvian depression and inferior frontal protuberance
	30—40. Good teeth, well ground. Front incisors slightly filed. Flattening of obelion. Flat nose. Long pointed mastoid processes. Frontal processes of squamous R. and L. Slight metopic crest rising to a very marked bregmatic eminence (as in No. 38)
	Frontal processes of squamous (Fronto-squamosal suture 22 mm. long on R., 17 mm. long on L.)
	Young adult. Third molar just appearing. Small epipteric on R. and ossicles at R. and L. asterion
	About 39—40 years. Sutures open. Very prognathous

No.	*C*	*F*	*L*	*B*	*B′*	*H*	*LB*	*Q*	*S*	S_1	S_2	S_3	S_3'	*U*	*G′H*	*GB*	*J*	*NH*	*NB*
1	1513	185	186	137	98	134	105	306	371	137	126	108	96	520	69	95	—	48	26
3	1622	186	188	141	104	142	103	315	387	149	123	115	102	525	66	99	139	48	29
4	1448	178	180	139	103	140	103	318	360	126	127	107	94	510	75	106	142	47	28
5	1453	183	184	140	106	141	106	308	377	130	130	117	98	513	69	92	134	50	29
14	1270	174	174	131	93	128	93	330	360	125	125	110	107	488	67	94	125	46	27
22	1330	177	179	134	94	128	100	305	365	125	120	115	102	498	—	98	126	45	25
23	1440	177	179	143	101	139	103	317	369	125	127	117	102	512	68	91	131	50	27
24	1335	178	181	132	100	136	106	298	357	120	122	115	101	492	—	99	136	51	29
26	1340	181	182	131	96	141	107	—	370	123	123	124	109	—	73	105	132	52	26
27	1520	180	182	148	98	143	103	315	382	143	120	119	100	518	55	95	136	44	27
28	1343	168	175	137	91	132	95	305	358	122	125	111	98	480	67	103	135	45	27
29	1640	185	186	143	97	138	101	320	382	130	139	113	101	524	60	98	132	47	27
30	1470	186	187	135	95	135	102	315	370	134	136	100	89	512	71	95	128	52	31
31	1350	175	176	145	101	131	98	312	361	119	119	123	102	509	70	93	127	47	25
32	1485	187	191	143	101	142	106	310	375	132	128	115	100	524	—	105	140	49	27
35	1600	190	190	134	101	142	107	302	385	130	138	117	103	518	71	101	133	51	27
12*a*	—	175	176	132	96	129	95	—	—	—	—	—	—	—	68	—	125	44	23
20	—	184	185	143	99	139	101	—	—	—	—	—	—	—	60	100	132	47	27

No.	*C*	*F*	*L*	*B*	*B′*	*H*	*LB*	*Q*	*S*	S_1	S_2	S_3	S_3'	*U*	*G′H*	*GB*	*J*	*NH*	*NB*
2	1195	168	168	126	89	133	100	293	345	120	112	113	101	468	67	—	—	45	25
6	1130	160	161	134	89	131	94	294	330	119	116	95	82	465	64	93	—	47	25
7	1190	166	168	132	89	130	94	285	345	115	130	100	90	473	57	89	118	46	27
8	—	162	162	122	91	—	—	282	333	113	114	106	81	452	55	82	106	35	21
9	1165	167	167	127	89	121	91	296	345	125	117	103	90	468	56	81	—	41	34
10	1134	168	169	123	90	129	96	293	333	115	115	103	89	463	56	84	114	43	24
12*b*	1095	166	167	130	97	128	93	298	347	121	115	111	99	475	65	91	124	44	24
15	1308	188	190	129	95	139	105	310	335	115	115	105	99	515	71	92	120	52	27
17	1150	161	164	124	88	131	93	285	343	120	121	102	92	462	60	93	118	41	24
18	—	174	175	131	92	126	96	302	355	122	123	110	93	493	70	—	—	—	—
19	1255	180	181	130	89	130	101	292	360	120	120	120	100	499	64	87	120	46	25
21	1405	176	176	132	97	137	102	308	358	120	135	103	91	490	71	96	127	47	27
33	1295	178	178	129	101	137	102	300	360	125	120	115	103	495	51	91	118	47	27
34	1268	163	163	133	98	129	96	300	342	115	123	106	96	480	65	86	123	48	25
36	1338	166	167	137	94	132	95	300	360	124	131	105	94	482	65	97	127	50	25
37	1444	176	179	141	91	141	102	320	370	123	140	107	101	500	64	93	122	46	27
38	1315	177	177	138	100	133	100	312	365	137	121	107	93	503	57	100	131	47	28
13	1235	167	167	131	97	134	98	300	347	126	112	109	97	478	68	96	123	47	25
39	1165	168	169	130	92	126	95	295	343	126	114	103	91	472	61	90	114	44	22

MALES.

	NH	NB	O_1	O_2	G_1'	G_1	G_2	GL	*fml*	*fmb*	Dacry	B_2	*St*
-	48	26	42	35	47	—	44	103	37	32	27	30	118
9	48	29	45	38	47	—	41	100	38	31	27	35	123
2	47	28	45	35	50	56	43	109	42	33	30	33	117
4	50	29	41	34	57	63	39	110	35	33	26	30	116
5	46	27	42	34	51	—	37	100	34	25	26	24	111
6	45	25	42	36	—	—	—	101	32	26	22	—	105
1	50	27	41	31	50	—	40	100	33	30	28	32	118
6	51	29	42	40	50	52	32	—	34	31	22	29	106
2	52	26	43	34	50	59	45	105	36	30	23	30	117
6	44	27	44	35	45	—	—	92	34	28	23	—	123
5	45	27	42	34	52	55	44	105	37	30	23	37	107
2	47	27	40	38	44	—	—	93	37	34	25	—	121
8	52	31	41	33	54	57	40	110	39	33	28	31	119
7	47	25	42	37	50	—	43	102	32	25	30	32	122
0	49	27	42	32	—	—	—	—	35	29	—	30	116
3	51	27	43	35	54	59	40	110	35	30	27	35	111
5	44	23	41	35	—	—	—	101	—	—	22	—	—
2	47	27	39	37	—	—	—	—	—	—	25	—	—

FEMALES.

J	NH	NB	O_1	O_2	G_1'	G_1	G_2	GL	*fml*	*fmb*	Dacry	B_2	*St*
—	45	25	38	33	45	51	36	100	36	31	20	30	90
—	47	25	41	36	—	—	37	96	36	31	22	27	103
18	46	27	39	32	—	—	40	82	34	28	23	37	102
06	35	21	35	32	41	45	39	—	—	—	23	31	99
—	41	34	38	32	37	40	31	90	28	24	21	27	103
14	43	24	40	34	45	—	—	95	35	27	22	28	103
24	44	24	41	34	47	52	—	102	30	25	23	—	113
20	52	27	41	37	52	56	35	109	31	30	23	32	108
18	41	24	39	33	50	56	40	100	36	28	26	31	109
—	—	—	40	37	—	—	—	102	41	30	23	—	101
20	46	25	40	36	46	51	36	100	36	28	20	29	97
27	47	27	42	37	49	51	42	106	34	32	25	26	109
18	47	27	42	35	—	—	—	89	32	27	24	—	113
23	48	25	44	34	47	—	43	96	35	30	23	29	108
27	50	25	39	34	44	52	36	96	34	31	22	29	106
22	46	27	41	32	52	58	33	109	33	29	20	30	112
31	47	28	43	31	—	—	—	97	32	28	23	—	110
23	47	25	39	35	—	—	—	99	30	28	25	—	111
14	44	22	40	34	52	57	—	99	34	26	23	—	104

1880 SERIES. CRANIA FROM FERNAND VAZ, GABO

MALES.

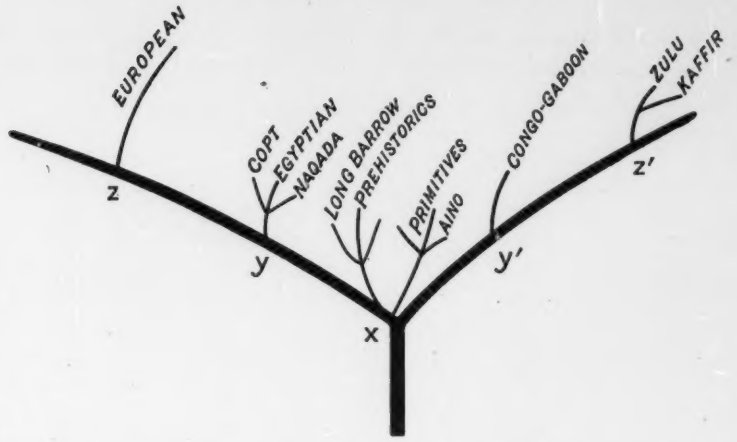
fml	*fmb*	Dacry	B_2	*St*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH*	O_2/O_1	G_2/G_1	W_1	W_2
37	32	27	30	118	73·7	72·0	102·2	72·6	54·2	83·3	—	121	96
38	31	27	35	123	75·0	75·5	99·3	66·7	60·4	84·4	—	129	110
42	33	30	33	117	77·2	77·8	99·3	70·8	59·6	77·8	76·8	125	102
35	33	26	30	116	76·1	76·6	99·3	75·0	58·0	82·9	61·9	123	95
34	25	26	24	111	75·3	73·6	102·3	71·3	58·7	81·0	—	113	93
32	26	22	—	105	74·9	71·5	104·7	—	55·6	85·7	—	—	—
33	30	28	32	118	79·9	77·7	102·9	74·7	54·0	75·6	—	120	114
34	31	22	29	106	72·9	75·1	97·1	—	56·9	95·2	61·5	115	83
36	30	23	30	117	72·0	77·5	92·9	69·5	50·0	79·1	76·3	—	—
34	28	23	—	123	81·3	78·6	103·5	57·9	61·4	79·6	—	—	—
37	30	23	37	107	78·3	75·4	103·8	65·1	60·0	81·0	80·0	—	—
37	34	25	—	121	76·9	74·2	103·6	61·2	57·5	95·0	—	—	—
39	33	28	31	119	72·2	72·2	100·0	74·7	59·6	80·5	70·2	—	—
32	25	30	32	122	82·4	74·4	110·7	75·3	53·2	88·1	—	—	—
35	29	—	30	116	74·9	74·4	100·7	—	55·1	76·2	—	—	—
35	30	27	35	111	70·5	74·7	94·4	70·3	52·9	81·4	67·8	—	—
—	—	22	—	—	75·0	73·3	102·3	—	52·3	85·4	—	—	—
—	—	25	—	—	76·9	74·7	102·9	60·0	57·5	94·9	—	—	—

FEMALES.

fml	*fmb*	Dacry	B_2	*St*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH*	O_2/O_1	G_2/G_1	W_1	W_2
36	31	20	30	90	75·0	79·2	94·7	—	55·6	86·8	70·6	104	85
36	31	22	27	103	83·2	81·4	102·3	68·8	53·2	87·8	—	102	93
34	28	23	37	102	78·6	77·4	101·5	64·0	58·7	82·1	—	—	—
—	—	23	31	99	75·3	—	—	67·1	60·0	91·4	86·7	90	82
28	24	21	27	103	76·1	72·5	105·0	69·1	82·9	84·2	77·5	—	—
35	27	22	28	103	72·8	76·3	95·4	66·7	55·8	85·0	—	107	80
30	25	23	—	113	77·8	76·7	101·6	71·4	54·6	82·9	—	109	91
31	30	23	32	108	67·9	73·2	92·8	77·2	51·9	90·2	62·5	—	—
36	28	26	31	109	75·6	79·9	94·7	64·5	58·5	84·6	71·4	116	99
41	30	23	—	101	74·9	72·0	104·0	—	—	92·5	—	—	—
36	28	20	29	97	71·8	71·8	100·0	73·6	54·4	90·0	70·6	—	—
34	32	25	26	109	75·0	77·8	96·4	74·0	57·5	88·1	82·4	—	—
32	27	24	—	113	72·5	77·0	94·2	56·0	57·5	83·3	—	—	—
35	30	23	29	108	81·6	79·1	103·1	75·6	52·1	77·3	—	—	—
34	31	22	29	106	82·0	79·0	103·8	67·0	50·0	87·2	69·2	—	—
33	29	20	30	112	78·8	78·8	100·0	68·8	58·7	78·1	56·9	112	98
32	28	23	—	110	78·0	75·1	103·8	57·0	59·6	72·1	—	113	91
30	28	25	—	111	78·4	80·2	97·8	70·8	53·2	89·7	—	104	92
34	26	23	—	104	76·9	74·6	103·2	67·8	50·0	85·0	—	—	—

G_1	W_1	W_2	h_1	f	Remarks
-	121	96	35	47	About 50 years old. Obliteration of sagittal suture. Flattening of obelion
-	129	110	30	40	About 50 years old. Obliteration of sagittal suture. Inion well marked. Slight post-coronal depression. Pterion in K
·8	125	102	27	42	Ossicles in lambdoid suture. Posterior sagittal ossicle. Subnasal prognathism
·9	123	95	33	50	Well marked inion and mastoids. Ossicles in lambdoid suture. Coronal and sagittal sutures nearly obliterated. Marked temporal ridges. Frontal portion of orbits slender. R. frontal process of squamous 7 mm. long
-	113	93	35	50	20—30 years old. Flattening of obelion. Epipteric bones, 28 mm. R., 19 mm. L. Very small mastoids. Ossicle in lambdoid suture
-	—	—	—	—	Sutures distinct, but teeth absent and alveolar margin absorbed. Lambdoid ossicles
-	120	114	30	48	Very small mastoids and inion. Ossicles in lambdoid suture and at asterion. Sylvian depression
·5	115	83	32	47	Obliteration of sagittal suture. Sylvian depression. Slight bathrocephaly
·3	—	—	—	—	Flattening of obelion. Temporal ridges marked
-	—	—	—	—	Sutures distinct, teeth absent, alveolar margin absorbed. Ossicles in lambdoid
·0	—	—	—	—	Very retreating forehead. Nasal bones depressed at root. Prominent overhanging frontal. Subnasal prognathism
-	—	—	—	—	40 years old?
·2	—	—	—	—	Flattening of obelion and pterion. Both orbits sliced
-	—	—	—	—	Flattening of obelion and pterion. R. pterion in K. Flat nasal bones
-	—	—	—	—	Squamous suture straight and nearly horizontal. Inion and mastoids immature. Frontal process of squamous extends beyond sphenoid R. and L. Central incisors drawn
·8	—	—	—	—	Wisdom tooth just appearing. Slight obliteration of coronal suture. Subnasal prognathism
-	—	—	—	—	Flattening of obelion. ♂? Aged about 30
-	—	—	—	—	Edentulous. Flattening of obelion. Sutures closed. Sylvian depression. Slight bathrocephaly

h_1/G_1	W_1	W_2	h_1	f	Remarks
0·6	104	85	30	46	Weatherworn. Young adult female
—	102	93	26	46	Middle aged
—	—	—	—	—	Squamous and coronal sutures obliterated. Aged. Four incisors drawn
6·7	90	82	30	45	About 14 years old. Sagittal suture obliterated. Large interparietal, 86 mm. broad, 43 mm. long
7·5	—	—	—	—	Under 14. Flattening of obelion. Very marked metopic eminence
—	107	80	22	47	Edentulous. Aged. Flattening of obelion. Ossicle in lambdoid. Sylvian depression. Epipteric on R.
—	109	91	27	56	Sutures open, except end of coronal at pterion. Subnasal prognathism
2·5	—	—	—	—	Coronal much and sagittal entirely obliterated. Edentulous. Aged. Marked
1·4	116	99	32	45	Middle aged. Flattening of obelion [frontal eminences
—	—	—	—	—	Middle aged. Sutures open [10 mm. long
0·6	—	—	—	—	Aged. Sagittal and coronal much obliterated; frontal process of squamous
2·4	—	—	—	—	Middle aged. Sutures distinct. Metopic eminence. Subnasal prognathism
—	—	—	—	—	Edentulous. Aged. Pterion in K
—	—	—	—	—	Middle aged. Sutures open. External angular process *gnawed* or filed
9·2	—	—	—	—	Middle aged. Medium incisors drawn. Flattened obelion
6·9	112	98	27	47	Old. Sagittal nearly obliterated. Lateral post-coronal depression. Frontal process of squamous R. and L.
—	113	91	21	44	Edentulous. Old. Flattening of obelion. Flat nose
—	104	92	38	51	Middle aged. Flattening of obelion
—	—	—	—	—	Four incisors drawn. Only one molar left on jaw. Sutures open. Sylvian depression. Ossicle in asterion, and on R. lambdoid. Very small mastoids. Aged female



Ancestral races on the main stems, as at x, y z, y' z', may be at present unknown. There may be many more branches to the main stems, and there are undoubtedly several other main stems, but the Congo-Gaboon crania serve a useful purpose in definitely linking up Egyptian and negroid crania, and thus in ultimately associating the Negro races with the European. If European and Egyptian admit of a common ancestry, the differentiation of Egyptian and Congo-Gaboon is no greater, and from the latter to the Kaffir-Zulu is in no sense a wider step; thus we no longer need to question a common ancestry for Negro and Caucasian, or to suppose a double origin for man. If in some respects the Kaffir-Zulu is closer to the European than the Congo-Gaboon group, this may be attributed to the development from y to z being to some extent similar to that from y' to z'. European and Kaffir may be far apart horizontally on the tree, but vertically their divergence may be less. In mere length of head the Zulu-Kaffir is closer to the European than the Congoese, but no one who studies carefully the tables given in this paper will believe the link lies otherwise than from Zulu-Kaffir to Congo-Gaboon, and from Congo-Gaboon to Egyptian, and Egyptian to European.

The main purpose of this memoir was stated to be one of suggestion; to suggest how much more extensive material is needed and how crying is the need for standardisation in the measurement of it when procured. If the hypothesis of this concluding section be contradicted by the careful measurement of larger series, this memoir will still have served most effectively its purpose for it will have led to a recognition of the need for longer series. And no more ambitious purpose ever occurred to the mind of Dr Crewdson Benington himself when he started at my suggestion the measurement of the few available crania from the Gaboon and Congo, than this one of indicating how suggestive the study of the negro skull is, and how incomplete it must remain, until more ample material is available.

Table to pass from measured index $\beta = 100\,(arc - chord)/chord$ *of a curve to the index and may be closely represented by a common catenary. Suggested use: to pass*

Values of α for given values of β as argument.

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
13	23·1	23·2	23·2	23·3	23·4	23·5	23·6	23·7	23·8	23·9
14	24·0	24·1	24·2	24·3	24·4	24·5	24·6	24·7	24·7	24·8
15	24·9	25·0	25·1	25·2	25·3	25·4	25·5	25·6	25·6	25·7
16	25·8	25·9	26·0	26·1	26·2	26·3	26·4	26·4	26·5	26·6
17	26·7	26·8	26·9	27·0	27·0	27·1	27·2	27·3	27·4	27·5
18	27·6	27·7	27·7	27·8	27·9	28·0	28·1	28·2	28·3	28·3
19	28·4	28·5	28·6	28·7	28·7	28·8	28·9	29·0	29·1	29·2
20	29·2	29·3	29·4	29·5	29·6	29·6	29·7	29·8	29·9	30·0
21	30·0	30·1	30·2	30·3	30·4	30·4	30·5	30·6	30·7	30·8
22	30·8	30·9	31·0	31·1	31·2	31·2	31·3	31·4	31·5	31·6
23	31·6	31·7	31·8	31·9	31·9	32·0	32·1	32·2	32·3	32·3
24	32·4	32·5	32·6	32·6	32·7	32·8	32·9	32·9	33·0	33·1
25	33·2	33·3	33·3	33·4	33·5	33·6	33·6	33·7	33·8	33·9
26	33·9	34·0	34·1	34·2	34·2	34·3	34·4	34·5	34·5	34·6
27	34·7	34·8	34·8	34·9	35·0	35·1	35·1	35·2	35·3	35·3
28	35·4	35·5	35·6	35·6	35·7	35·8	35·9	35·9	36·0	36·1
29	36·2	36·2	36·3	36·4	36·4	36·5	36·6	36·7	36·7	36·8
30	36·9	36·9	37·0	37·1	37·2	37·2	37·3	37·4	37·5	37·5
31	37·6	37·7	37·7	37·8	37·9	38·0	38·0	38·1	38·2	38·2
32	38·3	38·4	38·4	38·5	38·6	38·7	38·7	38·8	38·9	38·9
33	39·0	39·1	39·2	39·2	39·3	39·4	39·4	39·5	39·6	39·6
34	39·7	39·8	39·8	39·9	40·0	40·1	40·1	40·2	40·3	40·3
35	40·4	40·5	40·5	40·6	40·7	40·7	40·8	40·9	41·0	41·0
36	41·1	41·2	41·2	41·3	41·4	41·4	41·5	41·6	41·6	41·7
37	41·8	41·8	41·9	42·0	42·0	42·1	42·2	42·2	42·3	42·4
38	42·4	42·5	42·6	42·6	42·7	42·8	42·9	42·9	43·0	43·1
39	43·1	43·2	43·3	43·3	43·4	43·5	43·5	43·6	43·7	43·7
40	43·8	43·9	43·9	44·0	44·1	44·1	44·2	44·3	44·3	44·4
41	44·5	44·5	44·6	44·6	44·7	44·8	44·8	44·9	45·0	45·0
42	45·1	45·2	45·2	45·3	45·4	45·4	45·5	45·6	45·6	45·7
43	45·8	45·8	45·9	46·0	46·0	46·1	46·2	46·2	46·3	46·4
44	46·4	46·5	46·5	46·6	46·7	46·7	46·8	46·9	46·9	47·0
45	47·1	47·1	47·2	47·3	47·3	47·4	47·5	47·5	47·6	47·6
46	47·7	47·8	47·8	47·9	48·0	48·0	48·1	48·2	48·2	48·3
47	48·4	48·4	48·5	48·5	48·6	48·7	48·7	48·8	48·9	48·9
48	49·0	49·1	49·1	49·2	49·2	49·3	49·4	49·4	49·5	49·6
49	49·6	49·7	49·8	49·8	49·9	49·9	50·0	50·1	50·1	50·2
50	50·3	50·3	50·4	50·5	50·5	50·6	50·6	50·7	50·8	50·8
51	50·9	51·0	51·0	51·1	51·1	51·2	51·3	51·3	51·4	51·5
52	51·5	51·6	51·6	51·7	51·8	51·8	51·9	52·0	52·0	52·1
53	52·1	52·2	52·3	52·3	52·4	52·5	52·5	52·6	52·6	52·7
54	52·8	52·8	52·9	53·0	53·0	53·1	53·1	53·2	53·3	53·3
55	53·4	53·4	53·5	53·6	53·6	53·7	53·8	53·8	53·9	53·9
56	54·0	54·1	54·1	54·2	54·3	54·3	54·4	54·4	54·5	54·6
57	54·6	54·7	54·7	54·8	54·9	54·9	55·0	55·0	55·1	55·2
58	55·2	55·3	55·4	55·4	55·5	55·5	55·6	55·7	55·7	55·8
59	55·8	55·9	56·0	56·0	56·1	56·1	56·2	56·3	56·3	56·4
60	56·5	56·5	56·6	56·6	56·7	56·8	56·8	56·9	56·9	57·0
61	57·1	57·1	57 2	57·2	57·3	57·4	57·4	57·5	57·5	57·6
62	57·7	57·7	57·8	57·8	57·9	58·0	58·0	58·1	58·1	58·2
63	58·3	58·3	58·4	58·4	58·5	58·6	58·6	58·7	58·7	58·8
64	58·9	58·9	59·0	59·0	59·1	59·2	59·2	59·3	59·3	59·4

$\alpha = 100$ *subtense/chord, on the assumption that the curve is symmetrical about the subtense from callipers and tape measurements of the nasal bridge to the ratio of "rise" to "span."*

Values of α for given values of β as argument.

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
65	59·5	59·5	59·6	59·6	59·7	59·8	59·8	59·9	59·9	60·0
66	60·1	60·1	60·2	60·2	60·3	60·4	60·4	60·5	60·5	60·6
67	60·7	60·7	60·8	60·8	60·9	61·0	61·0	61·1	61·1	61·2
68	61·3	61·3	61·4	61·4	61·5	61·6	61·6	61·7	61·7	61·8
69	61·9	61·9	62·0	62·0	62·1	62·1	62·2	62·3	62·3	62·4
70	62·4	62·5	62·6	62·6	62·7	62·7	62·8	62·9	62·9	63·0
71	63·0	63·1	63·1	63·2	63·3	63·3	63·4	63·4	63·5	63·6
72	63·6	63·7	63·7	63·8	63·9	63·9	64·0	64·0	64·1	64·1
73	64·2	64·3	64·3	64·4	64·4	64·5	64·6	64·6	64·7	64·7
74	64·8	64·9	64·9	65·0	65·0	65·1	65·1	65·2	65·3	65·3
75	65·4	65·4	65·5	65·6	65·6	65·7	65·7	65·8	65·8	65·9
76	66·0	66·0	66·1	66·2	66·2	66·3	66·3	66·4	66·4	66·5
77	66·5	66·6	66·7	66·7	66·8	66·8	66·9	66·9	67·0	67·1
78	67·1	67·2	67·2	67·3	67·4	67·4	67·5	67·5	67·6	67·6
79	67·7	67·8	67·8	67·9	67·9	68·0	68·0	68·1	68·2	68·2
80	68·3	68·3	68·4	68·5	68·5	68·6	68·6	68·7	63·7	68·8
81	68·9	68·9	69·0	69·0	69·1	69·1	69·2	69·3	69·3	69·4
82	69·4	69·5	69·5	69·6	69·7	69·7	69·8	69·8	69·9	70·0
83	70·0	70·1	70·1	70·2	70·2	70·3	70·4	70·4	70·5	70·5
84	70·6	70·6	70·7	70·8	70·8	70·9	70·9	71·0	71·0	71·1
85	71·2	71·2	71·3	71·3	71·4	71·4	71·5	71·6	71·6	71·7
86	71·7	71·8	71·8	71·9	72·0	72·0	72·1	72·1	72·2	72·2
87	72·3	72·4	72·4	72·5	72·5	72·6	72·6	72·7	72·8	72·8
88	72·9	72·9	73·0	73·0	73·1	73·2	73·2	73·3	73·3	73·4
89	73·4	73·5	73·6	73·6	73·7	73·7	73·8	73·8	73·9	73·9
90	74·0	74·1	74·1	74·2	74·2	74·3	74·3	74·4	74·5	74·5
91	74·6	74·6	74·7	74·7	74·8	74·9	74·9	75·0	75·0	75·1
92	75·1	75·2	75·3	75·3	75·4	75·4	75·5	75·5	75·6	75·6
93	75·7	75·8	75·8	75·9	75·9	76·0	76·0	76·1	76·2	76·2
94	76·3	76·3	76·4	76·4	76·5	76·6	76·6	76·7	76·7	76·8
95	76·8	76·9	76·9	77·0	77·1	77·1	77·2	77·2	77·3	77·3
96	77·4	77·5	77·5	77·6	77·6	77·7	77·7	77·8	77·8	77·9
97	78·0	78·0	78·1	78·1	78·2	78·2	78·3	78·3	78·4	78·5
98	78·5	78·6	78·6	78·7	78·7	78·8	78·9	78·9	79·0	79·0
99	79·1	79·1	79·2	79·2	79·3	79·4	79·4	79·5	79·5	79·6
100	79·6	79·7	79·8	79·8	79·9	79·9	80·0	80·0	80·1	80·1
101	80·2	80·3	80·3	80·4	80·4	80·5	80·5	80·6	80·6	80·7
102	80·8	80·8	80·9	80·9	81·0	81·0	81·1	81·1	81·2	81·3
103	81·3	81·4	81·4	81·5	81·5	81·6	81·6	81·7	81·8	81·8
104	81·9	81·9	82·0	82·0	82·1	82·1	82·2	82·3	82·3	82·4
105	82·4	82·5	82·5	82·6	82·6	82·7	82·8	82·8	82·9	82·9
106	83·0	83·0	83·1	83·1	83·2	83·3	83·3	83·4	83·4	83·5
107	83·5	83·6	83·6	83·7	83·8	83·8	83·9	83·9	84·0	84·0
108	84·1	84·1	84·2	84·3	84·3	84·4	84·4	84·5	84·5	84·6
109	84·6	84·7	84·8	84·8	84·9	84·9	85·0	85·0	85·1	85·1
110	85·2	85·3	85·3	85·4	85·4	85·5	85·5	85·6	85·6	85·7
111	85·8	85·8	85·9	85·9	86·0	86·0	86·1	86·1	86·2	86·2
112	86·3	86·4	86·4	86·5	86·5	86·6	86·6	86·7	86·7	86·8
113	86·9	86·9	87·0	87·0	87·1	87·1	87·2	87·2	87·3	87·4
114	87·4	87·5	87·5	87·6	87·6	87·7	87·7	87·8	87·8	87·9
115	88·0	88·0	88·1	88·1	88·2	88·2	88·3	88·3	88·4	88·5
116	88·5	88·6	88·6	88·7	88·7	88·8	88·8	88·9	88·9	89·0

ON THE RELATION OF STATURE AND WEIGHT TO PIGMENTATION.

By ETHEL M. ELDERTON, Galton Eugenics Laboratory.

It is well-known that pigmentation differs widely from race to race. Further stature certainly and weight probably are racial characters. We might therefore anticipate that in investigations of stature and weight we should find a differentiation in these characters associated with the different pigmentation classes within the population of any town. This would arise not only because the chief classes of pigmentation are supposed to have originated from separate racial types, but because in many large towns there are considerable foreign elements not yet blended with the native population, e.g. in England and Scotland, we find Italian, Irish, Jewish and even Polish groups. In many cases these non-native elements form a considerable percentage of the population, and if, as often happens, they are the poorer section, then we may discover a relation between physique and environment which is not causal but racial in origin. Little attention seems to have been paid to this point in considering, for example, the effects of overcrowding on the weight and stature of children. The children who dwell in four-roomed homes are not necessarily of the same race as those who live in one or two-roomed homes. In a town like Glasgow, the Irish and Italian contingents are possibly more likely to be in one or two-roomed homes than the native Scottish, and a similar state of affairs probably occurs in those London school districts which contain large foreign contingents.

Before we have investigated the racial homogeneity of a population or before we have shown that racial differentiation does not indicate any economic differentiation, we ought not to lay much stress on the association of physique—measured by stature and weight—with environment. There is little doubt that a comparative study of native and foreign elements in our big towns, from the standpoints of both physique and economic condition would be of much service. Meanwhile it occurred to me that if an investigation were made of the relation of stature and weight to pigmentation a positive result would indicate the general importance of the problem. A negative result would show that within a homogeneous population pigmentation had little or no relation to stature and weight.

It would not demonstrate that a large foreign element might not modify the conclusion, unless we were absolutely certain that in the material considered we were really dealing with a heterogeneous population.

In the present inquiry I have worked with Glasgow data, which I selected because I thought that I should include considerable Italian and Irish elements. For the schools, however, where the stature and weight were known as well as the pigmentation, the surnames do not suggest that the foreign element is extensive. Hence I think my inquiry is concerned only with the problem of whether in a fairly homogeneous population* pigmentation is related to stature and weight.

Owing to the kindness of Sir John Struthers the Educational Board of Glasgow placed at our disposal schedules containing particulars of 72,857 Glasgow children attending the public schools of that town in 1905. Among the information given were the height and weight of these children. Now two years previously (1903) Mr Tocher obtained in his great pigmentation survey of the school children of Scotland the hair and eye colour of the Glasgow children (see *Biometrika*, Vol. VI. p. 131 *et seq.*). It occurred to me therefore that it might be feasible to combine these two records if we could identify the children in the two surveys. The colour of the eye and hair would not have changed much in the interval. Mr Tocher most kindly placed his material at my disposal.

The main difficulty was the identification of the children in the two surveys. In the 1905 records the ages of the children were given to the nearest year, but in the 1903 survey this does not appear to have been the case, the year of age being stated. Further, all the schools were not necessarily examined at the same time and the age was therefore only the roughest guide to identity. It was finally decided that only those children should be included whose initials or Christian names were given in the pigmentation inquiry†, and the first part of the work was to place the hair and eye records from the 1903 survey on the 1905 schedules containing the physical characters. Of the schools that sent returns of pigmentation in 1903 and physical measurements in 1905, only five of the earlier survey‡ gave the Christian names or initials of the children and my results are based on those schools only; the schools are Grove Street, Provanside, Alexandra Parade, Abbotsford and Wolseley Street.

Under the school board of Glasgow the City of Glasgow is divided into ten *educational districts* and the schools we are considering are included in the following districts:

* By homogeneous population I understand one which *may* be genetically a blend of several races, but has not unmixed foreign elements.

† In many schools only the child's surname had been written down, and even in the case of rare names, the age data were not close enough to admit of absolutely certain identification.

‡ The surnames were taken in the 1903 record with a view to some estimate of foreign parentage, and not for the purpose of identifying the child. It was naturally not foreseen how useful the full names might be at a later date and for a different inquiry.

Grove Street in Milton District; Provanside in St Rollox; Alexandra Parade in Dennistown; Abbotsford in Gorbals and Wolseley Street in Hutchesontown.

Hair was classified in five shades:

(1) all shades of red; light red, bright red, and sandy red, &c.

(2) all shades of fair; flaxen, white, and golden yellow.

(3) chestnut brown, dull brown, and all shades not red, fair, or dark.

(4) dark includes very dark brown (looking black at a moderate distance) and black.

(5) jet black.

Eyes were classified in four shades:

(1) deep blue or pure blue.

(2) light blue and light grey eyes.

(3) not blue, light grey, or brown; they are called medium eyes and include grey, green, orange and other mixed shades.

(4) hazel-brown, dark brown, and dark eyes generally.

Table I. gives the distribution of hair and eye colour in (*a*) Scotland, (*b*) Glasgow City, (*c*) the selected schools.

TABLE I. *Percentages.*

		Hair Colour					Eye Colour			
		Red	Fair	Medium	Dark	Black	Blue	Light	Medium	Dark
a	Boys ...	5·5	24·9	43·3	25·0	1·2	14·7	30·3	32·7	22·3
	Girls ...	5·1	27·4	40·9	25·4	1·2	14·9	30·3	32·1	22·8
b	Boys ...	5·3	21·8	45·7	26·1	1·1	10·7	30·7	34·1	24·5
	Girls ...	4·8	21·3	44·1	28·6	1·2	11·7	29·9	34·0	24·4
c	Boys ...	5·2	22·2	47·2	24·5	·9	11·0	31·0	37·2	20·8
	Girls ...	5·2	20·5	49·1	24·2	1·1	10·7	31·4	36·2	21·7

It will be noticed, if a comparison between the selected schools and the whole of Scotland be made, that the selected schools show a defect in the percentage of fair-haired children, which is greater in girls than in boys, and an excess in the percentage of medium-haired children also greater in girls than in boys; and further they show a defect of pure blue-eyed and an excess of medium coloured eyed children.

If we compare the selected schools with the schools in Glasgow, we find an excess of medium-haired and a defect of dark-haired children in the selected schools, and an excess of medium-coloured eyes, and a defect of dark-eyed children.

The number of children* with jet-black hair is too few to be considered separately.

The fact that the number of dark-haired children in the schools we have selected are not in excess of the number of dark-haired children in Scotland as a whole, leads me to think that the selected schools are schools without a large foreign element and consequently not the most satisfactory schools for a discussion of the influence of race, as measured by hair and eye colour on height and weight. It has been pointed out by Mr Tocher that excess of medium or brown hair, of medium eyes and dark eyes is associated with more thickly populated regions, which in turn are also regions of excess of foreigners.

TABLE II. *Means.*

Frequency			Boys			Girls			Boys		Girls	
Boys	Girls	Hair Colour	Inches of Height	Pounds of Weight	Age	Inches of Height	Pounds of Weight	Age	Height corrected for Age	Weight corrected for Age	Height corrected for Age	Weight corrected for Age
95	94	Red ...	51·11	62·9	10·44	50·50	59·34	10·41	51·35	63·7	50·67	59·8
410	367	Fair ...	50·85	61·8	10·36	50·84	61·34	10·36	51·24	63·0	51·08	62·0
869	880	Brown ...	51·25	62·5	10·51	50·97	61·58	10·45	51·37	62·9	51·10	61·9
451	434	Dark ...	51·95	65·3	10·90	51·57	63·66	10·91	51·38	63·5	51·09	62·3
18	19	Jet Black	52·93	65·2	11·50	51·32	61·86	10·74	51·29	60·1	51·06	61·1
		Eye Colour										
202	192	Blue ...	51·90	64·6	10·80	51·14	62·18	10·58	51·50	63·4	51·09	62·1
572	563	Light ...	51·21	62·7	10·51	50·84	61·42	10·42	51·33	63·1	51·01	61·9
685	650	Medium	51·36	63·3	10·56	51·11	61·77	10·55	51·39	63·4	51·10	61·8
384	389	Dark ...	51·22	62·4	10·59	51·28	62·02	10·70	51·20	62·3	51·07	61·4

The hair scale has been arranged roughly in order of pigmentation; red comes first since it contains fewer pigment granules, and black last since the darker the hair the more pigment granules, and it will be noticed that pigmentation increases slightly with age. Differences in height and weight are slight and an examination of the correlation coefficients in Table III. will show that when corrected, for age, the association between hair or eye colour and height and weight is insignificant.

A rough correction for age can be made on the average height and weight for each hair and eye colour. The deviation of the mean age of each eye and hair colour group from the mean age of the total is obtained and is multiplied by the differences in height and weight of children of 10 and 11, and the result so obtained is added or subtracted to the mean height or weight already found for each age group—the results so found are given in Table II.

* Only children of 7 to 14 years inclusive are considered.

TABLE III. *Correlations.*

Hair	Boys	Girls
Hair Colour and Height	·10	·07
Hair Colour and Weight	·11	·08
Hair Colour and Age	·11	·11
Height and Age	·76	·78
Weight and Age	·73	·75
Hair Colour and Height for Constant Age ...	·03 ± ·02	− ·03 ± ·02
Hair Colour and Weight for Constant Age ...	·04 ± ·02	− ·00 ± ·02
Eyes		
Eye Colour and Height	·05	·04
Eye Colour and Weight	·05	·03
Eye Colour and Age	·04	·06
Eye Colour and Height for Constant Age ...	·03 ± ·02	− ·01 ± ·02
Eye Colour and Weight for Constant Age ...	·03 ± ·02	− ·02 ± ·02

The differences are not significant as can be seen in the correlation coefficients*, except that boys with jet-black hair have decidedly less weight than other boys, but means based on 18 and 19 cases are of little value.

It was thought that the combination of hair and eye colour might prove a better criterion of racial differences than either separately, but a much larger number of cases would be required to test the point. For this purpose we grouped red and fair hair (1 and 2) together and dark and jet black (4 and 5), and the mean for each group is given below.

TABLE IV. *Corrected Means.*

Frequency	Hair	Eye	Boys					Girls					Frequency
			Mean			Corrected for Age		Mean			Corrected for Age		
			Height	Weight	Age	Height	Weight	Height	Weight	Age	Height	Weight	
97	1 and 2	with 1	51·16	65·8	10·49	51·31	66·3	50·61	60·8	10·15	51·14	62·3	76
221	1 and 2	„ 2	50·83	62·0	10·45	51·05	62·7	50·94	61·4	10·47	51·04	61·7	200
142	1 and 2	„ 3	50·97	62·4	10·32	51·42	63·8	50·64	60·5	10·39	50·85	61·1	147
45	1 and 2	„ 4	50·39	58·9	9·91	51·58	62·6	50·74	60·8	10·26	51·12	61·9	38
67	3	„ 1	52·07	65·3	10·94	51·42	63·1	51·23	62·0	10·75	50·95	61·2	84
265	3	„ 2	51·27	62·1	10·42	51·56	63·0	50·74	61·3	10·34	51·01	62·1	288
397	3	„ 3	51·39	63·2	10·55	51·43	63·3	51·23	62·5	10·52	51·27	62·6	352
140	3	„ 4	50·45	59·9	10·37	50·82	61·1	50·65	59·9	10·33	50·93	60·7	156
38	4 and 5	„ 1	53·50	67·7	11·34	52·14	63·5	52·18	66·0	11·16	51·36	63·7	32
86	4 and 5	„ 2	51·99	66·2	10·97	51·30	64·1	50·94	61·9	10·56	50·92	61·8	75
146	4 and 5	„ 3	51·65	64·5	10·82	51·24	63·2	51·30	63·5	10·78	50·98	62·6	151
199	4 and 5	„ 4	51·94	65·0	10·89	51·38	63·2	51·89	64·0	11·09	51·16	62·0	195

* Correlation ratio calculated when correlation coefficient not possible.

The only divergencies in this table which are of any, if slight, significance are in the group of dark-haired children with blue eyes (4 and 5, with 1), and in the group of brown-haired children with dark eyes (3, with 4). In the first case boys and girls with this colouring are distinctly above the average in each instance when corrected for age and in three instances out of four are more above the average than any other group; the exception is in the weights of boys. In the second case the lowest point in three out of four instances, corrected for age, is reached by the group of brown-haired dark-eyed children (3, with 4); the one exception is in the height of girls, fair-haired children with medium-coloured eyes (1 and 2, with 3) being the shortest.

As far as this material goes, we find that types of hair and eye colour are not associated to any substantially *significant* extent with divergencies in height and weight in children between the ages of seven and fourteen inclusive.

Mr Tocher has pointed out that an examination of two schools in Glasgow, Gorbals and Adelphi Terrace, where "about 500 children had distinctly foreign, mostly Jewish, surnames," shows that there is a decided excess of dark and jet black-haired and of dark-eyed children, and a corresponding defect in the other classes, but no such excess occurs in our selected schools, and it seems probable that the number of foreigners in these selected schools is small. At the same time if hair and eye colour were a satisfactory measure of racial descent, and height and weight differ in different races, we should *a priori* have expected to find significant differences in this material. That they do not occur suggests that in blended races there exists little correlation between pigmentation and weight or stature.

I A. *Eye Colour and Height of Girls.*

Eye Colour.

Height	*1*	*2*	*3*	*4*	Totals
38	—	—	1	—	1
39	—	3	—	—	3
40	—	1	—	2	3
41	2	—	2	1	5
42	1	4	5	5	15
43	6	16	13	7	42
44	3	22	20	10	55
45	8	33	20	18	79
46	14	32	43	13	102
47	14	37	50	27	128
48	20	59	51	29	159
49	18	42	62	41	163
50	20	53	67	43	183
51	14	50	59	33	156
52	8	42	59	31	140
53	9	27	34	26	96
54	11	34	38	21	104
55	7	29	33	26	95
56	8	18	24	15	65
57	10	18	20	11	59
58	7	11	18	7	43
59	4	15	11	8	38
60	5	8	8	4	25
61	2	3	4	4	13
62	1	1	5	4	11
63	—	3	2	1	6
64	—	1	1	2	4
65	—	—	—	—	—
66	—	1	—	—	1
Totals	192	563	650	389	1794

II A. *Hair Colour and Height of Girls.*

Hair Colour.

Height	*1*	*2*	*3*	*4*	*5*	Totals
38	—	—	1	—	—	1
39	1	—	1	1	—	3
40	—	—	2	1	—	3
41	1	—	3	1	—	5
42	4	—	6	5	—	15
43	11	4	17	10	—	42
44	8	1	31	14	1	55
45	19	7	41	12	—	79
46	25	3	55	18	1	102
47	31	5	59	32	1	128
48	35	12	80	30	2	159
49	28	12	84	37	2	163
50	41	14	89	39	—	183
51	30	8	83	33	2	156
52	27	6	62	40	5	140
53	18	3	44	29	2	96
54	14	3	59	27	1	104
55	18	4	39	34	—	95
56	12	6	27	19	1	65
57	17	2	26	14	—	59
58	7	3	24	9	—	43
59	7	1	19	10	1	38
60	8	—	10	7	—	25
61	1	—	9	3	—	13
62	2	—	4	5	—	11
63	1	—	3	2	—	6
64	1	—	1	2	—	4
65	—	—	—	—	—	—
66	—	—	1	—	—	1
Totals	367	94	880	434	19	1794

III A. *Eye Colour and Age of Girls.*

Eye Colour.

Age	*1*	*2*	*3*	*4*	Totals
7	10	26	14	9	59
8	28	94	81	40	243
9	33	75	122	63	293
10	35	97	128	71	331
11	17	91	88	60	256
12	18	83	86	77	264
13	27	70	101	46	244
14	24	27	30	23	104
Totals	192	563	650	389	1794

IV A. *Hair Colour and Age of Girls.*

Hair Colour.

Age	*1*	*2*	*3*	*4*	*5*	Totals
7	17	2	26	14	—	59
8	53	16	138	33	3	243
9	62	15	153	63	—	293
10	76	19	152	79	5	331
11	44	17	127	64	4	256
12	52	8	124	74	6	264
13	48	9	113	74	—	244
14	15	8	47	33	1	104
Totals	367	94	880	434	19	1794

I B. *Eye Colour and Height of Boys.*

Eye Colour.

Height	1	2	3	4	Totals
36	—	1	1	—	2
37	—	—	—	1	1
38	—	—	—	1	1
39	—	1	—	—	1
40	—	2	1	—	3
41	2	2	3	—	7
42	1	4	3	3	11
43	1	8	6	4	19
44	2	11	16	8	37
45	6	23	24	15	68
46	9	28	38	25	100
47	8	29	53	17	107
48	10	57	65	33	165
49	16	56	64	31	167
50	21	55	59	45	180
51	21	49	51	39	160
52	27	48	67	31	173
53	21	54	50	37	162
54	14	41	35	24	114
55	13	31	42	22	108
56	9	19	30	21	79
57	11	19	25	10	65
58	5	11	22	7	45
59	3	17	13	2	35
60	1	2	9	4	16
61	—	3	3	3	9
62	—	—	4	1	5
63	—	1	1	—	2
64	1	—	—	—	1
Totals	202	572	685	384	1843

II B *Hair Colour and Height of Boys.*

Hair Colour.

Height	1	2	3	4	5	Totals
36	1	1	—	—	—	2
37	—	—	—	1	—	1
38	—	—	1	—	—	1
39	—	—	1	—	—	1
40	2	—	1	—	—	3
41	2	—	3	2	—	7
42	3	—	7	1	—	11
43	8	2	7	2	—	19
44	10	4	15	8	—	37
45	22	2	33	11	—	68
46	23	7	46	24	—	100
47	20	7	48	31	1	107
48	43	8	89	24	1	165
49	42	6	81	38	—	167
50	46	8	79	44	3	180
51	31	9	76	42	2	160
52	28	9	91	44	1	173
53	30	7	81	40	4	162
54	25	9	52	27	1	114
55	22	2	50	32	2	108
56	18	3	33	24	1	79
57	16	2	26	20	1	65
58	6	6	20	12	1	45
59	5	2	18	10	—	35
60	3	—	8	5	—	16
61	1	1	1	6	—	9
62	2	—	2	1	—	5
63	1	—	—	1	—	2
64	—	—	—	1	—	1
Totals	410	95	869	451	18	1843

III B. *Eye Colour and Age of Boys.*

Eye Colour.

Age	1	2	3	4	Totals
7	8	26	29	16	79
8	19	87	91	49	246
9	26	88	112	55	281
10	34	73	114	73	294
11	36	101	107	52	296
12	36	91	87	63	277
13	30	74	95	55	254
14	13	32	50	21	116
Totals	202	572	685	384	1843

IV B. *Hair Colour and Age of Boys.*

Hair Colour.

Age	1	2	3	4	5	Totals
7	24	3	35	16	1	79
8	59	19	122	45	1	246
9	69	11	135	65	1	281
10	74	16	134	68	2	294
11	56	18	158	62	2	296
12	54	11	133	76	3	277
13	53	8	103	83	7	254
14	21	9	49	36	1	116
Totals	410	95	869	451	18	1843

V A. *Eye Colour and Weight of Girls.*

Eye Colour.

Weight	1	2	3	4	Totals
29—	—	—	—	1	1
33—	—	2	1	—	3
37—	3	5	3	2	13
41—	7	25	26	10	68
45—	20	53	50	32	155
49—	31	78	77	47	233
53—	22	87	94	57	260
57—	20	61	95	63	239
61—	19	59	65	44	187
65—	11	42	69	41	163
69—	16	50	52	30	148
73—	9	29	24	9	71
77—	8	20	27	17	72
81—	10	9	18	9	46
85—	3	13	14	7	37
89—	6	13	13	7	39
93—	4	5	9	2	20
97—	1	1	7	3	12
101—	1	3	—	2	6
105—	—	3	3	3	9
109—	1	4	1	—	6
113—	—	—	—	1	1
117—	—	—	2	1	3
121—	—	—	—	—	—
125—	—	1	—	1	2
Totals	192	563	650	389	1794

VI A. *Hair Colour and Weight of Girls.*

Hair Colour.

Weight	1	2	3	4	5	Totals
29—	—	—	1	—	—	1
33—	1	—	2	—	—	3
37—	3	—	7	3	—	13
41—	15	5	37	11	—	68
45—	37	12	73	31	2	155
49—	48	11	130	42	2	233
53—	56	13	123	66	2	260
57—	43	17	117	59	3	239
61—	40	9	86	48	4	187
65—	31	9	74	48	1	163
69—	30	8	69	38	3	148
73—	15	1	35	20	—	71
77—	12	3	41	16	—	72
81—	10	3	20	12	1	46
85—	4	1	22	9	1	37
89—	10	2	16	11	—	39
93—	5	—	9	6	—	20
97—	1	—	5	6	—	12
101—	3	—	2	1	—	6
105—	1	—	5	3	—	9
109—	2	—	4	—	—	6
113—	—	—	1	—	—	1
117—	—	—	—	3	—	3
121—	—	—	—	—	—	—
125—	—	—	1	1	—	2
Totals	367	94	880	434	19	1794

VII A. *Height and Age of Girls.*

Age of Girls.

Height	7	8	9	10	11	12	13	14	Totals
38	—	1	—	—	—	—	—	—	1
39	1	1	—	1	—	—	—	—	3
40	1	1	—	1	—	—	—	—	3
41	1	3	—	—	1	—	—	—	5
42	3	4	5	2	—	1	—	—	15
43	13	20	4	4	1	—	—	—	42
44	15	19	12	5	3	1	—	—	55
45	5	34	28	7	3	2	—	—	79
46	10	39	36	13	2	1	1	—	102
47	4	43	35	28	12	4	1	1	128
48	5	36	50	40	20	5	2	1	159
49	—	27	43	49	27	11	6	—	163
50	—	11	39	57	44	24	6	2	183
51	1	3	27	51	34	32	6	2	156
52	—	—	7	36	41	30	22	4	140
53	—	—	5	16	22	28	20	5	96
54	—	—	1	12	20	41	25	5	104
55	—	—	—	3	15	43	24	10	95
56	—	1	—	4	7	16	26	11	65
57	—	—	—	1	3	10	36	9	59
58	—	—	—	—	—	8	26	9	43
59	—	—	1	—	1	2	20	14	38
60	—	—	—	1	—	3	10	11	25
61	—	—	—	—	—	1	4	8	13
62	—	—	—	—	—	1	7	3	11
63	—	—	—	—	—	—	2	4	6
64	—	—	—	—	—	—	—	4	4
65	—	—	—	—	—	—	—	—	—
66	—	—	—	—	—	—	—	1	1
Totals	59	243	293	331	256	264	244	104	1794

VIII A. *Weight and Age of Girls.*

Age of Girls.

Weight	7	8	9	10	11	12	13	14	Totals
29	1	—	—	—	—	—	—	—	1
33	—	2	—	1	—	—	—	—	3
37	3	8	1	1	—	—	—	—	13
41	20	28	16	3	1	—	—	—	68
45	19	63	39	26	7	1	—	—	155
49	9	61	80	48	25	6	3	1	233
53	6	49	80	63	33	26	2	1	260
57	1	24	44	71	55	27	14	3	239
61	—	7	16	57	41	44	15	7	187
65	—	—	13	36	42	42	28	2	163
69	—	1	2	18	33	51	34	9	148
73	—	—	—	4	9	25	27	6	71
77	—	—	2	1	7	18	29	15	72
81	—	—	—	1	2	9	25	9	46
85	—	—	—	1	—	5	20	11	37
89	—	—	—	—	1	5	19	14	39
93	—	—	—	—	—	3	11	6	20
97	—	—	—	—	—	1	6	5	12
101	—	—	—	—	—	—	3	3	6
105	—	—	—	—	—	1	3	5	9
109	—	—	—	—	—	—	3	3	6
113	—	—	—	—	—	—	—	1	1
117	—	—	—	—	—	—	1	2	3
121	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	1	1	2
Totals	59	243	293	331	256	264	244	104	1794

V B. *Eye Colour and Weight of Boys.*

Eye Colour.

Weight	1	2	3	4	Totals
31	—	—	1	—	1
35	—	1	1	1	3
39	1	7	9	5	22
43	5	25	25	19	74
47	14	48	57	26	145
51	17	61	83	45	206
55	26	93	111	66	296
59	34	75	81	49	239
63	20	70	91	55	236
67	27	61	54	31	173
71	24	45	45	26	140
75	8	33	38	31	110
79	16	19	27	9	71
83	2	14	23	8	47
87	2	10	14	4	30
91	3	3	10	4	20
95	3	1	7	3	14
99	—	2	3	—	5
103	—	2	3	—	5
107	—	—	1	1	2
111	—	1	1	1	3
115	—	1	—	—	1
Totals	202	572	685	384	1843

VI B. *Hair Colour and Weight of Boys.*

Hair Colour.

Weight	1	2	3	4	5	Totals
31	—	—	1	—	—	1
35	—	—	3	—	—	3
39	7	—	11	4	—	22
43	22	5	34	13	—	74
47	49	10	65	21	—	145
51	39	14	106	45	2	206
55	64	15	144	69	4	296
59	56	9	115	58	1	239
63	43	10	115	63	5	236
67	35	6	90	40	2	173
71	32	7	55	45	1	140
75	24	6	46	33	1	110
79	18	5	29	19	—	71
83	11	4	19	12	1	47
87	4	—	16	9	1	30
91	1	2	10	7	—	20
95	4	1	4	5	—	14
99	—	1	2	2	—	5
103	—	—	2	3	—	5
107	—	—	1	1	—	2
111	—	—	1	2	—	3
115	1	—	—	—	—	1
Totals	410	95	869	451	18	1843

VII B. *Height and Age of Boys.*

Age of Boys.

Height	7	8	9	10	11	12	13	14	Totals
36	—	1	—	1	—	—	—	—	2
37	—	—	—	1	—	—	—	—	1
38	1	—	—	—	—	—	—	—	1
39	1	—	—	—	—	—	—	—	1
40	2	1	—	—	—	—	—	—	3
41	2	4	1	—	—	—	—	—	7
42	3	5	1	1	—	—	1	—	11
43	9	7	1	1	—	—	—	1	19
44	16	13	6	—	1	—	1	—	37
45	15	35	11	7	—	—	—	—	68
46	10	37	28	12	5	4	2	2	100
47	11	34	35	13	9	2	3	—	107
48	5	49	58	27	13	5	7	1	165
49	3	26	47	48	21	14	8	—	167
50	1	18	46	47	37	26	5	—	180
51	—	12	27	47	37	22	13	2	160
52	—	1	11	44	53	44	17	3	173
53	—	1	4	29	43	39	39	7	162
54	—	1	3	9	28	41	19	13	114
55	—	1	1	2	26	31	30	17	108
56	—	—	—	2	12	22	35	8	79
57	—	—	—	1	5	16	26	17	65
58	—	—	—	—	5	8	23	9	45
59	—	—	1	1	1	2	16	14	35
60	—	—	—	—	—	1	5	10	16
61	—	—	—	—	—	—	2	7	9
62	—	—	—	1	—	—	2	2	5
63	—	—	—	—	—	—	—	2	2
64	—	—	—	—	—	—	—	1	1
Totals	79	246	281	294	296	277	254	116	1843

VIII B. *Weight and Age of Boys.*

Age of Boys.

Weight	7	8	9	10	11	12	13	14	Totals
31	—	1	—	—	—	—	—	—	1
35	2	—	—	—	—	—	1	—	3
39	8	10	3	—	1	—	—	—	22
43	22	33	11	8	—	—	—	—	74
47	26	53	39	17	5	2	3	—	145
51	12	66	62	33	17	8	7	1	206
55	9	58	87	78	33	18	12	1	296
59	—	15	46	57	62	43	11	5	239
63	—	8	27	60	61	41	31	8	236
67	—	1	3	25	51	57	27	9	173
71	—	1	1	11	28	46	40	13	140
75	—	—	2	3	16	28	44	17	110
79	—	—	—	1	13	15	29	13	71
83	—	—	—	1	5	13	17	11	47
87	—	—	—	—	2	3	18	7	30
91	—	—	—	—	1	3	7	9	20
95	—	—	—	—	1	—	5	8	14
99	—	—	—	—	—	—	—	5	5
103	—	—	—	—	—	—	1	4	5
107	—	—	—	—	—	—	1	1	2
111	—	—	—	—	—	—	—	3	3
115	—	—	—	—	—	—	—	1	1
Totals	79	246	281	294	296	277	254	116	1843

IX A. *Hair and Eye Colour and Height of Girls.*

Hair and Eye Colour.

Height	1 and 2 with 1	1 and 2 with 2	1 and 2 with 3	1 and 2 with 4	3 with 1	3 with 2	3 with 3	3 with 4	4 and 5 with 1	4 and 5 with 2	4 and 5 with 3	4 and 5 with 4	Totals
38	—	—	—	—	—	—	1	—	—	—	—	—	1
39	—	1	—	—	—	1	—	—	—	1	—	—	3
40	—	—	—	—	—	1	—	1	—	—	—	1	3
41	1	—	—	—	1	—	2	—	—	—	—	1	5
42	1	2	1	—	—	1	3	2	—	1	1	3	15
43	3	8	3	1	1	6	6	4	2	2	4	2	42
44	—	5	4	—	3	13	10	5	—	4	6	5	55
45	6	11	6	3	2	19	11	9	—	3	3	6	79
46	7	11	8	2	4	20	24	7	3	1	11	4	102
47	3	17	14	2	10	13	26	10	1	7	10	15	128
48	10	20	14	3	7	32	24	17	3	7	13	9	159
49	6	12	19	3	9	24	34	17	3	6	9	21	163
50	12	20	15	8	8	25	39	17	—	8	13	18	183
51	4	15	13	6	7	29	31	16	3	6	15	11	156
52	1	17	15	—	3	22	28	9	4	3	16	22	140
53	4	7	8	2	3	15	14	12	2	5	12	12	96
54	2	9	5	1	8	20	25	6	1	5	8	14	104
55	3	12	3	4	1	13	20	5	3	4	10	17	95
56	1	8	8	1	5	5	12	5	2	5	4	9	65
57	3	11	5	—	6	5	11	4	1	2	4	7	59
58	5	3	1	1	2	7	12	3	—	1	5	3	43
59	—	5	2	1	2	8	8	1	2	2	1	6	38
60	3	3	2	—	2	4	4	—	—	1	2	4	25
61	1	—	—	—	—	3	3	3	1	—	1	1	13
62	—	1	1	—	—	—	3	1	1	—	1	3	11
63	—	1	—	—	—	1	1	1	—	1	1	—	6
64	—	1	—	—	—	—	—	1	—	—	1	1	4
65	—	—	—	—	—	—	—	—	—	—	—	—	—
66	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	76	200	147	38	84	288	352	156	32	75	151	195	1794

IX B. *Hair and Eye Colour and Height of Boys.*

Hair and Eye Colour.

Height	1 and 2 with 1	1 and 2 with 2	1 and 2 with 3	1 and 2 with 4	3 with 1	3 with 2	3 with 3	3 with 4	4 and 5 with 1	4 and 5 with 2	4 and 5 with 3	4 and 5 with 4	Totals
36	—	1	1	—	—	—	—	—	—	—	—	—	2
37	—	—	—	—	—	—	—	—	—	—	—	1	1
38	—	—	—	—	—	—	—	1	—	—	—	—	1
39	—	—	—	—	—	1	—	—	—	—	—	—	1
40	—	2	—	—	—	—	1	—	—	—	—	—	3
41	1	1	—	—	1	—	2	—	—	1	1	—	7
42	1	2	—	—	—	2	3	2	—	—	—	1	11
43	1	6	1	2	—	2	4	1	—	—	1	1	19
44	2	5	6	1	—	6	7	2	—	—	3	5	37
45	4	11	7	2	2	7	15	9	—	5	2	4	68
46	6	13	9	2	1	14	20	11	2	1	9	12	100
47	3	11	13	—	2	12	26	8	3	6	14	9	107
48	4	21	18	8	6	25	39	19	—	11	8	6	165
49	11	18	15	4	5	28	37	11	—	10	12	16	167
50	15	21	10	8	5	28	31	15	1	6	18	22	180
51	7	21	7	5	10	20	33	13	4	8	11	21	160
52	10	14	9	4	11	26	40	14	6	8	18	13	173
53	8	21	7	1	8	30	34	9	5	3	9	27	162
54	7	18	5	4	2	23	21	6	5	—	9	14	114
55	5	10	8	1	4	15	24	7	4	6	10	14	108
56	5	4	11	1	3	10	16	4	1	5	3	16	79
57	5	9	4	—	3	4	16	3	3	6	5	7	65
58	2	4	4	2	—	5	13	2	3	2	5	3	45
59	—	4	3	—	3	7	8	—	—	6	2	2	35
60	—	2	1	—	1	—	5	2	—	—	3	2	16
61	—	1	1	—	—	—	1	—	—	2	1	3	9
62	—	—	2	—	—	—	1	1	—	—	1	—	5
63	—	1	—	—	—	—	—	—	—	—	1	—	2
64	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	97	221	142	45	67	265	397	140	38	86	146	199	1843

X A. *Hair and Eye Colour and Weight of Girls.*

Hair and Eye Colour.

Weight	*1* and *2* with *1*	*1* and *2* with *2*	*1* and *2* with *3*	*1* and *2* with *4*	*3* with *1*	*3* with *2*	*3* with *3*	*3* with *4*	*4* and *5* with *1*	*4* and *5* with *2*	*4* and *5* with *3*	*4* and *5* with *4*	Totals
29	—	—	—	—	—	—	—	1	—	—	—	—	1
33	—	1	—	—	—	1	1	—	—	—	—	—	3
37	2	—	1	—	1	4	1	1	—	1	1	1	13
41	3	9	6	2	3	13	16	5	1	3	4	3	68
45	10	20	16	3	7	26	26	14	3	7	8	15	155
49	8	27	22	2	18	43	40	29	5	8	15	16	233
53	11	33	20	5	7	41	48	27	4	13	26	25	260
57	9	20	20	11	10	30	56	21	1	11	19	31	239
61	10	21	16	2	7	33	29	17	2	5	20	25	187
65	5	15	15	5	4	23	36	11	2	5	17	25	163
69	6	19	9	4	6	24	28	11	4	7	15	15	148
73	2	10	4	—	5	13	16	1	2	5	5	8	71
77	1	7	5	2	6	12	18	5	1	1	4	10	72
81	4	3	5	1	5	3	10	2	1	3	3	6	46
85	1	3	1	—	2	7	9	4	—	3	4	3	37
89	2	6	3	1	—	7	6	3	4	—	4	3	39
93	—	3	2	—	2	1	6	—	2	1	1	2	20
97	—	—	1	—	1	—	4	—	—	1	2	3	12
101	1	2	—	—	—	—	—	2	—	1	—	—	6
105	—	—	1	—	—	3	1	1	—	—	1	2	9
109	1	1	—	—	—	3	1	—	—	—	—	—	6
113	—	—	—	—	—	—	—	1	—	—	—	—	1
117	—	—	—	—	—	—	—	—	—	—	2	1	3
121	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	1	—	—	—	—	—	1	2
Totals	76	200	147	38	84	288	352	156	32	75	151	195	1794

XI A. *Hair and Eye Colour and Age of Girls.*

Hair and Eye Colour.

Age	*1* and *2* with *1*	*1* and *2* with *2*	*1* and *2* with *3*	*1* and *2* with *4*	*3* with *1*	*3* with *2*	*3* with *3*	*3* with *4*	*4* and *5* with *1*	*4* and *5* with *2*	*4* and *5* with *3*	*4* and *5* with *5*	Totals
7	7	9	2	1	1	13	7	5	2	4	5	3	59
8	14	34	18	3	11	51	53	23	3	9	10	14	243
9	10	28	29	10	18	37	68	30	5	10	25	23	293
10	17	30	37	11	13	52	58	29	5	15	33	31	331
11	7	30	20	4	10	49	45	23	—	12	23	33	256
12	7	31	19	3	8	42	48	26	3	10	19	48	264
13	7	27	18	5	12	33	56	12	8	10	27	29	244
14	7	11	4	1	11	11	17	8	6	5	9	14	104
Totals	76	200	147	38	84	288	352	156	32	75	151	195	1794

X B. *Hair and Eye Colour and Weight of Boys.*

Hair and Eye Colour.

Weight	1 and 2 with 1	1 and 2 with 2	1 and 2 with 3	1 and 2 with 4	3 with 1	3 with 2	3 with 3	3 with 4	4 and 5 with 1	4 and 5 with 2	4 and 5 with 3	4 and 5 with 4	Totals
31	—	—	—	—	—	—	1	—	—	—	—	—	1
35	—	—	—	—	—	1	1	1	—	—	—	—	3
39	1	6	—	—	—	1	7	3	—	—	2	2	22
43	4	12	7	4	1	11	14	8	—	2	4	7	74
47	11	25	19	4	2	19	30	14	1	4	8	8	145
51	7	23	17	6	7	30	45	24	3	8	21	15	206
55	14	29	27	9	9	46	66	23	3	18	18	34	296
59	14	29	13	9	15	33	48	19	5	13	20	21	239
63	9	25	13	6	5	39	54	17	6	6	24	32	236
67	10	21	9	1	11	34	37	8	6	6	8	22	173
71	10	16	10	3	6	22	20	7	8	7	15	16	140
75	6	15	7	2	2	14	23	7	—	4	8	22	110
79	9	6	8	—	4	6	16	3	3	7	3	6	71
83	1	6	7	1	1	5	12	1	—	3	4	6	47
87	—	3	1	—	—	3	10	3	2	4	3	1	30
91	1	2	—	—	2	—	7	1	—	1	3	3	20
95	—	1	4	—	2	—	2	—	1	—	1	3	14
99	—	1	—	—	—	—	2	—	—	1	1	—	5
103	—	—	—	—	—	—	2	—	—	2	1	—	5
107	—	—	—	—	—	—	—	1	—	—	1	—	2
111	—	—	—	—	—	1	—	—	—	—	1	1	3
115	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	97	221	142	45	67	265	397	140	38	86	146	199	1843

XI B. *Hair and Eye Colour and Age of Boys.*

Hair and Eye Colour.

Age	1 and 2 with 1	1 and 2 with 2	1 and 2 with 3	1 and 2 with 4	3 with 1	3 with 2	3 with 3	3 with 4	4 and 5 with 1	4 and 5 with 2	4 and 5 with 3	4 and 5 with 4	Totals
7	4	13	9	1	4	11	15	5	—	2	5	10	79
8	13	35	23	7	3	42	52	25	3	10	16	17	246
9	13	31	26	10	9	42	60	24	4	15	26	21	281
10	20	33	22	15	10	28	76	20	4	12	16	38	294
11	16	33	20	5	12	60	64	22	8	8	23	25	296
12	16	34	11	4	13	46	55	19	7	11	21	40	277
13	9	29	21	2	12	27	45	19	9	18	29	34	254
14	6	13	10	1	4	9	30	6	3	10	10	14	116
Totals	97	221	142	45	67	265	397	140	38	86	146	199	1843

PIGMENTATION IN RELATION TO SELECTION AND TO ANTHROPOMETRIC CHARACTERS.

By A. M. CARR SAUNDERS, M.A.

I. Pigmentation and Disease.

Introduction. The problem of the relation between pigmentation and disease has in recent years commanded a certain amount of attention; but, as yet, very little work has been done which has helped materially to elucidate the problem. In a recent number of *Biometrika* a memoir by Macdonald* deals with this question; the introduction to the memoir gives a summary of previous work in this subject, and the theories which have been put forward with regard to it. Readers of this summary will see how much of what is actually laid down about the relation between pigmentation and disease consists of little more than casual observations and vague suggestions. One theory that has gained currency is that the dark haired type is becoming more common in towns, and that this is due to the elimination of the lighter stocks owing to their greater susceptibility to disease. It was with the hope of bringing forward evidence to test such rather vague theories that the present work was undertaken. The problem is part of the larger subject—the action of natural selection on the human race. About this we know at present lamentably little. It has been shown that the great majority of deaths are selective. But we are almost entirely ignorant of the precise mode in which selection is influencing the race.

There exist some further contributions to the subject which are not mentioned by Macdonald. Shrubsall† in a memoir of 1909 finds that tuberculosis is more frequent in dark types, while the onset is earlier among the blondes. In another paper‡ he comes to the conclusion that blonde traits are associated with rheumatism, heart-disease, tonsilitis and osteo-arthritis, and that brunette traits are associated with nervous diseases, tuberculosis and malignant disease. Pfitzner§ found a marked increase of black hair with increasing age; the increase seems to be too

* D. Macdonald, *Biometrika*, Vol. VIII. p. 13.

† F. C. Shrubsall, *Proc. Roy. Soc. Med.* 1909, Vol. II.

‡ F. C. Shrubsall, *St Bartholomew Hospital Reports*, Vol. XXXIX.

§ Pfitzner, *Schwalbe's Zeitschrift für Morphologie*, Bd. I.

great to be accounted for by the normal darkening of the hair, and the most probable explanation is that the black haired type has been less subject to elimination*. But the most interesting result obtained is due to Pearson, which I am allowed to quote, but which has not yet been published elsewhere. He found a correlation of ·19 between health and hair colour, and of ·07 between health and eye colour in the case of data relating to 2317 boys. A very similar result was obtained from data relating to a nearly equal number of girls. These results of Pearson and Pfitzner tend to confirm the theory referred to above, that the darker types are less subject to disease. The data used by Macdonald are in some respects comparable with those employed in this paper, and they bear on very similar problems in both cases. All discussion and comparison must be reserved, however, until the results of the present investigation have been set forth.

Data and Methods. The data used in this work are derived from the information collected as the result of the Medical Survey of School Children carried out to meet the requirements of the Act of 1907. The data come entirely from Birmingham, and the fact that they were obtained for use in the Biometric Laboratory, is due to the kindness of Dr Auden, medical superintendent of the Birmingham Education Committee. The information with regard to each child is contained in a separate schedule, and includes a large number of particulars. The facts which have been of use in the present enquiry are those collected with regard to the hair and eye colour of the children, and to certain selected diseases. The hair colour was classified under six headings: red, fair, light brown, medium brown, dark brown and black, and the eye colour under four headings: blue, light, neutral and dark. The examination of the children was actually carried out by a number of doctors working with the help of nurses, all of whom Dr Auden had trained to estimate the pigmentation with the aid of a colour-meter. The classification of hair and eye colour always presents great difficulties; in the case of hair colour the most satisfactory method is to construct a scale with specimens of actual hair, and to compare the hair of each individual examined with the specimens on the scale. The colour-meter employed by Dr Auden consisted of different shades of coloured glass; and those responsible for the filling up of the schedules gained by its use a general idea of the categories of hair colour, but did not directly compare the hair of the children with their standard scale. Such a method of estimation is necessarily somewhat rough; but the results obtained are no doubt of sufficient exactness to enable broad conclusions to be drawn from them.

The six specified diseases are measles, whooping cough, scarlet fever, diphtheria, mumps and chicken pox. The schedules also contain information about other diseases, but this is not employed here.

Table I on the following page exhibits some of the weaknesses of the scheme of hair-classification employed. Taking the children of 13 years of age, it is clear

* See *Biometrika*, Vol. III. 1904, p. 463.

that a method of classification which divides the data into groups varying from 1·6 per cent. to 32·2 per cent. of the total is not entirely satisfactory.

TABLE I.

Percentage of Hair Colour Groups to the Total.

Hair Colour	Boys. Age 13		Girls. Age 13		Boys. Age 3—7		Girls. Age 3—7	
	Numbers	Per-centage	Numbers	Per-centage	Numbers	Per-centage	Numbers	Per-centage
Red	62	3·1	74	3·5	21	4·5	7	1·9
Fair	253	12·7	237	11·4	52	11·1	34	9·4
Light Brown ...	443	22·2	406	19·5	128	27·2	114	31·7
Medium Brown	626	31·4	652	31·2	189	40·2	144	40·0
Dark Brown ...	579	29·0	672	32·2	77	16·4	60	16·7
Black	31	1·6	46	2·2	3	·6	1	·3
Totals	1994	100·0	2087	100·0	470	100·0	360	100·0

But apart from this it is interesting to compare the percentage of the various hair colour groups so obtained in Birmingham with those obtained elsewhere. Table II is copied from Tocher's "Pigmentation Survey of School Children in Scotland*." The comparison between the hair colour schemes cannot be made in detail because the classification is not identical. The percentage of red haired

TABLE II.

Colour Distribution of Scottish Children.

		Hair					Eyes				
		Red	Fair	Medium	Dark	Black	Blue	Light	Medium	Dark	Totals
Boys.	Numbers	14162	64312	111569	64511	3212	37788	78140	84334	57504	257766
„	Percentage	5·5	24·9	43·3	25·0	1·3	14·7	30·3	32·7	22·3	—
Girls.	Numbers	12435	67036	99873	62073	2972	36347	74068	78357	55617	244389
„	Percentage	5·1	27·4	40·9	25·4	1·2	14·8	30·3	32·1	22·8	—

children is less in Birmingham, where it does not exceed 3·5 per cent., while in Scotland it is not less than 5 per cent.; the black haired class, however, would seem to be present in Birmingham in larger numbers. The two classes denominated Fair clearly do not correspond; many of the Scottish children placed as Fair go in the Birmingham arrangement under the head of Light Brown.

* *Biometrika*, Vol. vi. p. 147.

The comparison of the hair colour percentages of children aged 13 and those aged 3—7 shows no very considerable change. There is a certain darkening of the hair, but it is not very pronounced*. The percentage of fair haired children is actually higher among those of 13 years of age than among the younger children. The chief change is a gain of the Dark Brown class at the expense of the Light and Medium Brown classes.

TABLE III.

Percentage of Eye Colour Groups to the Total.

Eye Colour	Boys. Age 13		Girls. Age 13		Boys. Age 3—7		Girls. Age 3—7	
	Numbers	Per-centage	Numbers	Per-centage	Numbers	Per-centage	Numbers	Per-centage
Blue ...	354	17·76	385	18·45	208	44·26	173	48·06
Light ...	568	28·49	577	27·65	58	12·34	48	13·33
Neutral	544	27·28	564	27·02	89	18·94	64	17·78
Dark ...	528	26·47	561	26·88	115	24·46	75	20·83
Totals	1994	100·0	2087	100·0	470	100·0	360	100·0

Table III shows that for the older children the classification adopted for eye colour provides a fairly equal division into the four categories. A direct comparison with the Scottish figures is here possible since presumably the medium class in one case is identical with the neutral class in the other. The percentage of blue eyed children is distinctly higher in our data; and this is remarkable since the darker type of eye is more frequent in Scotland; and, further, Tocher's data give a higher percentage of blue eyes than is usually found. The suggestion is that at Birmingham the classification of blue eyes has not been very strict, and that a number of light blue eyes have been admitted into the blue eye category when they should have been placed in the light category. This idea is supported by the figures given for the children aged 3—7. In this case the blue eyed children amount to nearly 50 per cent. of the total. This is almost certainly too high a total, if the term blue eye is to be used in its usual meaning. Among the younger children the percentage of light eyes falls rather low; and probably a considerable number of the blue eyes should have been included in the light eyed class. The table is noteworthy as marking a very considerable change with age to the darker eye classes.

Table IV shows the average number of diseases per child in each hair category. The hair categories have also been combined into two larger groups—roughly representing light and dark hair—and the average number of diseases for these groups has been worked out. Further figures are also given to show the average number of diseases per boy and girl in the different age groups.

* For estimates of the correlation between hair colour and age, see *Biometrika*, Vol. III. p. 462.

If we consider first the average number of diseases to the different hair colour groups, we find no evidence of any significant connection between one hair colour and disease. The differences in the averages are not only slight, but occur apparently at random; we do not find one hair colour associated in both boys and girls with a distinctive average whether greater or less than the normal. Further it does not appear that dark hair shows any significant difference from light hair in this respect. Red, fair and light brown have been classed together as light hair, and the remaining categories as dark hair. In two cases the average number of diseases is exactly equal in the different groups; in the other two cases, the light haired group is in excess on one occasion, and the dark haired group in the other. So far, therefore, as this table goes, there is no reason for thinking that liability to disease is connected with any particular hair colour.

TABLE IV. (See Tables XV and XVI, pp. 378—379.)

Incidence of Disease in Hair Colour Groups.

Hair Colour	Boys. Age 13				Girls. Age 13			
	Cases of Disease		Average		Cases of Disease		Average	
Red	125	1317	2·01	1·74	148	1520	2·0	2·12
Fair	434		1·71		483		2·04	
Light Brown ...	758		1·71		889		2·19	
Medium Brown	1183	2323	1·88	1·88	1395	2908	2·13	2·12
Dark Brown ...	1071		1·84		1416		2·11	
Black	69		2·22		97		2·1	
Totals	3640		—		4428		—	

Average number of diseases to each boy 1·83. Average number of diseases to each girl 2·12.

Hair Colour	Boys. Age 3—7				Girls. Age 3—7			
	Cases of Disease		Average		Cases of Disease		Average	
Red	36	313	1·71	1·56	11	260	1·58	1·68
Fair	93		1·79		51		1·5	
Light Brown ...	184		1·44		198		1·74	
Medium Brown	296	414	1·57	1·54	217	286	1·5	1·40
Dark Brown ...	115		1·49		68		1·13	
Black	3		1·0		1		1·0	
Totals	727		—		546		—	

Average number of diseases to each boy 1·55. Average number of diseases to each girl 1·52.

When we turn to consider the average number of diseases per head in the two sexes, we do find a significant difference, but only in the children of the older group. Boys from 3—7 and girls of the same age have almost exactly the same number of diseases per head. The difference only occurs between the boys and girls aged 13, and between them it is marked, the boys having an average of 1·83 and the girls one of 2·12 diseases per child.

Now, the source of this fact is by no means clear; there are two possible explanations which may be at once put aside as not applicable here. In the first place the difference cannot be due to bias. For if there were any reason why the diseases of the girls should be remembered more vividly than those of the boys, then that reason ought to work in the case of the younger children as well. But it does not seem possible to find any such reason. Dr Auden has, indeed, pointed out to me that a parent is more often present during the examination of girls than of boys. Taking about 1500 of each sex, he calculated that a parent was present when 65 per cent. of the boys and 71 per cent. of the girls were being examined. But I agree with him in thinking that this could not possibly account for the difference in the average number of diseases per child in the two sexes. In the second place it is not due to an epidemic among the girls, since the girls are found to have had a higher average number of all the six diseases with which we are dealing.

The first explanation of this anomaly which promised to be at all sufficient is connected with the fact that, although more boys are born than girls, yet in a few years the numbers of the two sexes become equal, and later the girls outnumber the boys. So far these facts are a matter of common knowledge, and the manner in which it was at first thought that they might explain the above figures was as follows. If we suppose that the incidence rate for the special diseases with which we are concerned is of equal intensity for the two sexes, and that more boys die from these diseases than girls, then if a number of boys and girls aged 13 were examined, we might expect to find more girls who had diseases than boys; for supposing an equal number to have been attacked from each sex, and a greater percentage of boys to have died, there will clearly be present at a later age more girls living who have had diseases. Now, this explanation does not as a fact hold good, as will appear; but it was while examining data bearing on these points that certain facts were found which do help to elucidate the question.

The above reasoning rests on the assumptions that (1) the incidence rate is equal or at least not very dissimilar in the two cases, and (2) the death rate is higher among boys than girls. Firstly as to the incidence rate:—the age distributions of the cases of scarlet fever and diphtheria notified to the Metropolitan Asylums Board as given in the Board Report for 1900 were extracted. These figures alone do not give an accurate measure of the incidence, since an allowance must be made for the fact that the numbers of the two sexes are not equal at the different ages. The total numbers of the two sexes for the various age

groups were taken from the census of 1900 as given for London, and a calculation was then made on the basis of the number of cases notified among boys relative to the population totals thus obtained to find the expected number of cases among the girls relative to the population of that sex. The following figures give the cases actual and expected obtained in this way:

Scarlet Fever.

Age of cases notified		*0—5*	*5—10*	*10—15*
Number of cases.	Boys, actual	2,040	2,312	1,116
" "	Girls, actual	2,009	2,803	1,285
" " "	expected	2,038	2,311	1,138

Diphtheria.

Age of cases notified		*0—5*	*5—10*	*10—15*
Number of cases.	Boys, actual	2,322	1,782	607
" "	Girls, actual	2,218	2,075	759
" " "	expected	2,320	1,800	618

The incidence rate, therefore, in the case of these two diseases is not the same in the two sexes. Up to five years of age the incidence is nearly equal; if anything, the boys are attacked rather more often. But in the older age groups, the difference is most marked. The girls have both diseases far more often than the boys.

Secondly as to the death rate;—the following figures are taken from the Registrar General's Annual Report:

Annual Death Rate per Million living from 1861—1900.

Diphtheria.

Ages ...	*0—5*	*5—10*	*10—15*
Boys ...	827	396	94
Girls ...	821	499	131

Scarlet Fever.

Boys ...	2,734	1,225	254
Girls ...	2,596	1,190	278

Whooping Cough.

Boys ...	3,145	102	4
Girls ...	3,805	154	7

Measles.

Boys ...	3,092	225	21
Girls ...	2,890	247	24

Before we discuss the bearing of these figures on the problem, it will be of assistance to look at the following Table:

TABLE V.

Percentages of Boys and Girls who have had the following Diseases.

Age 13.

Measles,	Boys	75·2	Whooping Cough,	Boys	37·1	Scarlet Fever,	Boys	16·3
„	Girls	85·4	„ „	Girls	44·9	„ „	Girls	18·4
Diphtheria,	Boys	3·9	Mumps,	Boys	21·4	Chicken Pox,	Boys	28·6
„	Girls	4·7	„	Girls	23·2	„ „	Girls	35·5

This gives the percentages of boys and girls who have had the various diseases. It will be seen that not only do the older girls have, as before shown, a higher average number of diseases, but in each case a higher percentage of girls have had the disease than of boys; and further, the differences between the percentages as a fraction of the whole are roughly the same between boys and girls. The difference is rather greater in the case of diphtheria and rather less in the case of mumps; but on the whole there seems to be a consistently higher average among the girls for each disease. If we now turn back to the incidence and mortality rates, it is evident that they show the first explanation suggested to be impossible. The incidence rates are not the same for both sexes, and the mortality rates do not show that marked and persistent mortality among the younger boys which the explanation supposes. In the case of whooping cough there is a far higher death rate among girls, and here the explanation breaks down entirely; for as Table V shows, the difference between the two sexes is as marked as in any other disease. It must, of course, be remembered that we are here only dealing with six specified diseases; the elimination of boys, which we know to occur, must take place through the agency of other diseases.

The figures for the incidence rate suggest a further explanation. Unfortunately they are only obtainable for scarlet fever and diphtheria. But in these two cases the girls from 5 years of age and upwards are more liable to attack. Now, if the death rate were the same for both boys and girls, then this would be an explanation, in so far as the bare fact that girls are as a matter of fact more susceptible to disease than boys is an explanation. If we examine the death rates for diphtheria and scarlet fever, we see that they are not the same; the general tendency is for the rate among the younger boys to be higher than that among the younger girls, and the opposite way about for the older children. On the whole, however, up to the age of 15 diphtheria is a more fatal disease for girls than boys, while the case is reversed in scarlet fever. In such cases as that of diphtheria, in which the death rate is higher among girls, there must be a tendency to counteract the results of the higher incidence rate among girls, and in the girls of 13, as shown in Table V, we might expect to find such a large difference between the sexes as in the other cases. But, as already pointed out, the difference between the two sexes in the case of diphtheria is rather larger than for other diseases. The explanation is, it must be admitted, not very satisfactory; but granted the existence of the higher incidence rate among girls, we must suppose that the

differences in the death rate are not such as to seriously counteract the former factor. It would seem that the death rate can have very little influence on the figures in question, since the death rates for four out of the six specified diseases show no similarity. In whooping cough, for instance, the death rate is always higher for girls; while in scarlet fever it is almost always higher for boys. The only explanation, therefore, is that girls at the older ages suffer more from these specified diseases than boys; this is no real explanation, but a physiological fact. It does, however, rule out of account any explanation through selection by means of the six specified diseases which seemed at first the most probable

It is well known that the correlation between hair colour and eye colour is fairly high; it seems, in fact, to be something under ·4*. Now Table VI is constructed on exactly similar lines to Table IV, but with eye colour in place of hair

TABLE VI. (See Tables XV and XVI, pp. 378—379.)

Incidence of Disease in Eye Colour Groups.

Eye Colour	Boys. Age 13		Girls. Age 13	
	Cases of Disease	Average	Cases of Disease	Average
Blue... ...	623 | 1635	1·75 | 1·77	804 | 2027	2·08 | 2·11
Light ...	1012 |	1·78 |	1223 |	2·16 |
Neutral ...	997 | 2005	1·83 | 1·87	1226 | 2401	2·17 | 2·21
Dark ...	1008 |	1·90 |	1175 |	2·19 |
Totals	3640	—	4428	—

Eye Colour	Boys. Age 3—7		Girls. Age 3—7	
	Cases of Disease	Average	Cases of Disease	Average
Blue... ...	325 | 425	1·6 | 1·60	256 | 341	1·5 | 1·54
Light ...	100 |	1·7 |	85 |	1·8 |
Neutral ...	144 | 302	1·7 | 1·48	101 | 205	1·6 | 1·48
Dark ...	158 |	1·4 |	104 |	1·4 |
Totals	727	—	546	—

colour. There is no reason to suppose *a priori* that, owing to the correlation between hair and eye colour, this table must show a similar absence of connection between eye colour and disease. If there be three characters A, B and C of which

* *Biometrika*, Vol. III. p. 459.

A and *B* are correlated and *A* and *C* are correlated, there must be a correlation of over ·75 between *A* and *B*, and between *A* and *C* in order that *B* and *C* should be necessarily correlated. This table, however, does show a very similar result to the preceding table; and we do not find that any particular eye colour is associated with a greater or less average number of diseases; nor do we find, if we divide the classes of eye into light and dark, that there is any significant difference between them. In one case it might appear at first sight that there was some significant difference. The average number of diseases to both dark eyed boys and girls aged 3—7 falls somewhat below the general average. Calculations were made to test for random sampling according to the method given by Elderton*. *P* was calculated in each case. For the boys aged 3—7, *P* worked out at ·33 and for the girls at ·37, or in other words a less regular sample would occur more than 30 times in a hundred chances. There is, therefore, no importance whatsoever to be attributed to the drop in the average for the younger dark eyed boys and girls.

TABLE VII.

Incidence of Disease in Hair Colour Groups. Percentages.

Boys and Girls. Age 13.

Hair Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Red	79·0	87·8	38·7	40·5	17·7	9·5	1·6	4·0	27·6	28·4	37·1	29·7
Fair	73·1	87·4	31·7	44·4	14·0	16·2	2·8	5·5	20·2	21·1	30·1	29·6
Light Brown	71·9	86·0	34·2	44·7	14·5	21·5	3·2	4·4	19·8	26·9	27·9	35·7
Medium Brown	77·6	85·8	39·9	44·9	17·4	19·7	4·7	5·2	20·9	22·9	28·4	38·8
Dark Brown	75·5	84·5	37·8	44·2	17·7	17·4	4·2	4·5	23·3	22·3	27·2	38·8
Black ...	79·5	84·8	52·0	47·9	22·6	19·6	0·0	2·2	22·6	13·1	48·4	43·5

TABLE VIII.

Incidence of Disease in Eye Colour Groups. Percentages.

Boys and Girls. Age 13.

Eye Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Blue ...	73·7	85·6	36·7	41·3	15·6	22·2	4·9	3·1	19·5	21·3	25·8	35·7
Light...	75·2	85·9	36·8	45·9	15·0	18·6	2·5	6·9	18·3	22·3	30·4	32·8
Neutral	73·5	85·9	36·4	44·7	17·5	18·6	4·4	4·6	25·0	27·5	26·7	35·9
Dark ...	77·9	84·3	38·3	45·0	17·5	15·8	4·2	3·7	22·5	21·4	30·7	39·4

* *Biometrika*, Vol. I. p. 155.

These tables refer one to hair and the other to eye colour. They are both constructed on the same lines, and give the percentage out of the total of the various hair and eye colour categories which have had the diseases in question. They include only the older children. If any one hair or eye colour was definitely connected with a special liability to disease, these tables ought to show it. If, for example, dark hair was less liable to disease, we ought to find some indication of it here. But the variations that do occur appear wholly irregular, and give no indication of any systematic correlation between one shade of pigmentation and disease—whether we look at diseases as a whole or at any one disease alone. There are a few deviations from the general course of averages which appear at first sight to be somewhat remarkable; thus, 52 per cent. of the black haired boys had whooping cough, and only 13 per cent. of the black haired girls had mumps. But such cases always occur where the probable error is very large, due in this case to the fact that the number of black haired children out of the total is less than 2 per cent. In Table VIII, which deals with eye colour, the deviations are not so remarkable since the eye colour groups form more equal divisions of the total number than the hair colour groups do; in this table, where large deviations are found, it is due to the fact that in certain diseases, such as diphtheria, the total number of cases is very small.

In discussing the fact that girls were found to have a larger average number of diseases than boys, it was stated that this was not due to a special outbreak of one disease among the girls. These tables provide the justification for this statement; it can be seen that the girls in all six diseases have a larger average number than the boys.

Tables IX and X are constructed on exactly similar lines to Tables VII and VIII, but refer to the younger children aged 3—7. They show the percentage of each hair and eye colour group that have had each of the six specified diseases. It is again equally evident that no general coincidence can be traced—such as the

TABLE IX.

Incidence of Disease in Hair Colour Groups. Percentages.

Boys and Girls. Age 3—7.

Hair Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Red	71·4	85·7	38·6	42·9	9·5	14·3	4·8	0·0	23·8	0·0	23·8	14·3
Fair	78·9	64·7	36·5	44·1	5·7	8·8	0·0	0·0	17·3	0·0	40·4	32·4
Light Brown	63·3	64·0	31·2	43·0	7·0	6·1	0·8	2·6	10·2	18·4	30·5	39·5
Medium Brown	57·7	57·6	38·1	38·2	7·4	9·7	2·1	2·1	17·5	18·8	33·9	24·3
Dark Brown	59·7	45·0	41·6	33·3	5·2	5·0	1·3	0·0	13·0	13·3	28·6	16·7
Black ...	33·3	100·0	33·3	100·0	0·0	0·0	0·0	0·0	0·0	0·0	33·3	100·0

TABLE X.

Incidence of Disease in Eye Colour Groups. Percentages.

Boys and Girls. Age 3—7.

Eye Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Blue ...	62·0	58·4	37·0	38·2	8·2	5·8	1·5	1·2	14·9	17·3	31·7	27·2
Light...	72·4	68·8	41·4	52·1	5·2	10·4	1·7	4·2	10·3	8·3	41·4	33·3
Neutral	65·2	57·8	39·3	39·1	4·5	10·9	1·1	1·6	19·1	20·3	34·8	28·1
Dark ...	65·7	54·7	32·2	34·7	7·0	8·0	1·7	1·3	21·8	12·0	27·0	28·0

alliance of one hair colour with a general excess of disease or liability to one disease. There is a greater variation and irregularity in these percentages, due to the smaller number of children, and to the consequently larger probable errors.

TABLE XI.

Values of P.

		Measles	Whooping Cough	Scarlet Fever	Diphtheria	Mumps	Chicken Pox
Hair	Boys, Age 13	·298	·075	·506	·346	·611	·098
	„ „ 3—7	·171	·612	·661	·969	·522	·264
	Girls, „ 13	·787	·990	·123	·899	·164	·115
	„ „ 3—7	·097	·756	·416	·770	·082	·019
Eye	Boys, „ 13	·304	·909	·596	·238	·039	·215
	„ „ 3—7	·079	·825	·963	·651	·356	·679
	Girls, „ 13	·833	·571	·102	·023	·044	·143
	„ „ 3—7	·477	·256	·513	·543	·248	·858

In the above table all the data used in calculating all the preceding tables of percentages are employed in a different way. Percentages alone give no clear or accurate measure of such a point as we are investigating; we want to obtain some definite indication of the value and importance of the fluctuations which the percentages showed. This can be obtained by using Elderton's tables to test for random sampling. It involves calculating χ^2 for the various distributions; this is done by the use of Pearson's formula given in a recent number of *Biometrika**. The actual procedure is as follows: we are dealing with forty-eight groups and we may consider any one of them. We may, for instance, take the girls aged 13 arranged according to the eye colour classification, when the cases of measles among them are given. There are therefore two populations—an infected and a non-infected population; and we have two independent distributions of the frequency of eye colour, namely in an infected and in a non-infected population.

* *Biometrika*, 1911, Vol. vii. p. 186.

The method then enables us to test whether both are random samples of a general population. This is done in each of the forty-eight cases, and the χ^2 is found. Having calculated the χ^2, it is then possible to read off the P from Elderton's tables. P gives a measure of the probability of the occurrence of such a distribution. When P is found to be ·1 or more, it cannot be said that the distributions are significantly different. This indicates a probability of 1 in 10. ·01 would mean 1 in 100, and this would show that the distribution was almost certainly not a random sample. Any values of P between ·1 and ·01 which occur in the table are worth noticing. There are eight values of P which work out at less than ·1, but none of them indicative of probability of less than 1 in 50, which is in itself hardly significant in 48 trials. When, therefore, we examine the table, and find that these low values of P occur entirely at random, we cannot draw any conclusions from it. It would be reasonable to expect to find one particular age or one disease distinguished by a low value of P. In the former case it would indicate that children of that age group were subject to selection from disease; or in the latter case there would be reason to think that the disease in question at all ages exercised a selective influence. On looking at the columns giving the values of P for the six different diseases, it is clear that one column does not tend to show a lower average value of P than the others. As regards ages, however, there are some indications which might at first seem to show that the distribution of cases was not equally random between the older and younger children. In examining the measles column, it is clear that the values of P for the younger children are always less than those which we find for the older children. If the values of P were low enough to be really significant in the case of the younger children, we should have some grounds for thinking that the younger children were being selected by measles; or, in other words, that some type represented in one or more of the pigmentation classes was in process of elimination owing to a greater susceptibility to measles. But, as regards measles, the figures do not warrant any conclusion of the kind; for, although P is always larger for the older children, the difference in the value of P between the two age groups is sometimes only slight, as between ·298 and ·171; and further the lowest value of P is only ·079, which indicates a probability of 1 in 13, by no means an especially low value. All that can be said is that these figures for measles are possibly suggestive.

When we turn to the other diseases, we cannot discover any regular alternation of probabilities between older and younger children, such as there is an indication of in the case of measles. In the case of whooping cough, what we discovered in measles shows some signs of appearing again; but the series is spoilt by the fact that the value of P for boys 13 years old is considerably less than for boys 3—7 years old. The other columns do not even show an approach to a regular series. Therefore, from this method of regarding the problem, we are justified in saying that the distributions of the various diseases appear with these frequencies to be entirely random; and that this is equivalent to saying that there is no

correlation between any of the hair or eye colour classes and disease in general or any one disease in particular.

There is, however, another point of view from which we may look at this table. It shows no orderly system from the standpoint of age, sex and disease. But in another way the table seems to be improbable. In 48 cases we have enquired whether the samples are so divergent that they ought to occur. Therefore there ought to be one case that would only occur once on the average in 48 trials, or in other words with a probability of ·02. Now, there are actually two such cases ·023 and ·019. In the same way there should be five cases which occur once in ten trials—or with a probability roughly of ·1. Actually there are nine cases instead of five below ·1. There should be between nine and ten cases below ·2; actually there are fifteen. Thus it would seem that the whole table is somewhat improbable as a result of random sampling; but there are no systematic differences to be made out of it. Is the general irregularity a result of personal equation?

General Conclusions.

The data used in this section of my work consist of random samples of boys and girls at two different ages. We possess a classification of their pigmentation, and figures giving the number of cases of six specified diseases. An analysis of this data has shown that no obvious systematic connection can be discovered between any one category of pigmentation and disease. In other words, taking any child in our population we should not, starting from our knowledge of its pigmentation, be able to say that there was any probability of the child having had any more or less diseases than the average, or that there was a greater or less probability of the child having had one particular disease. Are we justified in going a step further and saying that in this population pigmentation is not a factor in natural selection? Suppose, for example, that the fair haired children were in process of elimination through selection, owing to their greater susceptibility to disease, would these figures show it? Now it is impossible to compare the proportions of the different categories of pigmentation, in the two age groups, and thus to estimate if these change as the children grow older, because the change in pigmentation cannot be corrected for with sufficient accuracy; and thus we cannot say how much of any change in the relative proportions of the classes of pigmentation is due to increasing age and how much to selection. We merely have the fact that boys and girls in this population, aged 3—7 and 13, do not show any connection between pigmentation and disease. Now, we argued before that if boys were being selected owing to their liability to disease, the girls should show a larger average number of diseases per child. In the same way we may argue that if the lighter haired children were being selected, the darker haired should show a greater average number of diseases per child. But we do not find anything of the kind. This argument, however, is by no means conclusive; it depends upon the assumption that, if one of two classes is in process of elimination through disease, that class differs from the other class simply in the fact that

the mortality is greater. Each class or category is, according to this argument, supposed to have the same number of diseases, but one class to show less resistance and to suffer a larger mortality; and, therefore, when the two classes are examined at a later period, one is found to have more individuals who have suffered from disease. But this assumption may be false; that class which has the higher mortality may very likely have also a larger number of cases of disease; for a higher mortality might indicate a smaller power, not only to recover from disease, but also to resist the attacks. Thus that class which had the higher mortality would also have the larger number of cases. Therefore when considering the two classes, we might expect to find an equal number of individuals in both categories who had had disease. If this reasoning be correct, such results, as those we have reached in this memoir, may fail entirely to show whether selection is taking place or not. It can only be said that these results make it unlikely that pigmentation is a basis of selection; but definite results can only be obtained from data which include not merely the surviving members of the population, but those which have died as well.

In his recent memoir Macdonald* deals with the same problem. His figures come from Glasgow; and in comparing his results with those reached in this paper, it must first be remembered that the populations in Glasgow and Birmingham are by no means of similar composition. The Birmingham population is probably as homogeneous as that of any large English city. The foreign element is extremely small; children with foreign names in the schools are found very rarely. In Glasgow, on the contrary, there are large Irish and foreign elements; and this fact may be of great importance in comparing the data derived from these two cities. Macdonald obtained figures from Fever Hospitals giving the pigmentation of children who were admitted for scarlet fever, diphtheria, measles and whooping cough. The pigmentation was measured by Tocher's method†. The children in the hospitals were then compared with Tocher's results from his "Pigmentation Survey of Scottish School Children." Differences in the proportions of various classes of pigmentation between these two populations were attributed to the greater liability of those classes of pigmentation, which were found to be in excess in the hospitals, to contract disease. Before examining Macdonald's results, it may be pointed out that the fact that the pigmentations of the general and fever populations were not measured by the same persons in the same survey makes it possible that the difference in the pigmentation classes in the two populations was due to personal equation. It is difficult to avoid such sources of error when the determination of such characters as hair and eye colour is not made by some very strict scale.

Putting aside the question whether this fact could affect the value of Macdonald's results, his conclusions are at variance with those we have reached. He finds that the medium haired and medium eyed child is more liable to be

* Macdonald, *Biometrika*, Vol. VIII. p. 13 *et seq.*

† Tocher, *Biometrika*, Vol. VI. p. 147.

infected than a child belonging to any other category. The dark type is the least liable to infection. It is unnecessary to quote his conclusions in detail; he has made out a number of similar points. The Birmingham and Glasgow data thus seem to come into conflict; but a possible explanation may be found in the fact above noted of the different composition of the two populations. The different racial groups in Glasgow may possibly represent types whose liability to disease is of unequal intensity, while no such racial differences exist in Birmingham. At this point we are approaching the question of local races in man, a subject about which, as yet, very little is known. It is quite uncertain whether we can justly use the term local races, and we are entirely in the dark as to how far those characteristics which are supposed to determine local races are to be attributed to the influence of heredity or of the environment.

Certain of Macdonald's results refer to the mortality in the hospitals and the power of the children to recuperate. These results, which are based on data collected in the hospitals alone, cannot be affected by the objection on the ground of personal equation which might be brought against his comparison of the hospital data with those collected for the school children outside. This latter series of results confirms his general conclusion that pigmentation is a factor in selection, since he finds that ability to resist disease when once contracted is connected with hair and eye colour.

To sum up: The conclusions reached in the first section of this paper would seem to indicate that pigmentation is not a factor in selection; but the data at our disposal do not enable any very decisive answer to be given. The apparent opposition between these results and those reached by Macdonald may possibly be explained by the fact that the data used in the two inquiries are derived from very differently composed populations.

II. The Relation of Pigmentation to Height and Weight.

Introduction. In the introduction to the first section of this paper attention was called to the small quantity and vague character of the work which had been done upon the problem with which that section was concerned. But in connection with this second problem we find an almost complete absence of exact investigation. The interest of the question in relation to local races and similar biological subjects is obvious; nor is its importance from other points of view less great. Until we have some exact knowledge of the relative value of hereditary and environmental influences, we are not justified in attributing greater importance to the one than to the other. In spite of this absence of knowledge it is the fashion at the present day to attribute all low averages of such characters as height and weight, when found in connection with poor social conditions, to those conditions no pause is made to enquire whether the low average of the physical measurements might not be racial characters which are developed without, or at least with only slight, reference to the environment. Now, it is a matter of common knowledge that races are distinguished by differences in pigmentation; if, therefore, a relation

between pigmentation and height and weight were established, there would no longer be any justification for attributing these characters entirely to the environment. In order to elucidate this question the ideal material to examine would be data collected from a city where there were known to be different racial elements. All the individuals examined should be of the same age, and preferably should have been born in the district, or at least have lived a long time there. But such material is not easy to get; the examination of school children, however, does provide useful data. It is better that the individuals should be of the same age, since in that case no correction is required for the change in pigmentation owing to increasing age. Although such a correction can be made, it introduces a disturbing factor which does not tend to increase the accuracy of the results. Birth and not only residence in the same district is a desirable feature for all the individuals, since the results are otherwise open to the criticism that the influence of an earlier environment has not been taken into account. If, with such material as is here indicated, a statistical examination were to show that pigmentation as a racial character was correlated with other physical characters, then we should have obtained an important result which would bear on the correct degree of influence to be attributed to social conditions.

The data which have been employed in the present work are for our immediate purposes unfortunately far from ideal. They are derived from two sources. Firstly, there is the medical survey of Birmingham School Children, which has been used in the first section of this paper. The one advantage which it offers is the fact that all the children are the same age; and therefore there is no need to correct for a change of pigmentation. But we do not know how long the children have been in Birmingham or where they were born. This, however, is not so serious a defect as the fact that the Birmingham population is a singularly homogeneous one. Foreign names are extremely rare among the children. This means that, if in such a population a connection between pigmentation and stature is found, it must be a very important feature indeed. The failure to find it here would not be surprising, and it might well exist in a more heterogeneous population. The Birmingham data give both height and weight for every child. Now, as has been explained in the first part of the paper, if two characters A and B are correlated and A is also correlated with another character C, then there is no reason to expect a correlation between B and C unless the correlation between A and B is very high. In other words, although we know that height and weight are correlated, if we find a correlation between pigmentation and height, there is no reason to expect a correlation between pigmentation and weight, unless the correlation between weight and height is a very high one. Therefore the investigations of height and weight and their relation to pigmentation are treated as entirely separate.

Secondly, there are the data collected by Tocher in his "Survey of the Pigmentation of the Scottish Insane*." In this case we have height and pigmentation

* *Biometrika*, Vol. v. p. 298.

only. Although many correlations were worked out and various points investigated by Tocher from his data, he did not use his material to throw light on this point. The Scottish data are more numerous than those from Birmingham, and the population more heterogeneous. But the environmental conditions are by no means the same, and moreover the population is a selected one. Therefore any positive results from such data, though interesting, would be far from conclusive, and would demand further investigation of the subject.

Birmingham Data. Height and Pigmentation. The method of pigmentation classification used at Birmingham has already been described. The height was taken to the nearest millimetre. This latter fact is of importance in calculating the means. In order to form a table the numbers of each hair colour group were classed into groups of two centimetres each. Now, owing to the fact that the height is taken to the nearest millimetre, the middle point of the group 150—152 centimetres will not be 151 but 150·95. The data were first treated by calculating the means and probable error for the total number of boys and girls and thus for each of the six hair colour groups. The result is given in the following table.

TABLE XII. (See Tables XVII and XVIII, pp. 380—381.)

Hair Colour and Height.

Mean Height in Centimetres.

Hair Colour	Boys. Age 13	Girls. Age 13
Red	142·86 ± ·64	144·15 ± ·51
Fair...	142·47 ± ·28	144·72 ± ·34
Light Brown ...	142·39 ± ·22	143·71 ± ·24
Medium Brown ...	142·41 ± ·19	144·69 ± ·19
Dark Brown ...	143·69 ± ·21	145·41 ± ·20
Black	142·69 ± ·93	144·56 ± ·60
General Mean ...	142·80 ± ·10	144·67 ± ·12

It is there seen that at this age girls are substantially taller than boys, i.e. by about two centimetres. When we look at the means of the hair colour groups, it is at once evident that none of them are going to show any marked deviation from the general mean. There are, however, some cases which demand closer attention; these are the dark brown hair groups in both cases, and the light brown hair group in the case of girls. In order to show this more clearly two figures have been constructed—one for the boys and the other for the girls.

The method adopted in both these cases is the same. A straight line shows the mean of the whole number of boys or girls as the case may be. The means of the various hair colour groups have been marked and joined together. Two other lines have been introduced and tabled as indicating twice the probable error. The

probable error here referred to was obtained in the following way. The formula for the probable error of the mean $67449 \frac{\sigma}{\sqrt{N}}$ was used; σ was taken in each case to represent the standard deviation of the mean of the whole, while N was the total of each hair colour group. Thus instead of calculating the probable error of each mean in the usual manner, the probable error of each mean with reference to the mean of the whole is obtained. This gives the easiest method of estimating the value of the separate means from the mean of the whole. Lines have been drawn in each table to mark twice the probable error thus calculated. On looking at Figure I A it is clear that the only group which need occupy our attention is that of the dark brown hair. All the other means fall well within the line representing twice the probable error; but the mean of the dark brown hair group stands outside it. A line representing four times the probable error would include it, and, therefore, the deviation is not a very serious one; on this evidence alone it

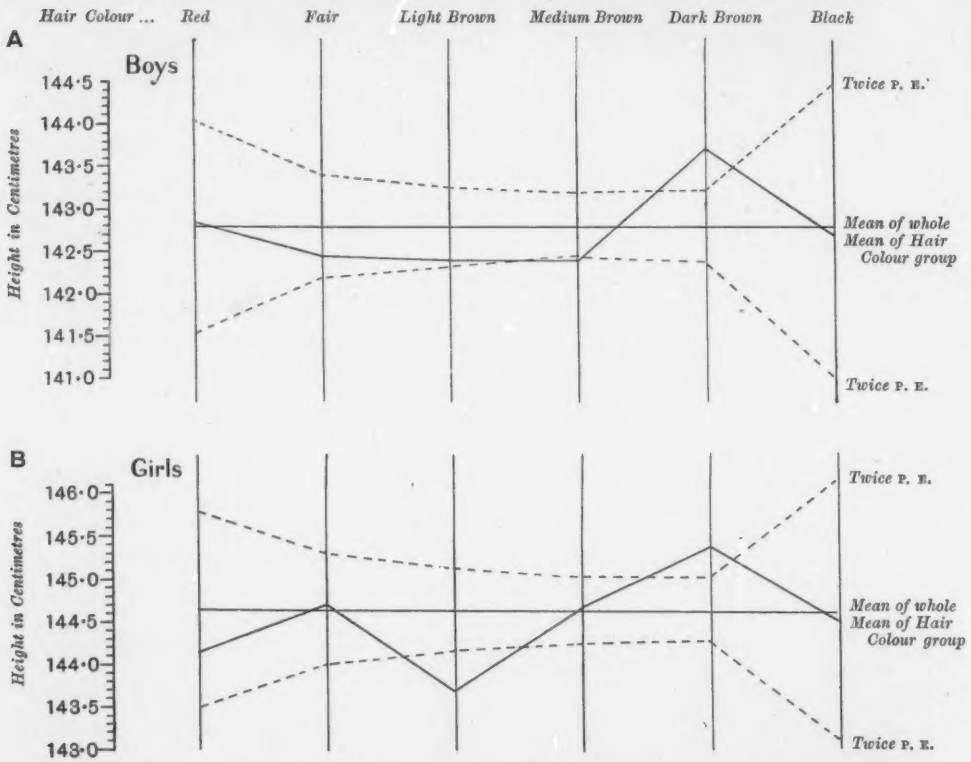


Fig. I. Age 13. Hair Colour and Height.

cannot be called a really significant difference. But Figure I B which is constructed in exactly the same manner for the girls gives reason for thinking that there may be some significance attaching to the greater height of the dark brown group; for in this case again the dark brown group shows a distinctly greater mean height. It is not quite, though it is nearly, as great a deviation from the general mean as in the case of the boys. One other hair colour group also calls for attention in Figure I B. The mean height of the light brown hair group is considerably less than the general mean; it would be included by a line drawn to represent about three and a half times the probable error. It seems unlikely that boys and girls would differ in this respect, and that the same pigmentation class should be associated with a significant difference in stature in one sex and not in the other. But apart from the improbability of this occurrence the deviation is not such as would be accounted very markedly significant in any case.

Scottish Hair Data. Height and Pigmentation. As has been already mentioned, the data collected by Tocher in his survey of the Scottish insane have also been employed to investigate the problem of the relation between pigmentation and

TABLE XIII. (See Tables XIX and XX, p. 332.)

Pigmentation and Height. Scottish Insane.

Mean Height in Feet.

	Men	Women
Hair Colour:		
Red	5 feet 7 inches ·31 ± ·27	5 feet 1 inches ·92 ± ·21
Fair	5 „ 6 „ ·42 ± ·1	5 „ 1 „ ·49 ± ·24
Medium	5 „ 6 „ ·33 ± ·04	5 „ 1 „ ·51 ± ·04
Dark	5 „ 6 „ ·28 ± ·05	5 „ 1 „ ·72 ± ·05
Eye Colour:		
Light	5 „ 6 „ ·30 ± ·05	5 „ 1 „ ·53 ± ·08
Medium	5 „ 6 „ ·46 ± ·05	5 „ 1 „ ·70 ± ·05
Dark	5 „ 6 „ ·28 ± ·07	5 „ 1 „ ·60 ± ·06
Hair and Eye Colour:		
Red Light ...	5 „ 7 „ ·10 ± ·31	5 „ 0 „ ·76 ± ·39
Medium...	5 „ 7 „ ·63 ± ·74	5 „ 2 „ ·95 ± ·24
Dark ...	5 „ 7 „ ·60 ± ·76	5 „ 0 „ ·71 ± ·56
Fair Light ...	5 „ 6 „ ·51 ± ·18	5 „ 1 „ ·27 ± ·31
Medium...	5 „ 6 „ ·58 ± ·21	5 „ 2 „ ·02 ± ·39
Dark ...	5 „ 7 „ ·08 ± ·47	5 „ 1 „ ·75 ± ·66
Medium Light ...	5 „ 6 „ ·27 ± ·15	5 „ 2 „ ·35 ± ·17
Medium...	5 „ 6 „ ·49 ± ·07	5 „ 1 „ ·41 ± ·08
Dark ...	5 „ 6 „ ·12 ± ·1	5 „ 1 „ ·60 ± ·11
Dark Light ...	5 „ 6 „ ·21 ± ·11	5 „ 1 „ ·47 ± ·10
Medium...	5 „ 6 „ ·34 ± ·09	5 „ 1 „ ·89 ± ·08
Dark ...	5 „ 6 „ ·29 ± ·09	5 „ 1 „ ·69 ± ·07
General Mean ...	5 „ 6 „ ·32 ± ·03	5 „ 1 „ ·6 ± ·03

height. Both the methods used and the colour categories adopted to classify pigmentation are in this case different from those employed at Birmingham. There are four hair colour groups—red, fair, medium and dark, and three eye colour groups, light, medium and dark. In this case again the usefulness of the data suffers from lack of precision in classification. The height was taken to the nearest inch.

The data have been treated in the same way as those derived from Birmingham. Table XIII corresponds to Table XII; but the mean height and probable error have here been found not only for the hair colour groups, but also for the eye colour groups, and every combination of hair and eye colour. An inspection of this Table shows how little deviation there is from the general mean. When we look to see whether there is any indication of the same deviations from the general mean commented upon in connection with the Birmingham data, we fail to find any correspondence whatever. In this case, on the whole, there is less deviation than at Birmingham, and there is nothing to call for attention.

In order to make this more clear, figures have been constructed on the same principle as that followed to form Figures I A and B. Here again in Figure II A, B, C and D the mean of the whole is represented together with the means of the various groups and twice the probable error calculated with reference to the mean of the whole.

Figure II A and B gives the height and hair colour shown according to this method. In this case we have the only mean which exceeds twice the probable error. The mean of the red haired men is considerably in excess of the general mean, but not so much as to be of itself very markedly significant; and further, when it is found that among the women in Figure II B no such very marked deviation is indicated, then we are probably justified in thinking that it does not represent any true connection between red hair and stature in excess of the mean. A glance at Figure II is enough to show that the differences are, except for red-haired men, absolutely insignificant, and that there is no indication whatever to be found of those deviations which we observed in the Birmingham data.

Height and Pigmentation. Summary. The data, therefore, which have been examined in this paper do not afford any grounds for the belief that pigmentation and height are correlated. As has been pointed out, there are some vague indications of possible association between a certain pigmentation class, namely dark brown, and a stature in excess of the general mean. But this scarcely does more than to provide hints for future research; and the main essential for such research is that pigmentation should be measured with greater accuracy. With data such as have been at our disposal, which in the first place are derived from populations that would not be deliberately chosen for such an investigation, and which are lacking in the accuracy of pigmentation classification that is desirable, it would be surprising to reach any markedly definite result if the relation be really small; the failure to do so, therefore, does not make it less probable than before that,

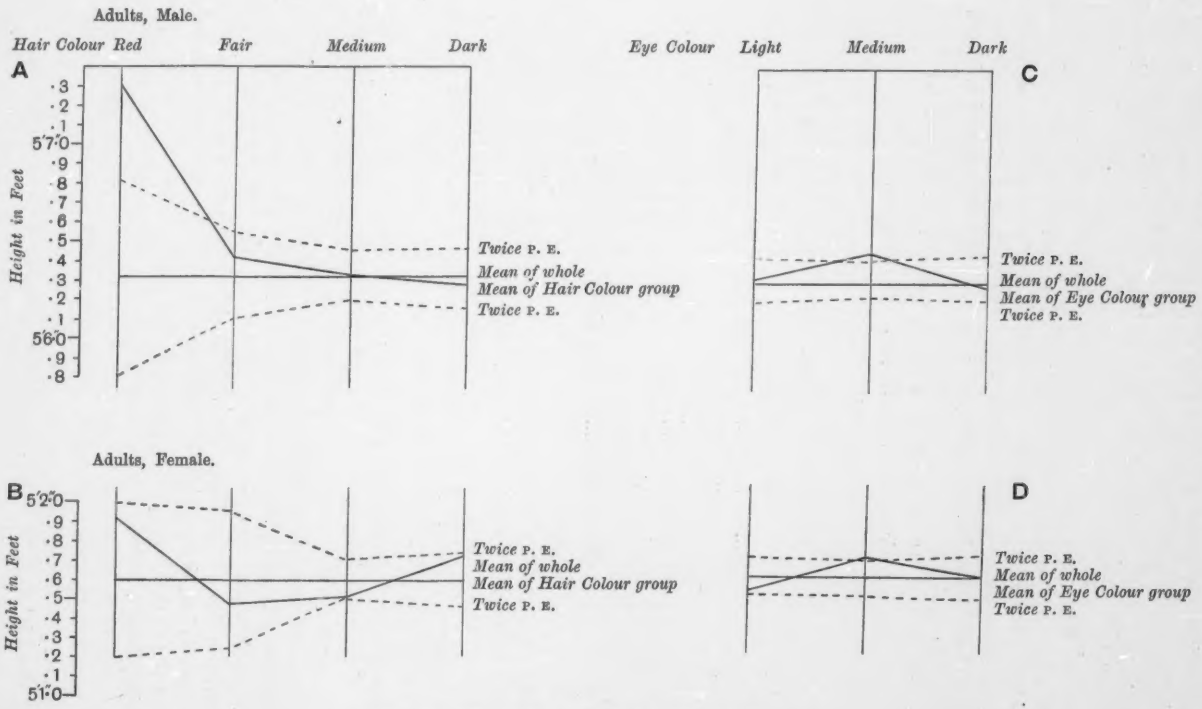


Fig. II. Pigmentation and Height. Scottish Insane.

with more suitable means, some *not very marked* connection between pigmentation and height may ultimately be found.

Birmingham Data. Pigmentation and Weight. The Birmingham data also give the weight of the children, and therefore afford an opportunity of investigating the relation between pigmentation and weight. As has been explained, this is a problem separate from that of the relation between pigmentation and height, and there might possibly be a correlation between pigmentation and weight although there were none between pigmentation and height. The hair and eye colour method of classification has been described; the weight was measured in kilogs.

TABLE XIV. (See Tables XXI and XXII, pp. 383—384.)

Hair Colour and Weight. Birmingham.

Mean Weight in Kilogs.

Hair Colour	Boys. Age 13	Girls. Age 13
Red	33·69 ± ·38	36·22 ± ·43
Fair	34·28 ± ·21	35·71 ± ·26
Light Brown ...	34·40 ± ·14	35·59 ± ·19
Medium Brown ...	34·15 ± ·12	36·28 ± ·15
Dark Brown ...	35·08 ± ·15	36·62 ± ·15
Black	34·95 ± ·57	35·92 ± ·49
General Mean ...	34·39 ± ·04	36·22 ± ·04

The data have been treated in exactly the same way as in the previous section of the paper. Table XIV gives the mean weight for the total number of both boys and girls and their probable errors; the means and probable errors are also given for each hair colour group. Again we notice no striking deviation from the mean; but certain groups differ from the remainder enough to require special attention.

Figure III is constructed in the same manner as before in the case of the pigmentation and height data. With regard to the boys, the only group which differs in any remarkable way is the dark brown group. The deviation is not very large, and would be covered by four times the probable error. It is interesting to find that among the girls the same deviation is apparent, though to a much smaller extent. In the latter case the mean of the group only falls just above the line representing twice the probable error. These deviations are similar to, but not quite so marked as, those which we have observed in the pigmentation and height table for the dark brown group. The coincidence is certainly interesting and suggestive, though it cannot fairly be called more. The coincidence is, however, further increased by the fact shown in Figure III B that the light brown group have a lower mean weight than the mean of the whole. It is impossible to estimate the true value, if there be any, of these results; further research alone can show whether there is anything in these possible connections between pigmentation and

height and weight. The general value of these latter figures with reference to pigmentation and weight is depreciated by just the same circumstances as those already discussed in connection with the data as a whole, and it is not necessary to repeat them here.

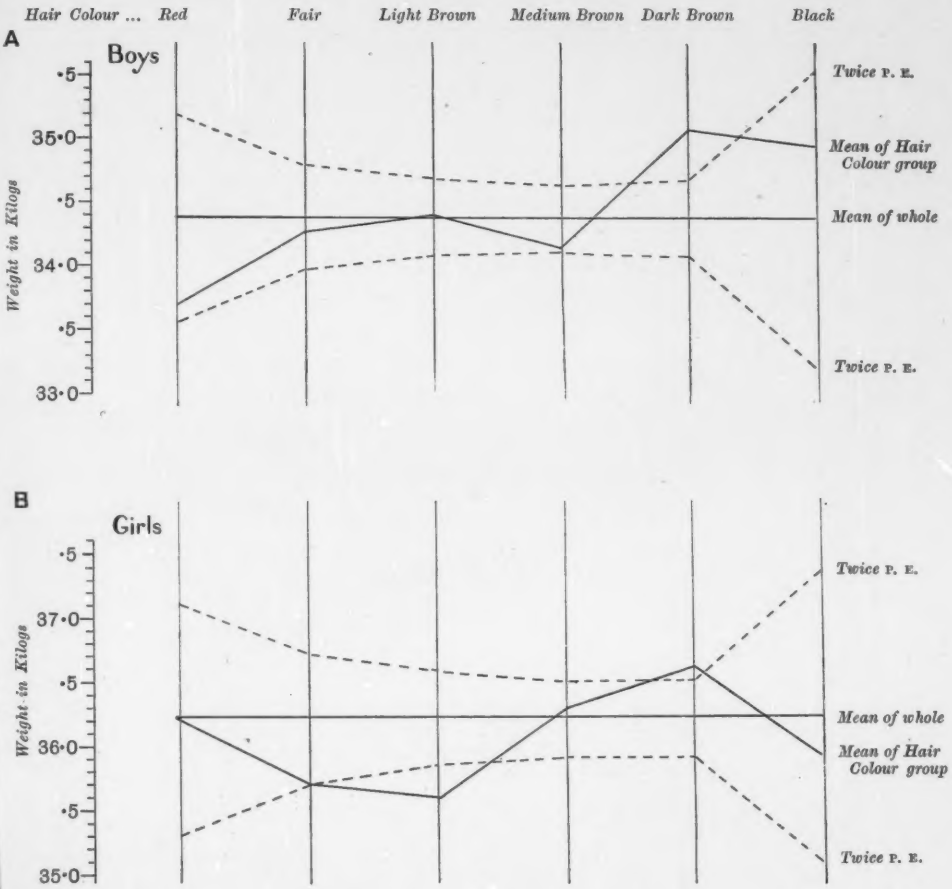


FIG. III. Age 13. Hair Colour and Weight.

TABLE XV.

Boys. Age 13.

Hair Colour	Eye Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox		Totals	Totals of Hair Colour
		n. a.	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*		
Red	Blue ...	5	2	1	6	—	7	—	7	1	6	2	5	7	62
„	Light ...	17	4	7	14	3	18	—	21	2	19	6	15	21	
„	Neutral	17	4	8	13	7	14	—	21	10	11	9	12	21	
„	Dark ...	10	3	8	5	1	12	1	12	4	9	6	7	13	
Fair	Blue ...	53	14	25	42	12	55	4	63	13	54	25	42	67	253
„	Light ...	75	34	33	76	14	95	—	109	23	86	36	73	109	
„	Neutral	48	19	19	48	8	59	3	64	13	54	13	54	67	
„	Dark ...	9	1	3	7	1	9	—	10	2	8	2	8	10	
Light Brown ...	Blue ...	84	36	43	77	14	106	3	117	21	99	26	94	120	443
„ ...	Light ...	116	42	59	99	25	133	3	155	29	129	44	114	158	
„ ...	Neutral	75	34	33	76	19	90	6	103	26	83	35	74	109	
„ ...	Dark ...	43	13	16	40	6	50	2	54	11	45	19	37	56	
Medium Brown	Blue ...	88	25	43	70	18	95	5	108	26	87	28	85	113	626
„ ...	Light ...	119	36	65	90	24	131	6	149	30	125	53	102	155	
„ ...	Neutral	152	47	77	122	37	162	9	190	44	155	52	147	199	
„ ...	Dark ...	127	32	65	94	29	130	11	148	31	128	44	115	159	
Dark Brown ...	Blue ...	30	16	18	28	11	35	5	41	8	38	9	37	46	579
„ ...	Light ...	99	25	45	79	19	105	5	119	20	104	33	91	124	
„ ...	Neutral	103	39	58	84	24	118	6	136	41	101	34	108	142	
„ ...	Dark ...	205	62	97	170	48	219	8	259	65	202	80	187	267	
Black	Blue ...	1	—	—	1	—	1	—	1	—	1	1	—	1	31
„	Light ...	1	—	—	1	—	1	—	1	—	1	1	—	1	
„	Neutral	4	2	3	3	—	6	—	6	2	4	2	4	6	
„	Dark ...	18	5	13	10	7	16	—	23	5	18	11	12	23	
Totals ...	...	1499	495	739	1255	327	1667	77	1917	427	1567	571	1423	1994	1994

n. a. = not-attacked; *a.* = attacked.

Totals of Eye Colour

Blue	354
Light	568
Neutral ...	544
Dark	528
Total ...	1994

Totals of Diseases

Measles ...	1499
Whooping Cough	739
Scarlet Fever ...	327
Diphtheria ...	77
Mumps ...	427
Chicken Pox ...	571
Total ...	3640

TABLE XVI.

Girls. Age 13.

Hair Colour	Eye Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox		Totals	Totals of Hair Colour
		n. a.	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*		
Red	Blue ...	16	2	7	11	1	17	1	17	4	14	3	15	18	74
„	Light ...	19	2	12	9	3	18	1	20	9	12	8	13	21	
„	Neutral	17	1	7	11	2	16	1	17	5	13	7	11	18	
„	Dark ...	13	4	4	13	1	16	—	17	3	14	4	13	17	
Fair	Blue ...	53	9	26	36	10	52	1	61	11	51	21	41	62	237
„	Light ...	100	16	55	61	18	98	9	107	21	95	30	86	116	
„	Neutral	40	4	17	27	8	36	2	42	13	31	12	32	44	
„	Dark ...	14	1	7	8	2	13	1	14	5	10	7	8	15	
Light Brown ...	Blue ...	77	15	43	49	23	69	2	90	21	71	38	54	92	406
„ ...	Light ...	137	23	71	89	34	126	11	149	46	114	47	113	160	
„ ...	Neutral	90	11	46	55	20	81	4	97	32	69	41	60	101	
„ ...	Dark ...	45	8	21	32	10	43	1	52	10	43	19	34	53	
Medium Brown	Blue ...	118	21	52	87	33	106	5	134	34	105	48	91	139	652
„ ...	Light ...	127	15	75	67	27	115	13	129	25	117	54	88	142	
„ ...	Neutral	188	35	101	122	40	183	11	212	60	163	77	146	223	
„ ...	Dark ...	126	22	64	84	28	120	5	143	30	118	54	94	148	
Dark Brown ...	Blue ...	61	9	29	41	17	53	3	67	12	58	24	46	70	672
„ ...	Light ...	111	23	51	83	23	111	6	128	26	108	49	85	134	
„ ...	Neutral	140	25	73	92	34	131	8	157	41	124	63	102	165	
„ ...	Dark ...	252	51	144	159	42	261	13	290	70	233	124	179	303	
Black	Blue ...	4	—	2	2	1	3	—	4	—	4	3	1	4	46
„	Light ...	2	2	—	4	2	2	—	4	—	4	1	3	4	
„	Neutral	10	3	8	5	1	12	—	13	4	9	3	10	13	
„	Dark ...	23	2	12	13	5	20	1	24	2	23	13	12	25	
Totals ...	...	1783	304	937	1150	385	1702	99	1988	484	1603	740	1347	2087	2087

n. a. = not-attacked ; *a.* = attacked.

Totals of Eye Colour

Blue	385
Light	577
Neutral ...	564
Dark	561
Total ...	2087

Totals of Diseases

Measles ...	1783
Whooping Cough	937
Scarlet Fever ...	385
Diphtheria ...	99
Mumps ...	484
Chicken Pox ...	740
Total ...	4428

TABLE XVII.

Boys. Age 13. *Birmingham. Hair and Eye Colour with Height.*

Height in Centimetres.

Hair and Eye Colour.		112	114	116	118	120	122	124	126	128	130	132	134	136	138	140	142	144	146	148	150	152	154	156	158	160	162	164	166	168	170	172	Totals
Red	Blue ...	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	3	1	1	—	—	—	—	—	—	—	—	—	—	—	7
„	Light	—	—	—	—	—	—	—	1	2	—	1	1	2	1	3	1	5	—	1	—	2	—	—	1	—	—	—	—	—	—	—	21
„	Neutral	—	—	—	—	—	—	—	—	—	2	1	2	1	—	3	3	—	—	4	1	2	1	1	—	—	—	—	—	—	—	—	21
„	Dark ...	—	—	—	—	—	—	—	—	—	—	1	—	2	3	—	1	3	1	1	—	1	—	—	—	—	—	—	—	—	—	—	13
Fair	Blue ...	—	—	—	—	—	—	—	—	—	5	5	4	6	8	12	3	7	5	3	6	2	—	—	—	1	—	—	—	—	—	—	67
„	Light	—	—	—	—	—	—	—	1	—	3	4	7	11	11	8	17	11	14	10	4	3	4	1	—	—	—	—	—	—	—	—	109
„	Neutral	—	—	—	—	—	—	—	2	1	1	4	1	9	4	6	7	8	8	12	—	1	—	—	1	1	1	—	—	—	—	—	67
„	Dark ...	—	—	—	—	—	—	—	—	1	1	1	—	—	1	2	1	—	—	1	—	1	—	—	—	1	—	—	—	—	—	—	10
Light Brown	Blue ...	—	—	—	—	—	—	—	—	1	4	5	7	19	6	13	15	10	16	10	3	5	2	2	2	—	—	—	—	—	—	—	120
„	Light	—	—	—	—	—	—	1	1	3	10	7	10	14	13	16	14	15	20	13	8	3	2	1	4	1	1	—	1	—	—	—	158
„ ...	Neutral	—	—	—	—	—	—	—	—	2	3	3	7	16	15	6	16	12	12	7	7	—	2	—	1	—	—	—	—	—	—	—	109
„ ...	Dark ...	—	—	—	—	—	—	—	2	1	2	4	5	5	2	5	4	7	5	3	5	4	1	—	1	—	—	—	—	—	—	—	56
Medium Brown	Blue ...	—	—	—	—	—	—	—	1	—	1	5	5	14	14	18	15	7	12	5	9	4	3	—	—	—	—	—	—	—	—	—	113
„ ...	Light	—	—	—	—	—	—	—	2	5	5	7	8	12	15	19	18	18	17	10	4	8	3	1	1	2	—	—	—	—	—	—	155
„ ...	Neutral	—	—	—	1	—	—	1	2	6	8	6	5	15	18	26	23	32	13	11	13	10	4	2	—	—	1	—	1	—	—	1	199
„ ...	Dark ...	—	—	—	—	1	—	1	1	3	4	5	14	18	22	13	22	18	7	9	8	6	1	2	2	—	—	—	1	—	—	1	159
Dark Brown ...	Blue ...	—	—	—	—	—	—	—	—	1	1	1	1	3	5	5	3	7	5	2	2	4	5	1	—	—	—	—	—	—	—	—	46
„ ...	Light	1	—	—	—	—	—	2	—	1	4	8	8	9	10	16	10	14	12	6	6	5	4	3	3	2	—	—	—	—	—	—	124
„ ...	Neutral	—	—	—	—	—	—	—	—	4	5	7	10	11	15	12	10	16	10	11	11	6	5	3	3	1	1	1	—	—	—	—	142
„ ...	Dark ...	1	—	—	—	1	1	1	3	1	2	6	13	18	25	30	34	36	25	18	14	17	4	5	7	5	—	—	—	—	—	—	267
Black	Blue ...	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
„	Light	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
„	Neutral	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	1	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	6
„	Dark ...	—	—	—	—	—	—	—	—	—	1	1	2	4	1	3	1	2	3	1	3	1	—	—	—	—	—	—	—	—	—	—	23
Totals ...	...	2	—	—	1	2	1	6	17	32	62	83	111	189	189	217	221	228	189	139	105	85	42	22	26	14	5	1	3	—	—	2	1994

TABLE XVIII.

Girls. Age 13. Birmingham. Hair and Eye Colour with Height.

Height in Centimetres.

Hair and Eye Colour.		106	112	114	116	118	120	122	124	126	128	130	132	134	136	138	140	142	144	146	148	150	152	154	156	158	160	162	164	166	168	170	172	184	Totals
Red	Blue ...	—	—	—	—	—	—	—	—	—	—	—	—	1	—	3	3	4	1	2	2	2		—	—	—	1	—	—	—	—	—	—	—	19
„	Light	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	2	2	1	4	4	—	2	1	1	—	—	1	—	—	—	—	—	—	21
„	Neutral	—	—	—	—	—	—	—	—	—	—	—	2	1	2	3	2	2	3	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	18
„	Dark ...	—	—	—	—	—	—	—	—	—	—	—	—	3	—	3	3	2	1	2	—	—	—	2	1	—	—	—	—	—	—	—	—	—	17
Fair	Blue ...	—	—	—	—	—	1	—	—	2	—	2	3	4	2	6	7	8	7	4	2	8	3	1	1	1	—	—	—	—	—	—	—	—	62
„	Light	—	—	—	—	—	—	—	2	—	4	—	1	6	7	6	9	12	12	10	13	12	6	4	5	1	—	3	1	1	1	—	—	—	116
„	Neutral	—	—	—	—	—	—	—	1	—	—	—	1	1	3	—	6	8	2	6	3	4	3	2	2	2	—	—	—	—	—	—	—	—	44
„	Dark ...	—	—	—	—	—	—	—	—	—	1	1	—	—	2	3	1	2	2	—	2	—	—	1	—	—	—	—	—	—	—	—	—	—	15
Light Brown	Blue ...	—	—	—	—	—	—	—	—	—	1	4	4	6	5	5	12	13	12	7	4	8	6	3	1	—	1	—	—	—	—	—	—	—	92
„ ...	Light	—	—	—	—	—	—	—	1	1	—	6	2	9	10	16	12	23	21	16	11	12	6	8	2	1	1	—	—	—	—	—	—	—	158
„ ...	Neutral	—	—	—	—	1	—	—	—	1	2	2	2	8	6	12	8	10	12	11	6	8	5	3	3	1	—	—	—	—	—	—	—	—	101
„ ...	Dark ...	—	—	—	1	—	—	1	—	2	—	—	1	—	4	3	7	6	3	3	6	3	4	6	1	—	1	—	—	—	—	—	—	—	52
Medium Brown	Blue ...	—	—	—	—	—	—	—	—	1	2	3	5	9	9	9	10	16	14	15	11	10	10	5	5	2	2	1	—	—	—	—	—	—	139
„ ...	Light	—	—	—	—	—	—	—	—	3	1	1	2	10	10	8	10	15	21	13	9	16	10	9	2	2	—	—	—	—	—	—	—	—	142
„ ...	Neutral	—	—	—	—	—	1	—	1	1	2	2	5	14	9	20	13	29	22	25	22	18	10	8	10	3	7	1	—	—	—	—	—	—	223
„ ...	Dark ...	—	1	—	—	1	—	—	1	—	1	—	6	7	12	10	16	18	18	20	14	8	7	3	3	1	2	—	—	—	—	—	—	—	149
Dark Brown	Blue ...	—	—	—	—	—	—	—	—	2	2	2	3	4	5	5	3	5	7	12	8	2	6	3	1	—	—	—	—	—	—	—	—	—	70
„ ...	Light	—	—	—	—	—	—	1	—	2	2	1	4	4	10	12	7	11	14	9	15	11	11	9	5	2	2	—	1	—	—	1	—	—	134
„ ...	Neutral	—	—	—	—	—	—	—	—	1	1	1	3	7	4	9	19	14	13	25	20	10	17	5	9	5	2	—	—	—	—	—	—	—	165
„ ...	Dark ...	1	—	1	1	—	—	1	—	1	3	3	9	15	12	25	23	29	26	31	41	30	20	15	4	3	3	2	2	—	—	—	—	1	302
Black	Blue ...	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
„	Light	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
„	Neutral	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	2	4	3	—	1	—	—	—	—	—	—	—	—	—	—	—	13
„	Dark ...	—	—	—	—	—	—	—	—	—	—	1	2	—	2	1	4	1	6	3	2	2	—	—	1	—	—	—	—	—	—	—	—	—	—	25
Totals ...	...	1	1	1	2	2	2	3	6	18	22	29	57	109	114	161	180	230	221	221	202	168	127	89	57	24	22	8	4	1	1	1	—	1	2085

TABLE XIX. *Men. Scottish Insane.*

Height in feet.

Hair and Eye Colour.		Four Feet												Five Feet												Six Feet				Totals
		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	*2*	*3*	
Red ...	Light	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	3	—	3	5	9	4	3	4	1	1	—	—	—	36
„ ...	Medium	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	2	3	3	1	2	—	1	1	—	—	16
„ ...	Dark ...	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	—	1	1	—	—	2	2	—	—	—	—	10
Fair ...	Light	—	—	—	—	—	—	—	—	—	—	—	2	—	4	5	16	21	20	26	26	26	9	8	1	4	—	—	—	168
„ ...	Medium	—	—	—	—	—	—	—	—	—	—	—	1	—	2	2	7	14	11	11	7	5	7	5	3	3	—	—	—	78
„ ...	Dark ...	—	—	—	—	—	—	—	—	—	1	1	—	—	2	2	1	3	2	5	—	6	—	1	—	—	—	—	—	24
Medium	Light	—	1	—	1	—	—	1	1	1	6	11	6	20	40	73	99	146	180	215	172	148	96	62	36	15	3	4	—	1337
„ ...	Medium	—	—	1	—	—	—	1	—	2	1	3	9	7	24	39	40	74	113	110	124	95	54	34	22	13	4	1	1	772
„ ...	Dark ...	—	—	1	—	—	—	—	—	1	1	2	2	6	12	29	31	44	51	60	44	32	30	15	14	5	1	—	—	381
Dark ...	Light	—	1	1	—	—	—	—	—	—	1	—	2	9	13	19	24	41	46	51	48	30	21	21	10	3	4	—	—	345
„ ...	Medium	—	—	—	—	—	—	—	—	2	2	1	5	8	11	23	37	49	68	76	70	52	27	28	7	8	3	1	—	478
„ ...	Dark ...	1	—	—	—	—	—	1	1	—	1	4	4	9	11	28	39	54	73	78	78	52	33	20	7	10	6	1	—	511
Totals	...	1	2	3	1	—	—	3	2	6	13	23	31	61	120	222	299	448	567	640	582	453	281	202	103	63	22	7	1	4156

TABLE XX. *Women. Scottish Insane.*

Height in Feet.

Hair and Eye Colour.		Three Feet				Four Feet												Five Feet												Six Feet		Totals
		8	*9*	*10*	*11*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	
Red ...	Light	—	—	—	—	—	1	—	—	—	—	—	—	—	2	2	4	6	1	6	2	2	1	—	—	—	—	—	—	—	—	27
„ ...	Medium	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	3	3	9	9	8	6	1	3	2	—	—	1	—	—	—	47
„ ...	Dark ...	—	—	—	—	—	—	—	—	—	1	—	—	1	—	2	1	2	—	4	2	—	1	—	—	—	—	—	—	—	—	14
Fair ...	Light	—	—	—	—	1	1	—	2	—	1	—	—	1	2	9	4	11	9	12	6	3	6	2	3	—	—	—	1	—	—	74
„ ...	Medium	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	5	1	3	6	3	6	1	—	1	—	—	—	—	—	—	29
„ ...	Dark ...	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	4
Medium	Light	—	—	—	—	1	—	1	1	1	3	2	11	16	43	62	104	139	178	171	117	71	42	19	13	4	1	—	—	1	1	1002
„ ...	Medium	1	—	1	—	—	5	1	1	—	—	1	7	7	30	49	67	88	91	95	76	45	33	11	5	2	2	—	—	—	—	618
„ ...	Dark	—	—	—	—	—	—	—	—	1	3	1	3	6	18	25	39	56	65	60	45	23	14	9	4	1	1	1	—	—	—	375
Dark ...	Light	—	—	—	—	1	—	—	—	—	1	—	4	13	13	20	33	63	54	44	44	20	15	9	3	2	—	—	—	—	1	340
„ ...	Medium	—	—	—	—	—	—	1	—	2	3	1	3	5	18	40	51	69	89	90	81	33	32	19	10	3	2	—	1	—	—	553
„ ...	Dark ...	—	—	—	—	2	1	—	2	1	—	2	8	10	20	46	58	66	98	100	63	49	30	14	11	6	—	—	—	2	—	589
Totals	...	1	—	1	—	5	8	3	6	5	12	8	36	59	147	259	369	504	597	599	448	258	176	86	52	18	6	2	2	3	2	3672

TABLE XXI.

Boys. Age 13. Hair and Eye Colour with Weight.

Weight in Kilogs.

Hair and Eye Colour.		20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	Totals
Red	Blue ...	—	—	1	—	—	—	3	—	2	—	1	—	—	—	—	—	—	—	—	7
„	Light ...	—	—	2	1	2	—	5	6	2	1	2	—	—	—	—	—	—	—	—	21
„	Neutral	—	—	1	2	1	6	2	2	1	2	2	2	—	—	—	—	—	—	—	21
„	Dark ...	—	—	—	—	1	3	3	4	2	—	—	—	—	—	—	—	—	—	—	13
Fair	Blue ...	—	—	2	2	12	7	17	13	6	3	2	—	1	1	1	—	—	—	—	67
„	Light ...	—	1	1	3	15	15	21	18	12	11	7	3	—	2	—	—	—	—	—	109
„	Neutral	—	—	1	3	6	9	12	11	7	10	3	2	1	—	1	1	—	—	—	67
„	Dark ...	—	—	1	1	2	—	1	1	—	1	1	—	1	—	—	—	—	—	1	10
Light Brown	Blue ...	—	—	1	7	15	17	25	14	20	9	9	—	1	2	—	—	—	—	—	120
„ ...	Light ...	—	—	2	11	18	24	29	24	21	7	10	7	2	2	—	—	—	1	—	158
„ ...	Neutral	—	1	1	6	8	18	32	15	17	3	5	1	—	1	1	—	—	—	—	109
„ ...	Dark ...	—	—	4	4	4	10	5	10	7	7	2	2	1	—	—	—	—	—	—	56
Medium Brown	Blue ...	—	—	2	6	11	12	35	17	11	8	5	5	—	1	—	—	—	—	—	113
„ ...	Light ...	—	—	3	6	23	15	30	35	20	8	9	5	—	1	—	—	—	—	—	155
„ ...	Neutral	—	—	5	10	15	36	39	25	23	23	6	10	5	1	1	—	—	—	—	199
„ ...	Dark ...	1	1	3	8	13	26	27	30	17	13	9	5	2	1	1	—	—	—	1	158
Dark Brown...	Blue ...	—	—	1	2	3	6	5	7	5	6	3	3	1	3	1	—	—	—	—	46
„ ...	Light ...	—	1	—	11	16	16	16	20	16	9	8	6	1	1	1	—	—	1	1	124
„ ...	Neutral	—	—	2	10	15	18	23	20	19	16	5	4	4	—	3	2	—	1	—	142
„ ...	Dark ...	2	—	2	9	17	39	44	49	34	35	14	8	5	5	1	—	2	1	—	267
Black... ...	Blue ...	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
„	Light ...	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
„	Neutral	—	—	—	1	1	1	1	1	—	—	—	—	—	1	—	—	—	—	—	6
„	Dark ...	—	—	—	—	1	7	3	3	2	3	4	—	—	—	—	—	—	—	—	23
Totals	...	3	4	35	103	199	285	378	326	244	175	107	64	25	22	11	3	2	4	3	1993

TABLE XXII.

Girls. Age 13. *Hair and Eye Colour with Weight.*

Weight in Kilogs.

Hair and Eye Colour.		*20*	*22*	*24*	*26*	*28*	*30*	*32*	*34*	*36*	*38*	*40*	*42*	*44*	*46*	*48*	*50*	*52*	*54*	*56*	*58*	*60*	Totals
Red	Blue ...	—	—	—	1	2	2	4	3	3	—	3	1	—	—	—	—	—	—	—	—	—	19
„	Light ...	—	—	—	—	1	1	1	4	3	1	6	2	—	1	—	—	1	—	—	—	—	21
„	Neutral	—	—	—	1	2	2	3	2	3	2	2	—	—	1	—	—	—	—	—	—	—	18
„	Dark ...	—	—	—	1	3	2	3	2	—	1	2	1	1	—	—	1	—	—	—	—	—	17
Fair	Blue ...	—	1	1	4	7	7	14	8	6	2	3	4	4	1	—	—	—	—	—	—	—	62
„	Light ...	—	1	2	7	5	11	10	17	15	17	8	8	5	4	3	1	—	2	—	—	—	116
„	Neutral	—	1	—	1	4	5	10	7	3	3	3	3	2	1	1	—	—	—	—	—	—	44
„	Dark ...	—	—	1	2	—	3	2	—	4	3	—	—	—	—	—	—	—	—	—	—	—	15
Light Brown	Blue ...	—	—	2	10	5	12	13	15	12	9	6	2	3	1	—	1	1	—	—	—	—	92
„ ...	Light ...	—	—	1	5	10	31	20	16	24	22	10	11	2	2	1	2	—	1	—	1	—	159
„ ...	Neutral	1	—	1	8	8	12	15	14	15	8	8	4	4	1	2	—	—	—	—	—	—	101
„ ...	Dark ...	—	—	1	3	4	2	11	6	7	4	3	1	1	4	3	1	1	1	—	—	—	53
Medium Brown	Blue ...	—	1	—	9	5	18	14	23	15	17	10	16	1	3	4	2	—	—	—	—	1	139
„ ...	Light ...	—	—	4	3	13	11	17	27	14	19	10	10	5	4	1	2	2	—	—	—	—	142
„ ...	Neutral	—	1	2	7	20	20	28	25	34	28	17	11	13	10	1	2	—	—	1	—	—	220
„ ...	Dark ...	2	—	2	5	10	15	25	35	14	11	11	7	5	1	4	1	—	1	—	—	—	149
Dark Brown ...	Blue ...	—	—	—	6	7	7	9	7	12	5	10	2	3	2	—	—	—	—	—	—	—	70
„ ...	Light ...	—	—	1	2	11	13	20	21	16	14	14	4	7	2	3	2	—	1	1	1	—	133
„ ...	Neutral	—	—	1	6	9	13	18	23	27	18	18	9	10	7	3	2	1	—	—	—	—	165
„ ...	Dark ...	—	2	3	8	20	24	40	46	41	42	28	17	16	6	3	1	2	—	2	2	—	303
Black ...	Blue ...	—	—	1	—	—	—	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	4
„	Light ...	—	—	—	—	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	4
„	Neutral	—	—	—	1	—	1	1	5	1	3	—	1	—	—	—	—	—	—	—	—	—	13
„	Dark ...	—	—	—	1	2	1	8	2	1	4	2	2	3	—	—	—	—	—	—	—	—	26
Totals ...	...	3	7	23	91	148	213	289	309	271	233	175	116	86	51	29	18	8	6	4	4	1	2085

SUPPLEMENTARY TABLES FOR FINDING THE CORRELATION COEFFICIENT FROM TETRACHORIC GROUPINGS.

By P. F. EVERITT, B.Sc.

Object of the Tables.

These tables have been prepared to facilitate the determination of the correlation coefficient by the method of the fourfold table in those cases where the correlation has a very high value. The usual method involves the solution of an equation in ascending powers of the correlation coefficient r; the numerical coefficients of these powers are the products of the corresponding terms of two series, alternate terms of these series being ultimately absolutely convergent. When r is large, these alternate terms will not in general converge rapidly, so that it usually becomes necessary to take a large number of terms into consideration and the work becomes very long and tedious. The present tables are designed to allow the value of r to be obtained easily and rapidly by a series of simple interpolations, thus avoiding the lengthy and rather intricate calculations formerly necessary and consequently decreasing the opportunities of making slips in the numerical work. It is precisely in the case of those fourfold tables for which r is large that we must go beyond the number of terms provided for in my paper on the tetrachoric functions.

Method of Construction.

The nomenclature used will be that of Pearson's* paper describing the method of the fourfold table, and explained at length in the Tables of Tetrachoric Functions for Fourfold Correlation Tables†.

* *Phil. Trans.* A, Vol. 195, pp. 1—47.

† *Biometrika*, Vol. VII. pp. 437—451, 1910. The notation is given by the scheme:

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

It has been shown by Elderton* that

$$\frac{d}{N}=\frac{1}{2\pi\sqrt{1-r^2}}\int_h^\infty\int_k^\infty e^{-\frac{1}{2}\cdot\frac{1}{1-r^2}(x^2+y^2-2rxy)}\,dxdy$$

may be transformed into

$$\frac{d}{N}=\frac{1}{2\pi}\int_k^\infty e^{-\frac{1}{2}y^2}\left\{\int_t^\infty e^{-\frac{1}{2}X^2}\,dX\right\}dy,$$

where $t=\dfrac{h-yr}{\sqrt{1-r^2}}$ and this form is utilised to compute d as a check for the value of r obtained by the usual method of solution. Now $\dfrac{1}{\sqrt{2\pi}}\int_t^\infty e^{-\frac{1}{2}X^2}\,dX$ and $\dfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}$ are known quantities, being the $\frac{1}{2}(1+\alpha)$ and z of Sheppard's Tables†. Putting $\frac{1}{2}(1+\alpha)=I$ for convenience, we now have $\dfrac{d}{N}=\int_k^\infty zI\,dy$ and this form was used for constructing the tables.

The work was carried out in columns; the first column contained the values of y or k rising from zero by intervals of ·1, while the second column contained the corresponding values of z as found from Sheppard's Tables. These two columns remained constant during the whole of the work. The remaining columns were grouped in threes and contained the values of t, I and zI, the headings having suffixes attached showing the value of h to which they applied. The values of $\int_k^\infty zI\,dy$ were now obtained by a quadrature formula, first using all the values of zI in the column and then striking out each in turn from the top of the column downwards. Considerable difficulty was found in choosing a suitable quadrature formula and after many trials Weddle's Rule was adopted as being the most suitable‡. The table for $r=1$ was formed directly from Sheppard's Tables.

Checking.

Since $$\frac{d}{N}=\frac{1}{2\pi\sqrt{1-r^2}}\int_h^\infty\int_k^\infty e^{-\frac{1}{2}\cdot\frac{1}{1-r^2}(x^2+y^2-2rxy)}\,dxdy$$

is perfectly symmetrical in x and y, it is obvious that the value of d/N is unaltered when the values of h and k are interchanged. Consequently (excluding the case $h=k$) each value of d/N in the tables occurs twice and by the above method of construction each of the two values is obtained independently; it will also be noticed that in the final quadrature, the number of ordinates employed will also

* *Frequency Curves and Correlation.* Layton. London.

† New Tables of the Probability Integral. W. F. Sheppard. *Biometrika*, Vol. II. pp. 174 ff.

‡ It was always possible to make the ordinates number a multiple of six, as required by Weddle's Rule, by the simple process of adding sensibly zero ordinates at the asymptotic tail of the curve.

differ, so that the checking not only applies to the actual numerical work but also tests the accuracy of the quadrature formula. The whole of the work was carried out using five places of decimals, and the resulting values of d/N taken to five places gave a maximum discrepancy of one unit in the last place; this discrepancy could always be traced to the effect of cutting off at the fifth place in the formation of the product zI and only occurred when, as must occasionally happen, a number of values grouped together by the quadrature formula happened to be rounded off in the same direction.

In the special case $h = k$, the value of d/N only occurs once in each table and these cases were checked by differences.

Method of using the Tables.

In using the Tables to find the value of r the quantities $(b+d)/N$, $(c+d)/N$ and d/N are first found by division and then the values of h and k taken out from Sheppard's Tables. Using these values of h and k as arguments and comparing the values of d/N given in the Tables with the actual value of d/N in the particular case, the two values of r, between which the required value probably lies, can be readily found by inspection.

For each of these two values of r, the value of d/N is next found by interpolation, using the actual values of h and k as arguments; for each d/N there will be two such interpolations, one for h and one for k, and it is preferable as a general rule to perform that interpolation first for which the correction to d/N is the smaller. Having now obtained the value of d/N for the actual h and k for the two assumed values of r, the actual value of r is easily found by one more simple interpolation. The process may, at first sight, seem rather complicated but in actual practice the result is obtained with ease in a very few minutes.

Example.

Consider the fourfold table given below.

608	45	653
9	48	57
617	93	710

By division $\frac{b+d}{N} = \cdot 13099$, $\frac{c+d}{N} = \cdot 08028$, $\frac{d}{N} = \cdot 0676$,

whence from Sheppard's Tables $h = 1 \cdot 1218$, $k = 1 \cdot 4032$.

The values between which r will probably lie are next found by inspection of the tables, and it will at once be evident that the value of r lies between $\cdot 9$ and $\cdot 95$.

We accordingly proceed to find d/N for these values of r by interpolating in these two tables for the above values of h and k; in accordance with the general rule previously given, the interpolation is first carried out for k and afterwards for h.

From the tables we have

			$h=1{\cdot}1$	$h=1{\cdot}2$
	$r={\cdot}9$	$k=1{\cdot}4$	·0686	·0645
		$k=1{\cdot}5$	·0591	·0562
			$h=1{\cdot}1$	$h=1{\cdot}2$
which gives	$r={\cdot}9$	$k=1{\cdot}4032$	·0683	·0642

and finally $r={\cdot}9$, $k=1{\cdot}4032$, $h=1{\cdot}1218$, $d/N={\cdot}0674$.

Similarly we obtain

$r={\cdot}95$, $k=1{\cdot}4032$, $h=1{\cdot}1218$, $d/N={\cdot}0745$.

Interpolating for r, bearing in mind that the interval of r from one table to the next is ·05, and using $d/N={\cdot}0676$ as argument, we obtain the result $r={\cdot}903$.

Accuracy of the Result.

In order to test the accuracy of the result, the value of r in the above example was calculated by the usual method, using the Tables of Tetrachoric Functions and including terms up to the twelfth power of r.

The equation obtained was

$$\begin{aligned}{\cdot}057090 = {\cdot}010516r + {\cdot}024948r^2 + {\cdot}01322r^3 + {\cdot}003733r^4 + {\cdot}003868r^5 - {\cdot}000225r^6 \\ + {\cdot}002923r^7 - {\cdot}000121r^8 + {\cdot}001597r^9 + {\cdot}000442r^{10} + {\cdot}000630r^{11} + {\cdot}000017r^{12},\end{aligned}$$

whence solving by Newton's rule the value of r is found to be ·904.

It is not suggested that this remarkably close agreement between the two values ·903 and ·904 is always to be expected; but the difference between the two values will always, I think, be found very much less than the probable error of r and will therefore be without any significance.

Further Note on the Tables.

If D_{hk} be the tabulated value of d/N for arguments h and k for any one of the values of r for which the Tables are constructed, then the volume of the frequency solid on the area bounded by the lines corresponding to the values h_1, h_2, k_1, k_2, is given by $D_{h_1k_1} + D_{h_2k_2} - D_{h_1k_2} - D_{h_2k_1}$. Consequently within the limits $r={\cdot}8$ to $r=1$ the distribution of the frequency within the one quadrant, for which the Tables are constructed, may be readily found.

Cases where the Method of the Fourfold Table fails.

A careful examination of the Tables shows that, when the values of h and k differ widely and r is large, then the corresponding values of d/N differ very little from one value of r to the next; in such a case the probable error of r will be

large. It may also quite easily happen in such cases, that the value of d/N does not vary for values of r between ·8 and 1 and in such a case the method of the fourfold table fails, as the magnitude of the probable error would render the determination by the ordinary method quite idle as regards a quantitative result although it would generally be possible to say whether the correlation was large or small. Unfortunately it is not generally possible to classify the data, on which the fourfold table has to be based, in more than one way, or else by a suitable choice of classification it might be possible to avoid such cases.

In conclusion I desire to express my gratitude to Prof. Pearson for many valuable suggestions, especially in connection with the choice of a suitable quadrature formula.

Supplementary Tables for determining High

$r = \cdot 80$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0·0$	·3976	·3766	·3538	·3294	·3039	·2778	·2515	·2254	·2001	·1759	·1531	·1320	·1127
0·1	·3766	·3583	·3380	·3162	·2930	·2689	·2445	·2200	·1960	·1728	·1509	·1304	·1116
0·2	·3538	·3380	·3204	·3011	·2804	·2586	·2361	·2134	·1909	·1690	·1481	·1284	·1102
0·3	·3294	·3162	·3011	·2843	·2661	·2466	·2263	·2056	·1848	·1643	·1446	·1258	·1083
0·4	·3039	·2930	·2804	·2661	·2503	·2332	·2152	·1965	·1775	·1587	·1402	·1226	·1060
0·5	·2778	·2689	·2586	·2466	·2332	·2186	·2028	·1862	·1692	·1520	·1351	·1187	·1031
0·6	·2515	·2445	·2361	·2263	·2152	·2028	·1893	·1748	·1598	·1444	·1291	·1140	·0995
0·7	·2254	·2200	·2134	·2056	·1965	·1862	·1748	·1625	·1494	·1359	·1222	·1086	·0954
0·8	·2001	·1960	·1909	·1848	·1775	·1692	·1598	·1494	·1383	·1266	·1146	·1025	·0906
0·9	·1759	·1728	·1690	·1643	·1587	·1520	·1444	·1359	·1266	·1167	·1064	·0958	·0852
1·0	·1531	·1509	·1481	·1446	·1402	·1351	·1291	·1222	·1146	·1064	·0976	·0886	·0794
1·1	·1320	·1304	·1284	·1258	·1226	·1187	·1140	·1086	·1025	·0958	·0886	·0809	·0731
1·2	·1127	·1116	·1102	·1083	·1060	·1031	·0995	·0954	·0906	·0852	·0794	·0731	·0665
1·3	·0953	·0946	·0936	·0923	·0906	·0885	·0859	·0828	·0791	·0749	·0702	·0652	·0597
1·4	·0798	·0793	·0787	·0778	·0766	·0751	·0733	·0710	·0682	·0650	·0614	·0574	·0530
1·5	·0662	·0659	·0655	·0649	·0641	·0631	·0618	·0601	·0581	·0557	·0529	·0498	·0464
1·6	·0545	·0543	·0540	·0536	·0531	·0524	·0515	·0503	·0489	·0471	·0451	·0427	·0401
1·7	·0444	·0443	·0441	·0438	·0435	·0430	·0424	·0416	·0406	·0394	·0379	·0362	·0342
1·8	·0358	·0357	·0357	·0355	·0353	·0350	·0346	·0341	·0334	·0325	·0315	·0302	·0287
1·9	·0287	·0286	·0286	·0285	·0283	·0281	·0279	·0275	·0271	·0265	·0258	·0249	·0238
2·0	·0227	·0227	·0227	·0226	·0225	·0224	·0223	·0220	·0217	·0213	·0209	·0202	·0195
2·1	·0178	·0178	·0178	·0178	·0177	·0177	·0176	·0174	·0172	·0170	·0167	·0163	·0158
2·2	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0137	·0135	·0134	·0132	·0129	·0126
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104	·0103	·0101	·0099
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0080	·0079	·0078	·0077
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061	·0061	·0060	·0059
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046	·0046	·0045	·0045

$r = \cdot 85$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0·0$	·4117	·3905	·3670	·3417	·3149	·2873	·2595	·2319	·2052	·1798	·1560	·1341	·1141
0·1	·3905	·3723	·3518	·3292	·3050	·2796	·2537	·2277	·2022	·1777	·1546	·1332	·1136
0·2	·3670	·3518	·3342	·3145	·2930	·2702	·2464	·2222	·1983	·1748	·1527	·1319	·1127
0·3	·3417	·3292	·3145	·2978	·2791	·2588	·2374	·2154	·1931	·1712	·1501	·1301	·1116
0·4	·3149	·3050	·2930	·2791	·2632	·2457	·2268	·2070	·1867	·1665	·1467	·1277	·1099
0·5	·2873	·2796	·2702	·2588	·2457	·2309	·2146	·1972	·1790	·1606	·1423	·1246	·1078
0·6	·2595	·2537	·2464	·2374	·2268	·2146	·2008	·1859	·1700	·1535	·1370	·1206	·1049
0·7	·2319	·2277	·2222	·2154	·2070	·1972	·1859	·1733	·1597	·1453	·1306	·1158	·1014
0·8	·2052	·2022	·1983	·1931	·1867	·1790	·1700	·1597	·1483	·1360	·1232	·1101	·0971
0·9	·1798	·1777	·1749	·1712	·1665	·1606	·1535	·1453	·1360	·1258	·1149	·1035	·0920
1·0	·1560	·1546	·1527	·1501	·1467	·1423	·1370	·1306	·1232	·1149	·1058	·0962	·0862
1·1	·1341	·1332	·1319	·1301	·1277	·1246	·1206	·1158	·1101	·1035	·0962	·0882	·0798
1·2	·1141	·1136	·1127	·1116	·1099	·1078	·1049	·1014	·0971	·0920	·0862	·0798	·0729
1·3	·0963	·0959	·0954	·0947	·0936	·0921	·0901	·0876	·0845	·0807	·0763	·0712	·0656
1·4	·0805	·0803	·0800	·0795	·0788	·0778	·0765	·0748	·0725	·0698	·0665	·0626	·0583
1·5	·0666	·0665	·0664	·0661	·0656	·0650	·0642	·0630	·0615	·0595	·0571	·0543	·0510
1·6	·0547	·0547	·0546	·0544	·0541	·0538	·0532	·0525	·0514	·0501	·0484	·0464	·0439
1·7	·0445	·0445	·0444	·0443	·0442	·0440	·0436	·0432	·0425	·0416	·0405	·0390	·0373
1·8	·0359	·0359	·0359	·0358	·0357	·0356	·0354	·0351	·0347	·0341	·0334	·0324	·0312
1·9	·0287	·0287	·0287	·0287	·0286	·0285	·0284	·0283	·0280	·0276	·0272	·0265	·0257
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0224	·0221	·0218	·0214	·0209
2·1	·0179	·0179	·0179	·0178	·0178	·0178	·0178	·0177	·0176	·0175	·0173	·0171	·0167
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0136	·0135	·0133
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0081	·0080
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046

Correlations from Tetrachoric Groupings.

$r = \cdot 80$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0953	·0798	·0662	·0545	·0444	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·1	·0946	·0793	·0659	·0543	·0443	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·2	·0936	·0787	·0655	·0540	·0441	·0357	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·3	·0923	·0778	·0649	·0536	·0438	·0355	·0285	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0906	·0766	·0641	·0531	·0435	·0353	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·5	·0885	·0751	·0631	·0524	·0430	·0350	·0281	·0224	·0177	·0138	·0107	·0082	·0062	·0047
0·6	·0859	·0733	·0618	·0515	·0424	·0346	·0279	·0223	·0176	·0137	·0106	·0082	·0062	·0046
0·7	·0828	·0710	·0601	·0503	·0416	·0341	·0275	·0220	·0174	·0137	·0106	·0081	·0062	·0046
0·8	·0791	·0682	·0581	·0489	·0406	·0334	·0271	·0217	·0172	·0135	·0105	·0081	·0061	·0046
0·9	·0749	·0650	·0557	·0471	·0394	·0325	·0265	·0213	·0170	·0134	·0104	·0080	·0061	·0046
1·0	·0702	·0614	·0529	·0451	·0379	·0315	·0258	·0209	·0167	·0132	·0103	·0079	·0061	·0046
1·1	·0652	·0574	·0498	·0427	·0362	·0302	·0249	·0202	·0163	·0129	·0101	·0078	·0060	·0045
1·2	·0597	·0530	·0464	·0401	·0342	·0287	·0238	·0195	·0158	·0126	·0099	·0077	·0059	·0045
1·3	·0541	·0484	·0427	·0372	·0319	·0271	·0226	·0186	·0151	·0121	·0096	·0075	·0058	·0044
1·4	·0484	·0436	·0388	·0341	·0295	·0252	·0212	·0176	·0144	·0117	·0093	·0073	·0057	·0043
1·5	·0427	·0388	·0348	·0309	·0270	·0232	·0197	·0165	·0136	·0111	·0089	·0070	·0055	·0042
1·6	·0372	·0341	·0309	·0276	·0243	·0211	·0181	·0153	·0127	·0104	·0084	·0067	·0053	·0041
1·7	·0319	·0295	·0270	·0243	·0216	·0190	·0164	·0140	·0117	·0097	·0079	·0063	·0050	·0039
1·8	·0271	·0252	·0232	·0211	·0190	·0168	·0146	·0126	·0107	·0089	·0073	·0059	·0047	·0037
1·9	·0226	·0212	·0197	·0181	·0164	·0146	·0129	·0112	·0096	·0081	·0067	·0055	·0044	·0035
2·0	·0186	·0176	·0165	·0153	·0140	·0126	·0112	·0098	·0085	·0072	·0060	·0050	·0040	·0032
2·1	·0151	·0144	·0136	·0127	·0117	·0107	·0096	·0085	·0074	·0064	·0054	·0045	·0037	·0030
2·2	·0121	·0117	·0111	·0104	·0097	·0089	·0081	·0072	·0064	·0055	·0047	·0040	·0033	·0027
2·3	·0096	·0093	·0089	·0084	·0079	·0073	·0067	·0060	·0054	·0047	·0041	·0035	·0029	·0024
2·4	·0075	·0073	·0070	·0067	·0063	·0059	·0055	·0050	·0045	·0040	·0035	·0030	·0025	·0021
2·5	·0058	·0057	·0055	·0053	·0050	·0047	·0044	·0040	·0037	·0033	·0029	·0025	·0022	·0018
2·6	·0044	·0043	·0042	·0041	·0039	·0037	·0035	·0032	·0030	·0027	·0024	·0021	·0018	·0016

$r = \cdot 85$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0963	·0805	·0666	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0959	·0803	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0954	·0800	·0664	·0546	·0444	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0947	·0795	·0661	·0544	·0443	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0936	·0788	·0656	·0541	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·5	·0921	·0778	·0650	·0538	·0440	·0356	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·6	·0901	·0765	·0642	·0532	·0436	·0354	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·7	·0876	·0748	·0630	·0525	·0432	·0351	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·8	·0845	·0725	·0615	·0514	·0425	·0347	·0280	·0224	·0176	·0138	·0107	·0082	·0062	·0047
0·9	·0807	·0698	·0595	·0501	·0416	·0341	·0276	·0221	·0175	·0137	·0106	·0081	·0062	·0046
1·0	·0763	·0665	·0571	·0484	·0405	·0334	·0272	·0218	·0173	·0136	·0106	·0081	·0062	·0046
1·1	·0712	·0626	·0543	·0464	·0390	·0324	·0265	·0214	·0171	·0135	·0105	·0081	·0061	·0046
1·2	·0656	·0583	·0510	·0439	·0373	·0312	·0257	·0209	·0167	·0133	·0104	·0080	·0061	·0046
1·3	·0597	·0535	·0473	·0411	·0352	·0297	·0247	·0202	·0163	·0130	·0102	·0079	·0060	·0046
1·4	·0535	·0485	·0432	·0380	·0329	·0280	·0234	·0194	·0157	·0126	·0100	·0078	·0060	·0045
1·5	·0473	·0432	·0390	·0346	·0302	·0260	·0220	·0183	·0150	·0121	·0097	·0076	·0058	·0045
1·6	·0411	·0380	·0346	·0311	·0274	·0239	·0204	·0172	·0142	·0116	·0093	·0073	·0057	·0044
1·7	·0352	·0329	·0302	·0274	·0245	·0216	·0186	·0159	·0133	·0109	·0088	·0070	·0055	·0043
1·8	·0297	·0280	·0260	·0239	·0216	·0192	·0168	·0144	·0122	·0102	·0083	·0067	·0053	·0041
1·9	·0247	·0234	·0220	·0204	·0186	·0168	·0149	·0129	·0111	·0093	·0077	·0063	·0050	·0039
2·0	·0202	·0194	·0183	·0172	·0159	·0144	·0129	·0114	·0099	·0084	·0070	·0058	·0047	·0037
2·1	·0163	·0157	·0150	·0142	·0133	·0122	·0111	·0099	·0087	·0075	·0063	·0053	·0043	·0034
2·2	·0130	·0126	·0121	·0116	·0109	·0102	·0093	·0084	·0075	·0065	·0056	·0047	·0039	·0032
2·3	·0102	·0100	·0097	·0093	·0088	·0083	·0077	·0070	·0063	·0056	·0049	·0042	·0035	·0029
2·4	·0079	·0078	·0076	·0073	·0070	·0067	·0063	·0058	·0053	·0047	·0042	·0036	·0031	·0025
2·5	·0060	·0060	·0058	·0057	·0055	·0053	·0050	·0047	·0043	·0039	·0035	·0031	·0026	·0022
2·6	·0046	·0045	·0045	·0044	·0043	·0041	·0039	·0037	·0034	·0032	·0029	·0025	·0022	·0019

Supplementary Tables for determining High

$r = \cdot 90$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0\cdot0$	·4282	·4067	·3822	·3552	·3266	·2969	·2670	·2377	·2094	·1827	·1579	·1353	·1149
0·1	·4067	·3887	·3678	·3441	·3183	·2910	·2630	·2350	·2077	·1817	·1574	·1350	·1147
0·2	·3822	·3678	·3504	·3302	·3076	·2830	·2573	·2311	·2052	·1801	·1564	·1345	·1144
0·3	·3552	·3441	·3302	·3135	·2943	·2728	·2498	·2258	·2016	·1778	·1550	·1336	·1140
0·4	·3266	·3183	·3076	·2943	·2784	·2602	·2401	·2187	·1966	·1744	·1528	·1322	·1132
0·5	·2969	·2910	·2830	·2728	·2602	·2453	·2284	·2097	·1900	·1698	·1497	·1302	·1119
0·6	·2670	·2630	·2573	·2498	·2401	·2284	·2145	·1988	·1817	·1637	·1454	·1274	·1101
0·7	·2377	·2350	·2311	·2258	·2187	·2097	·1988	·1860	·1717	·1561	·1399	·1236	·1075
0·8	·2094	·2077	·2052	·2016	·1966	·1900	·1817	·1717	·1600	·1470	·1331	·1186	·1041
0·9	·1827	·1817	·1801	·1778	·1744	·1698	·1637	·1561	·1470	·1365	·1249	·1124	·0997
1·0	·1579	·1574	·1564	·1550	·1528	·1497	·1454	·1399	·1331	·1249	·1155	·1052	·0942
1·1	·1353	·1350	·1345	·1336	·1322	·1302	·1274	·1236	·1186	·1124	·1052	·0969	·0878
1·2	·1149	·1147	·1144	·1140	·1132	·1119	·1101	·1075	·1041	·0997	·0942	·0878	·0806
1·3	·0967	·0966	·0965	·0962	·0958	·0950	·0939	·0923	·0900	·0869	·0830	·0783	·0727
1·4	·0807	·0807	·0806	·0805	·0802	·0798	·0792	·0782	·0767	·0747	·0720	·0686	·0645
1·5	·0668	·0668	·0667	·0667	·0665	·0663	·0660	·0654	·0645	·0632	·0614	·0591	·0562
1·6	·0548	·0548	·0548	·0547	·0547	·0546	·0544	·0540	·0535	·0528	·0516	·0501	·0481
1·7	·0446	·0446	·0446	·0445	·0445	·0445	·0444	·0442	·0439	·0435	·0428	·0418	·0405
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0356	·0353	·0350	·0344	·0338
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0286	·0284	·0282	·0279	·0274
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0223	·0220
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178	·0178	·0178	·0177	·0176	·0175
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·00₄,	·0047	·0047	·0047	·0047	·0047

$r = \cdot 95$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0\cdot0$	·4495	·4271	·4005	·3705	·3385	·3055	·2729	·2414	·2116	·1840	·1586	·1357	·1151
0·1	·4271	·4099	·3880	·3622	·3333	·3026	·2713	·2407	·2113	·1839	·1586	·1356	·1151
0·2	·4005	·3880	·3712	·3500	·3252	·2976	·2685	·2392	·2106	·1835	·1585	·1356	·1150
0·3	·3705	·3622	·3500	·3338	·3135	·2898	·2637	·2365	·2092	·1829	·1582	·1355	·1150
0·4	·3385	·3333	·3252	·3135	·2980	·2787	·2564	·2320	·2067	·1816	·1576	·1352	·1149
0·5	·3055	·3026	·2976	·2898	·2787	·2640	·2459	·2250	·2024	·1792	·1563	·1346	·1147
0·6	·2729	·2713	·2685	·2627	·2564	·2459	·2321	·2153	·1960	·1753	·1542	·1335	·1141
0·7	·2414	·2407	·2392	·2365	·2320	·2250	·2153	·2025	·1870	·1694	·1506	·1315	·1131
0·8	·2116	·2113	·2106	·2092	·2067	·2024	·1960	·1870	·1753	·1611	·1452	·1283	·1113
0·9	·1840	·1839	·1835	·1829	·1816	·1792	·1753	·1694	·1611	·1505	·1377	·1234	·1084
1·0	·1586	·1586	·1585	·1582	·1576	·1563	·1542	·1506	·1452	·1377	·1281	·1167	·1041
1·1	·1357	·1356	·1356	·1355	·1352	·1346	·1335	·1315	·1283	·1234	·1167	·1082	·0981
1·2	·1151	·1151	·1150	·1150	·1149	·1147	·1141	·1131	·1113	·1084	·1041	·0981	·0906
1·3	·0968	·0968	·0968	·0968	·0967	·0966	·0964	·0959	·0950	·0934	·0908	·0870	·0818
1·4	·0808	·0808	·0808	·0808	·0807	·0807	·0806	·0804	·0800	·0792	·0778	·0755	·0721
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0667	·0665	·0661	·0654	·0642	·0622
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0547	·0545	·0542	·0536	·0525
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0445	·0445	·0445	·0443	·0440	·0435
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0355
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0285
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = \cdot 90$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0965	·0806	·0667	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0962	·0805	·0667	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0958	·0802	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0950	·0798	·0663	·0546	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0939	·0792	·0660	·0544	·0444	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0923	·0782	·0654	·0540	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·8	·0900	·0767	·0645	·0535	·0439	·0356	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·9	·0869	·0747	·0632	·0528	·0435	·0353	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·0	·0830	·0720	·0614	·0516	·0428	·0350	·0282	·0225	·0177	·0138	·0107	·0082	·0062	·0047
1·1	·0783	·0686	·0591	·0501	·0418	·0344	·0279	·0223	·0176	·0138	·0107	·0082	·0062	·0047
1·2	·0727	·0645	·0562	·0481	·0405	·0338	·0274	·0220	·0175	·0137	·0106	·0082	·0062	·0047
1·3	·0664	·0596	·0526	·0456	·0388	·0325	·0267	·0216	·0173	·0136	·0106	·0081	·0062	·0046
1·4	·0596	·0543	·0485	·0426	·0367	·0310	·0258	·0211	·0169	·0134	·0105	·0081	·0062	·0046
1·5	·0526	·0485	·0439	·0391	·0341	·0292	·0246	·0203	·0164	·0131	·0103	·0080	·0061	·0046
1·6	·0456	·0426	·0391	·0353	·0312	·0271	·0231	·0193	·0158	·0127	·0101	·0079	·0060	·0046
1·7	·0388	·0367	·0341	·0312	·0281	·0247	·0214	·0181	·0150	·0122	·0098	·0077	·0059	·0045
1·8	·0325	·0310	·0292	·0271	·0247	·0221	·0194	·0167	·0140	·0116	·0094	·0074	·0058	·0044
1·9	·0267	·0258	·0246	·0231	·0214	·0194	·0173	·0151	·0129	·0108	·0088	·0071	·0056	·0043
2·0	·0216	·0211	·0203	·0193	·0181	·0167	·0151	·0134	·0116	·0099	·0082	·0067	·0053	·0042
2·1	·0173	·0169	·0164	·0158	·0150	·0140	·0129	·0116	·0102	·0088	·0075	·0062	·0050	·0040
2·2	·0136	·0134	·0131	·0127	·0122	·0116	·0108	·0099	·0088	·0078	·0067	·0056	·0046	·0037
2·3	·0106	·0105	·0103	·0101	·0098	·0094	·0088	·0082	·0075	·0067	·0058	·0050	·0042	·0034
2·4	·0081	·0081	·0080	·0079	·0077	·0074	·0071	·0067	·0062	·0056	·0050	·0044	·0037	·0031
2·5	·0062	·0062	·0061	·0060	·0059	·0058	·0056	·0053	·0050	·0046	·0042	·0037	·0032	·0027
2·6	·0046	·0046	·0046	·0046	·0045	·0044	·0043	·0042	·0040	·0037	·0034	·0031	·0027	·0024

$r = \cdot 95$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0964	·0806	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0959	·0804	·0667	·0548	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0950	·0800	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0934	·0792	·0661	·0545	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0908	·0778	·0654	·0542	·0443	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0870	·0755	·0642	·0536	·0440	·0357	·0286	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0818	·0721	·0622	·0525	·0435	·0355	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
1·3	·0752	·0676	·0593	·0508	·0426	·0350	·0283	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·4	·0676	·0619	·0554	·0483	·0411	·0342	·0279	·0224	·0177	·0139	·0107	·0082	·0062	·0047
1·5	·0593	·0554	·0505	·0450	·0390	·0330	·0273	·0221	·0176	·0138	·0107	·0082	·0062	·0047
1·6	·0508	·0483	·0450	·0409	·0362	·0312	·0263	·0215	·0173	·0137	·0106	·0082	·0062	·0047
1·7	·0426	·0411	·0390	·0362	·0328	·0289	·0248	·0207	·0169	·0135	·0105	·0081	·0062	·0047
1·8	·0350	·0342	·0330	·0312	·0289	·0261	·0229	·0195	·0162	·0131	·0104	·0080	·0062	·0046
1·9	·0283	·0279	·0273	·0263	·0248	·0229	·0205	·0179	·0152	·0125	·0101	·0079	·0061	·0046
2·0	·0226	·0224	·0221	·0215	·0207	·0195	·0179	·0160	·0139	·0117	·0096	·0077	·0060	·0046
2·1	·0178	·0177	·0176	·0173	·0169	·0162	·0152	·0139	·0124	·0107	·0090	·0073	·0058	·0045
2·2	·0139	·0139	·0138	·0137	·0135	·0131	·0125	·0117	·0107	·0095	·0082	·0068	·0055	·0043
2·3	·0107	·0107	·0107	·0106	·0105	·0104	·0101	·0096	·0090	·0082	·0072	·0062	·0051	·0041
2·4	·0082	·0082	·0082	·0082	·0081	·0080	·0079	·0077	·0073	·0068	·0062	·0054	·0046	·0038
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0060	·0058	·0055	·0051	·0046	·0040	·0034
2·6	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0045	·0043	·0041	·0038	·0034	·0030

Supplementary Tables for determining High

$r = 1{\cdot}00$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0{\cdot}0$	·5000	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·1	·4602	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·2	·4207	·4207	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·3	·3821	·3821	·3821	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·4	·3446	·3446	·3446	·3446	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·5	·3085	·3085	·3085	·3085	·3085	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·6	·2743	·2743	·2743	·2743	·2743	·2743	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·7	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2119	·1841	·1587	·1357	·1151
0·8	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·1841	·1587	·1357	·1151
0·9	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1587	·1357	·1151
1·0	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1357	·1151
1·1	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1151
1·2	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151
1·3	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968
1·4	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = 1{\cdot}00$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0{\cdot}0$	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·4	·0808	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·5	·0668	·0668	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·6	·0548	·0548	·0548	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·7	·0446	·0446	·0446	·0446	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0139	·0107	·0082	·0062	·0047
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0107	·0082	·0062	·0047
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0082	·0062	·0047
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0062	·0047
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0047
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

NOTE ON THE EXTENT TO WHICH THE DISTRIBUTION OF CASES OF DISEASE IN HOUSES IS DETERMINED BY THE LAWS OF CHANCE.

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There is a problem of some importance in connection with medical investigations which we venture to think has not received sufficient attention from the mathematical standpoint. Stated briefly it is this: If n cases of a certain disease occur in a town with m houses, what are the probabilities that a houses will be affected with one case, b houses with two cases, and so on? In actual practice the problem presents itself in this way: a houses are known to have contained one case, b houses two cases, c houses three cases, and so on: what evidence is to be got of house infection or other disturbing factor from such a distribution, or can it be shown that such a distribution may have arisen by chance and be explicable by the laws of probability?

In order to render the problem suitable for mathematical treatment, we assume for the present that each house is equally liable to infection, i.e., that each house contains the same number of inhabitants, all of whom are equally liable to infection, irrespective of age and sex. We also assume that any particular house is equally likely to be infected at any time during the period under consideration; in practice this involves the exclusion from the data of known instances of case-to-case infection.

With these assumptions the problem now becomes equivalent to the following. If n balls are thrown into m equal compartments which are so arranged that it is equally likely that any ball will fall into any compartment, then the probability P that they will distribute themselves so that,

p_0 compartments will contain 0 balls,
p_1 " " " 1 ball,
p_2 " " " 2 balls,
p_3 " " " 3 "
.....
p_s " " " s "
and so on,

is

$$P = \frac{m!\, n!}{m^n p_0!\, p_1! \ldots p_s! \ldots (1!)^{p_1} (2!)^{p_2} \ldots (s!)^{p_s} \ldots} \quad \ldots\ldots\ldots\ldots\text{(i).}$$

In the above expression it should be noted that both series of factors in the denominator can be carried to infinity, for as there are only n balls, p_{n+1}, p_{n+2}, &c., are all zero, and therefore $p_{n+1}! = 1$, $(\overline{n+1}!)^{p_{n+1}} = 1$, &c. The importance of this will appear later.

Now let $$\eta = \frac{m!}{p_0!\, p_1! \dots p_s! \dots (1!)^{p_1} (2!)^{p_2} \dots (s!)^{p_s} \dots} \quad \dots\dots\dots\dots\dots\dots\text{(ii)}.$$

Then η is the general term in the expansion of

$$\left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^s}{s!} + \dots\right)^m$$

for all positive integral values of p_0, p_1, p_2, &c., subject to the conditions:

$$p_0 + p_1 + p_2 + \dots + p_s + \dots = m \quad \dots\dots\dots\dots\dots\dots\text{(iii)},$$

$$p_1 + 2p_2 + \dots + rp_r + \dots = n \dots\dots\dots\dots\dots\dots\text{(iv)},$$

and $$S(\eta) = \text{coeft. of } x^n \text{ in } \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^s}{s!} + \dots\right)^m$$

$$= \text{coeft. of } x^n \text{ in } e^{mx} = \frac{m^n}{n!} \quad \dots\dots\dots\dots\dots\dots\text{(v)},$$

or $$S\left(\frac{m!\, n!}{m^n\, p_0!\, p_1! \dots p_s! \dots (1!)^{p_1} (2!)^{p_2} \dots (s!)^{p_s} \dots}\right) = 1 \dots\dots\dots\dots\text{(vi)},$$

where the summation is taken for all values of p_0, p_1, ... subject to the conditions (iii) and (iv). This merely expresses the fact that the sum of all the probabilities in the case is unity.

Now let $$\bar{p}_0 = \frac{S(\eta p_0)}{S(\eta)}, \quad \bar{p}_1 = \frac{S(\eta p_1)}{S(\eta)}, \dots \bar{p}_s = \frac{S(\eta p_s)}{S(\eta)}, \dots,$$

and let $$\sigma_0^2 = \frac{S[\eta (p_0 - \bar{p}_0)^2]}{S(\eta)}, \dots \sigma_s^2 = \frac{S[\eta (p_s - \bar{p}_s)^2]}{S(\eta)}, \dots;$$

we have $$\bar{p}_s = S\left(\frac{m!\, p_s}{p_0!\, p_1! \dots (p_s)! \dots (1!)^{p_1} (2!)^{p_2} \dots (s!)^{p_s} \dots}\right) \Big/ S(\eta)$$

$$= S\left(\frac{m!}{p_0!\, p_1! \dots (p_s - 1)! \dots (1!)^{p_1} (2!)^{p_2} \dots (s!)^{p_s} \dots}\right) \Big/ \frac{m^n}{n!}$$

$$= \frac{m}{s!} S\left(\frac{(m-1)!}{p_0!\, p_1! \dots (p_s - 1)! \dots (1!)^{p_1} (2!)^{p_2} \dots (s!)^{p_s - 1} \dots}\right) \Big/ \frac{m^n}{n!}$$

$$= \frac{m}{s!} \cdot \frac{(m-1)^{n-s}}{(n-s)!} \cdot \frac{n!}{m^n} \quad \dots\dots\dots\dots\dots\dots\text{(vii)},$$

for the final summation is equivalent to (v) with $p_s - 1$ substituted for p_s, $m - 1$ for m, and $n - s$ for n, the conditions becoming

$$p_0 + p_1 + p_2 + \dots + (p_s - 1) + \dots = m - 1,$$

$$p_1 + 2p_2 + \dots + s(p_s - 1) + \dots = n - s.$$

It is, therefore, the summation in the case when there are $(n-s)$ balls and $(m-1)$ compartments, and it is to be noticed that p_{n-s+1}, p_{n-s+2}, &c., are all zero, so that it is unnecessary to eliminate the factors containing these expressions.

To find σ_s^2, assume

$$(p_s - \bar{p}_s)^2 = Ap_s(p_s-1) + Bp_s + C$$

for all values of p_s. Then, putting p_s successively $= 0, 1, 2,$

$$\begin{aligned} \bar{p}_s^2 &= C, \\ (1-\bar{p}_s)^2 &= B + C, \\ (2-\bar{p}_s)^2 &= 2A + 2B + C, \\ \therefore \quad A &= 1, \quad B = 1 - 2\bar{p}_s, \quad C = \bar{p}_s^2, \end{aligned}$$

and therefore

$$(p_s - \bar{p}_s)^2 \equiv p_s(p_s-1) + (1-2\bar{p}_s)p_s + \bar{p}_s^2 \quad \text{.....(viii),}$$

$$\begin{aligned} \therefore \quad S[\eta(p_s-\bar{p}_s)^2] &= S[\eta p_s(p_s-1)] + (1-2\bar{p}_s)S(\eta p_s) + \bar{p}_s^2 S(\eta) \\ &= S[\eta p_s(p_s-1)] + (1-2\bar{p}_s)\bar{p}_s S(\eta) + \bar{p}_s^2 S(\eta) \\ &= S[\eta p_s(p_s-1)] + (\bar{p}_s - \bar{p}_s^2).S(\eta). \end{aligned}$$

Now proceeding as in the previous case, we have

$$\begin{aligned} S[\eta p_s(p_s-1)] &= S\left(\frac{m!}{p_0!\,p_1!\ldots(p_s-2)!\ldots(1!)^{p_1}(2!)^{p_2}\ldots(s!)^{p_s}\ldots}\right) \\ &= \frac{m(m-1)}{(s!)^2} S\left(\frac{(m-2)!}{p_0!\,p_1!\ldots(p_s-2)!\ldots(1!)^{p_1}(2!)^{p_2}\ldots(s!)^{p_s-2}\ldots}\right) \\ &= \frac{m(m-1)}{(s!)^2} \cdot \frac{(m-2)^{n-2s}}{(n-2s)!} \quad \text{.....(ix).} \end{aligned}$$

For the final summation is equivalent to (v), with $p_s - 2$ substituted for p_s, $m-2$ for m, and $n-2s$ for n, the conditions becoming

$$\begin{aligned} p_0 + p_1 + p_2 + \ldots + (p_s - 2) + \ldots &= m-2, \\ p_1 + 2p_2 + \ldots + s(p_s-2) + \ldots &= n-2s. \end{aligned}$$

We have therefore,

$$\sigma_s^2 = \frac{m(m-1)(m-2)^{n-2s}}{(s!)^2(n-2s)!}\,\frac{n!}{m^n} + \bar{p}_s - \bar{p}_s^2 \quad \text{.....(x),}$$

and

$$\bar{p}_s = \frac{m(m-1)^{n-s}\,n!}{s!\,(n-s)!\,m^n} \quad \text{.....(vii).}$$

It is only necessary to calculate the first term and the others are rapidly obtained as follows:

$$\frac{\bar{p}_s}{\bar{p}_{s-1}} = \frac{m(m-1)^{n-s}}{s!\,(n-s)!} \times \frac{(s-1)!\,(n-s+1)!}{m(m-1)^{n-s+1}} = \frac{n-s+1}{s(m-1)},$$

$$\therefore \quad \bar{p}_1 = \bar{p}_0 \frac{n}{m-1}, \quad \bar{p}_2 = \bar{p}_1 \frac{n-1}{2(m-1)} \quad \text{.....(xi),}$$

and for successive terms in σ^2 if α_s be the first term in σ_s^2 of equation (x),

$$\frac{\alpha_s}{\alpha_{s-1}} = \frac{(n-2s+2)(n-2s+1)}{s^2(m-2)^2},$$

$$\therefore \quad \alpha_1 = \alpha_0 \times \frac{n(n-1)}{(m-2)^2}, \quad \alpha_2 = \alpha_1 \frac{(n-2)(n-3)}{2^2(m-2)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xii)}.$$

We proceed now to deal with an example taken from Newsholme's *Vital Statistics*, p. 344. In a town with an average of 106,721 houses there were in the course of 7 years 3512 cases of Enteric Fever. By means of the above formulae we can calculate the mean number of houses that would be affected once, twice, thrice, on the supposition that the distribution is a purely chance one, and compare them with the observed numbers, viz., 3350, 78 and 2.

The values of $\bar{p}_1$, $\bar{p}_2$, $\bar{p}_3$ to the nearest unit are

$$\bar{p}_1 = 3398, \quad \bar{p}_2 = 56, \quad \bar{p}_3 = 1.$$

If we had a large number of samples of this size and the laws of chance alone were at work, the mean number of houses affected twice would be 56. By means of the Standard Deviation we can determine what variations from this value may reasonably be expected.

Now $\sigma_2^2 = 51$ and therefore $\sigma_2 = 7$ (to the nearest unit). This gives a probable error of 5, and it would therefore be an even chance that in any sample the number of houses affected twice would be as many as 61 or as few as 51.

We now proceed to obtain by a simple method a close approximation to the modal values for p_0, p_1, ... p_s, &c., and as the values thus obtained are very closely the same as those found for the mean, the formula forms a useful method for quickly calculating the approximate values of $\bar{p}_0$, $\bar{p}_1$, ... $\bar{p}_s$, &c.

To find the values of p_0, p_1, ... p_s, ... which give a maximum value to the probability

$$\frac{m!\,n!}{m^n\, p_0!\, p_1! \ldots p_s! \ldots (1!)^{p_1} (2!)^{p_2} \ldots (s!)^{p_s} \ldots}.$$

The variable part of the above expression is

$$p_0!\, p_1!\, p_2! \ldots (1!)^{p_1}\, (2!)^{p_2} \ldots\ldots\ldots\ldots\ldots\ldots\text{(xiii)}.$$

If this is a minimum it will be nearly equal to each of the expressions got by substituting successively

$$p_0 - 1,\ p_1 + 2,\ p_2 - 1 \text{ for } p_0,\ p_1,\ p_2;$$
$$p_0 - 2,\ p_1 + 3,\ p_3 - 1 \text{ for } p_0,\ p_1,\ p_3;$$
$$p_0 - 3,\ p_1 + 4,\ p_4 - 1 \text{ for } p_0,\ p_1,\ p_4;$$

and so on,

where it is to be observed that the conditions (iii) and (iv) are satisfied by the new values. The expression (xiii) is a little less than each of the following:

$$(p_0-1)!\,(p_1+2)!\,(p_2-1)!\,p_3!\ldots(1!)^{p_1+2}(2!)^{p_2-1}(3!)^{p_3} \quad \ldots\ldots\text{(xiv)},$$

$$(p_0-2)!\,(p_1+3)!\,p_2!\,(p_3-1)!\ldots(1!)^{p_1+3}(2!)^{p_2}(3!)^{p_3-1}\ldots,$$

and so on.

Dividing each of these by the original expression (xiii), we get

$$\frac{(p_1+2)(p_1+1)}{p_0 p_2 2!}=1, \text{ nearly},$$

$$\frac{(p_1+3)(p_1+2)(p_1+1)}{p_0(p_0-1)p_3 3!}=1, \text{ nearly},$$

and so on;

whence

$$p_2=\frac{(p_1+2)(p_1+1)}{p_0 2!}, \text{ nearly},$$

$$p_3=\frac{(p_1+3)(p_1+2)(p_1+1)}{p_0(p_0-1)3!}, \text{ nearly},$$

and so on.

Now where m is considerably greater than n, as in most of the problems of this nature with which we have to deal, p_0 will obviously be considerably greater than p_1, and the values of p_2, p_3, &c., found above will rapidly diminish so that we may put for a first approximation,

$$p_0=p_0,$$

$$p_1=p_0\frac{p_1}{p_0} \qquad \text{and} \qquad p_1=p_1,$$

$$p_2=p_0\left(\frac{p_1}{p_0}\right)^2\frac{1}{2!}, \qquad 2p_2=p_1\frac{p_1}{p_0},$$

$$p_3=p_0\left(\frac{p_1}{p_0}\right)^3\frac{1}{3!}, \qquad 3p_3=p_1\left(\frac{p_1}{p_0}\right)^2\frac{1}{2!},$$

$$\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots$$

therefore adding,

$$m=p_0\left[1+\frac{p_1}{p_0}+\left(\frac{p_1}{p_0}\right)^2\frac{1}{2!}+\left(\frac{p_1}{p_0}\right)^3\frac{1}{3!}+\ldots\right]=p_0e^{\frac{p_1}{p_0}} \quad \ldots\ldots\ldots\text{(xv)},$$

and

$$n=p_1\left[1+\frac{p_1}{p_0}+\left(\frac{p_1}{p_0}\right)^2\frac{1}{2!}+\ldots\right]=p_1e^{\frac{p_1}{p_0}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xvi)}.$$

Dividing (xv) by (xvi), we get

$$\frac{m}{n}=\frac{p_0}{p_1},$$

therefore

$$m=p_0e^{\frac{n}{m}}, \quad n=p_1e^{\frac{n}{m}}.$$

Writing p_0', p_1', p_2', &c., for these values of p_0, p_1, p_2, &c.

$$\left.\begin{aligned} p_0' &= me^{-\frac{n}{m}} \\ p_1' &= ne^{-\frac{n}{m}} = \frac{n}{m}p_0' \\ p_2' &= \frac{n^2}{m\,2!}e^{-\frac{n}{m}} = \frac{n}{2m}p_1' \\ p_3' &= \frac{n^3}{m^2\,3!}e^{-\frac{n}{m}} = \frac{n}{3m}p_2' \\ &\cdots\cdots\cdots\cdots \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(xvii).}$$

The close correspondence between (xi) and (xvii) will be noticed. Referring to the example dealt with above, we find the numerical values to the nearest whole number for p_1', p_2', &c., as found by this method to be: $p_1' = 3398$, $p_2' = 56$, $p_3' = 1$, an identical result to that found above for $\bar{p}_1$, $\bar{p}_2$, $\bar{p}_3$.

We thus see that the mean and the approximate modal values closely correspond when m is large.

Turning now to the assumption that all houses have the same number of inhabitants; there are of course very few towns in which the distribution of the population is so homogeneous as to admit of this approximation being used with safety. It is clear that where some houses contain only 2 or 3 inhabitants, while others have from 20 to perhaps 50, a considerable error might be introduced by grouping them together. It is, therefore, necessary to investigate a method of obtaining the values of $\bar{p}_0$, $\bar{p}_1$, &c., when the distribution of the population is not homogeneous.

Keeping to our assumption that all individuals are equally likely to contract the disease, it follows that a house containing (say) six people is three times as likely to be attacked as a house containing only two. Let us suppose then that there are

m_1 houses containing an average of a_1 inhabitants,

m_2 „ „ „ „ „ a_2 „

and so on; where each house in the group contains approximately the same number of inhabitants.

Then the total population $N = m_1a_1 + m_2a_2 + m_3a_3 + \ldots$,

and the number of houses $m = m_1 + m_2 + m_3 + m_4 + \ldots$.

If n is as before the total number of cases of disease, the m_1 houses will on the average contain $m_1a_1\frac{n}{N}$ cases, the m_2 houses $m_2a_2\frac{n}{N}$ cases, and so on.

Writing $n_1 = m_1 a_1 n/N$, $n_2 = m_2 a_2 n/N$, &c., and dealing separately with each class of house as in the original problem, we get, by adding the results for the various classes together (taking the approximation $\bar{p}_0 = p_0'$ and writing $\bar{q}_0$ for the new value of $\bar{p}_0$),

$$\bar{q}_0 = m_1 e^{-\frac{n_1}{m_1}} + m_2 e^{-\frac{n_2}{m_2}} + m_3 e^{-\frac{n_3}{m_3}} + \ldots$$

$$= m_1 e^{-\frac{a_1 n}{N}} + m_2 e^{-\frac{a_2 n}{N}} + \ldots,$$

$$\bar{q}_1 = n_1 e^{-\frac{a_1 n}{N}} + n_2 e^{-\frac{a_2 n}{N}} + \ldots,$$

$$\bar{q}_2 = \frac{n_1^2}{m_1 2!} e^{-\frac{a_1 n}{N}} + \frac{n_2^2}{m_2 2!} e^{-\frac{a_2 n}{N}} + \ldots.$$

It is thus a simple matter to calculate the values of $\bar{q}_0$, $\bar{q}_1$, $\bar{q}_2$, when we have given the number of cases of disease, the total population and the number of houses averaging (say) 3, 5, 10, &c., inhabitants. An imaginary example will show that the change may quite reasonably be a 33 °/₀ increase on the value of $\bar{p}_2$.

If in a town with 50,000 inhabitants and 10,000 houses there are 500 cases of a certain disease, then we find $\bar{p}_0 = 9512$, $\bar{p}_1 = 476$, $\bar{p}_2 = 12$, and $\bar{p}_3 = 0$ to the nearest whole number; and $p_0' = 9512$. $p_1' = 476$, $p_2' = 12$, and $p_3' = 0$. If we now assume 3000 houses to have 2 inhabitants, 5000 to have 5, 1000 to have 7, 800 to have 10, and 200 to have 20, then $\bar{q}_0 = 9516$, $\bar{q}_1 = 467$, $\bar{q}_2 = 16$, and $\bar{q}_3 = 1$.

Returning now to our original example we are unfortunately unable to obtain a distribution of houses according to number of inhabitants in the town dealt with (viz., Manchester). We, therefore, assume that the town would show a somewhat similar distribution to that of Baltimore, U.S.A., for which we were able to obtain the necessary figures. The following table shows the values found for $\bar{q}_2$ on this assumption. While it is probably only approximately accurate it will give an idea of the change in value of $\bar{p}_2$ which may be reasonably expected.

Columns in Table.

(1) Number of people to a house.

(2) Number of houses out of 10,000 with 1, 2, 3, &c., persons (Baltimore).

(3) Number of houses with 1, 2, 3, &c., people in Manchester if the distribution had been the same as that in Baltimore.

(4) Number of persons in each group of houses.

(5) Number of cases of Enteric occurring in each group.

(6) Calculated number of houses affected twice in each group.

a	Baltimore	m	am	n	$\bar{p}_2$
(1)	(2)	(3)	(4)	(5)	(6)
1	162	1730	1730	10	0·029
2	875	9340	18680	109	0·629
3	1313	14010	42030	245	2·105
4	1570	16760	67040	391	4·455
5	1564	16690	83450	487	6·901
6	1380	14730	88380	516	8·727
7	1056	11270	78890	460	8·807
8	757	8080	64640	377	8·394
9	510	5440	48960	286	7·133
10	314	3350	33500	196	5·409
13	416	4440	57720	337	11·855
17	52	550	9350	55	2·461
22	31	330	7260	42	2·300
Totals	10000	106720	$601630 = N$	3511	$69 = \bar{q}_2$

The probable error of $\bar{q}_2$ may be assumed to be at least as great as that of $\bar{p}_2$, which we have found to be 5. Hence it is an even chance that the value of $\bar{q}_2$ will lie between 74 and 64. The observed value 78 would occur on the average at least once in four observations on a similar scale. There is, therefore, on the above-mentioned assumption, no clear evidence of house infection in this example.

Neither the conclusions drawn from the results, nor the results themselves, as found by the method adopted by Dr Niven, seem to us to be justified. Dr Newsholme while approving of the method does not give any mathematical proof of its validity, and the results differ considerably from the results found by the method described in this paper.

Although we have been able to eliminate the error arising from variations in number of persons to a house, there still remains the disturbing factor introduced by variations in age and sex. It is difficult to see how this can be dealt with statistically, as it is almost impossible to obtain the necessary information. In some diseases it will not be of so much importance as in others, viz. those which are most prone to attack the very young, e.g., scarlet fever, or the aged, as, for instance, cancer. Statistics of "cancer-houses," if suitably collected and obtainable in sufficient quantity, would form a useful and interesting subject for mathematical investigation.

In conclusion, we would observe that while the formulae given in this paper have, we consider, a field of practical utility in connection with some public health problems, yet the results obtained must be interpreted with caution. The unknown factors will tend to increase the value of the probable error, that is to say, observed variations from the calculated values may be greater than would be expected, without justifying the conclusion that house infection, for instance, is a factor in their causation.

PRETORIA, *March* 26, 1911.

ON THE APPEARANCE OF MULTIPLE CASES OF DISEASE IN THE SAME HOUSE.

By KARL PEARSON, F.R.S.

1. *Introductory.*

This problem is one of surpassing interest both from the standpoint of the mathematician and of the public health officer. Contributions to the theory of the problem have recently appeared from Dr Greenwood, Jnr*, Drs Troup and Maynard†, and Captain McKendrick‡. Each of the papers referred to have thrown light on the subject, but cannot be considered as giving a final solution. The solution I am proposing in the present paper is at best only an approximate one. The fundamental condition requisite for its sufficiency is that the number of deaths should be small as compared with the number of houses among which they are distributed. This condition is amply fulfilled by the data actually used by Troup and Maynard. Hence to such cases my theory applies. It would not apply without including higher terms in the approximation to cases in which there was a heavy epidemic visiting a large percentage of all houses. It is, however, applicable to cancer distributions.

Troup and Maynard have been the first, so far as I am aware, to reach perfectly general mathematical expressions for (i) the probable number, $\bar{p}_s$, of houses with s cases, supposing the same number of inhabitants to each house, and each house to be equally likely to be attacked, and (ii) the standard-deviation, σ_{p_s}, due to random sampling of the frequency p_s under the same conditions.

They do not go further than this and they do not proceed to find the correlation in deviations of p_s and p_t due to the variations of random sampling. This stage seems to me needful, if we are to provide an answer to the general problem: Is the observed distribution of houses with multiple cases compatible with the theoretical distribution on the assumption that the cases are distributed at random? Troup and Maynard confine their attention to the divergence between the expected and observed numbers of houses with *two* cases, and conclude that for enteric fever in

* See *Journal of Hygiene*, Vol. x. p. 416.
† See *Biometrika*, Vol. viii. p. 396.
‡ *Ibid.* p. 413.

a case from Newsholme's *Vital Statistics* (p. 344) there was not a very remarkable divergence. I admit the difficulty of finding the correlation of deviations in p_s and p_t in the general case (n commensurable with m). But this difficulty does not exist when m is large as compared with n; and I venture to think that in any case the *whole* distribution of houses must be compared in the observed and theoretical instances. If my analysis be correct—I frankly admit the matter is obscure and difficult—the case of enteric fever when treated from this broader standpoint indicates that the observed distribution of frequency was *not*, on the hypothesis of equivalent houses, compatible with a mere random scattering of the cases.

Greenwood in his paper on plague distribution has seen the importance of comparing the *whole* of the observed and theoretical distributions. He has found the "goodness of fit" of the observed to the theoretical frequency for the whole system of villages (not houses) in his data. I feel confident that the "goodness of fit" method is the correct method to be applied in such cases, but I doubt if it can be applied without weighting the terms of the χ^2, as, I think, Greenwood has omitted to do. In other words, I do not consider that

$$\chi^2 = S\left\{\frac{(p_s' - \bar{p}_s)^2}{\bar{p}_s}\right\} \quad \text{.....(i)}$$

is the correct expression to use*. This expression is only true when

$$S(p_s) = m,$$

but we have to remember that in our present case

$$S(sp_s) = n,$$

or the number of cases as well as the number of houses is fixed. Accordingly the fundamental relations on which (i) is based, i.e.

$$\left.\begin{aligned} \sigma^2_{p_s} &= p_s\left(1 - \frac{p_s}{m}\right) \\ \sigma_{p_s}\sigma_{p_t}r_{p_sp_t} &= -\frac{p_sp_t}{m} \end{aligned}\right\} \quad \text{.....(ii),}$$

are no longer correct and must be replaced before we can determine the true form of χ^2. Greenwood is correct in his view that we must use χ^2, and I think this has been overlooked by Troup and Maynard. They on the other hand give the full values for p_s and $\sigma^2_{p_s}$, showing that the first equation of (ii) is incorrect, but they do not attempt the second equation of (ii).

The present writer only offers the approximately correct values of (ii) and the correspondingly corrected value of (i). These results apply absolutely to the enteric data, but would only apply as an approximation to houses in plague data. It has been recently half suggested† that the application of mathematics to medicine does

* Greenwood's case is not exactly the present case, but the need for weighting the contributions to χ^2 still I think remains true.

† *British Medical Journal*, June 17, 1911, p. 1431.

not need the highest mathematical powers and the most complete mathematical training. Now here is a case which absolutely confutes such a suggestion—just as it is confuted by almost every medico-biometric problem that arises. None of us, who are biometricians or medical men, have yet succeeded in fully solving the problem—a most vital one in many respects. We are simply pottering round it and nibbling off little corners of it. And why is this? Solely because we have no transcendent mathematical power at our service. Those with such power have not the faintest notion of the existence of these statistico-medical problems, and those without it do not recognise the essential difficulties of the analysis. Thus we go on nibbling off one bit after another of such hard nuts, where the highly trained mathematician—if he could be made to grasp our problems—might reach the kernels in a few hours. Strange as the confession may seem, when I come to these medico-statistical problems, my regret is not for my want of medical training, but for the extreme defectiveness of my powers of mathematical analysis.

This must be my apology for adding still another incomplete solution to those of my colleagues.

2. *Mean Values.*

Let it be required to drop n balls into m compartments. Then fixing all attention on one compartment, the chance that a ball falls into it is $\frac{1}{m}$, and that it fails to fall into it $\frac{m-1}{m}$. Repeating the process n times, we have the binomial

$$\left(\frac{m-1}{m}+\frac{1}{m}\right)^n$$

as giving the theoretically expected frequency or

$$m\left(\frac{m-1}{m}+\frac{1}{m}\right)^n$$

gives by its terms the distribution of compartments with 0, 1, 2, ... s ... balls in them, i.e.

$$\bar{p}_s = m\,\frac{n!}{s!\,(n-s)!}\left(\frac{m-1}{m}\right)^{n-s}\left(\frac{1}{m}\right)^s \quad \text{.....(iii).}$$

This is the result reached as equation (vii) of Troup and Maynard's memoir.

Now let us apply Stirling's Theorem

$$x! = x^x e^{-x}\sqrt{2\pi x}\left(1+\frac{1}{12x}+\frac{1}{288x^2}+\dots\right)$$

to (iii) for $n!$ and $(n-s)!$ and also taking logarithms evaluate

$$(n-s)\log_e\left(1-\frac{1}{m}\right)$$

by the well-known formula

$$\log_e(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\dots$$

and neglect terms of the orders n/m^2 (and therefore of course s/m^2) and s^2/n^2. We find

$$\bar{p}_s = \frac{m}{s!}\left(\frac{n}{m}\right)^s e^{-\frac{n-s}{m} - \frac{1}{2}\frac{s(s-1)}{n}} \text{ approximately } \ldots\ldots\ldots\ldots\text{(iv)}.$$

Since in many cases s is a very small number as compared with n or m, this may frequently be written

$$\bar{p}_s = \frac{m}{s!}\left(\frac{n}{m}\right)^s e^{-\frac{n}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)},$$

which is the approximate form given by Troup and Maynard (see p. 402).

3. *Correlations.*

I now turn to the correlation in deviations of p_s and p_t. I reserve $s\mu$ deaths out of my total of n deaths; I intend to put these deaths into μ houses, s apiece, so that I reduce my number of available houses to $m-\mu$. I have accordingly to distribute $n-s\mu$ deaths among $m-\mu$ houses. The most probable distribution will be

$$(m-\mu)\left(\frac{m-\mu-1}{m-\mu} + \frac{1}{m-\mu}\right)^{n-s\mu},$$

i.e. by (iii): $\bar{p}_s' = (m-\mu)\dfrac{(n-s\mu)!}{s!\,(n-s\mu-s)!}\left(\dfrac{m-\mu-1}{m-\mu}\right)^{n-s\mu-s}\left(\dfrac{1}{m-\mu}\right)^s$

and $\bar{p}_t' = (m-\mu)\dfrac{(n-s\mu)!}{t!\,(n-s\mu-t)!}\left(\dfrac{m-\mu-1}{m-\mu}\right)^{n-s\mu-t}\left(\dfrac{1}{m-\mu}\right)^t.$

Clearly we shall now, introducing the $s\mu$ deaths in μ houses, have

$$\left.\begin{aligned}\bar{p}_s + \delta p_s &= \mu + \bar{p}_s' \\ \bar{p}_t + \delta p_t &= \bar{p}_t'\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)}.$$

Now if we suppose μ small as compared with m or n, we can expand by the same theorems as we have used before and determine $\bar{p}_s'$ and $\bar{p}_t'$. This will enable us to find the ratio of the *mean* change in p_t to an arbitrary change in p_s, or if the relation turn out *linear*, we have

$$\frac{\delta p_t}{\delta p_s} = \frac{\sigma_{p_t}}{\sigma_{p_s}} R_{p_t p_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)},$$

where $R_{p_t p_s}$ is the correlation between an arbitrary change in p_s and the resulting change in p_t.

It will be simplest to work by making the requisite changes in $\bar{p}_s$ as given by (iv). We have

$$\frac{\bar{p}_t'}{\bar{p}_t} = \frac{\left(1-\dfrac{s\mu}{n}\right)^t}{\left(1-\dfrac{\mu}{m}\right)^{t-1}} e^{-\frac{n\mu}{m^2} + \frac{s\mu}{n} - \frac{1}{2}\frac{s\mu}{n}\frac{t(t-1)}{n}}.$$

But μ being small compared with n and m, and remembering that we have neglected terms of order s^2/n^2 and n/m^2, we have on expanding the exponential

$$\frac{\bar{p}_t'}{\bar{p}_t} = 1 - \mu\left(\frac{st}{n} - \frac{s+t-1}{m}\right).$$

For the special case of $t = s$:

$$\frac{\bar{p}_s'}{\bar{p}_s} = 1 - \mu\left(\frac{s^2}{n} - \frac{2s-1}{m}\right).$$

Accordingly from (vi) we find,—showing a linear relation—

$$\delta p_s = \mu\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\},$$

$$\delta p_t = -\mu\bar{p}_t\left(\frac{st}{n} - \frac{s+t-1}{m}\right).$$

Whence from (vii):

$$\sigma_{p_t}\sigma_{p_s}R_{p_tp_s} = -\frac{\sigma^2_{p_s}\bar{p}_t\left(\frac{st}{n} - \frac{s+t-1}{m}\right)}{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)} \quad \text{.....(viii).}$$

It remains to find $\sigma^2_{p_s}$ to the same order of approximation.

Clearly $\sigma_{p_t}\sigma_{p_s}R_{p_tp_s}$ must be a function perfectly symmetrical in s and t. Hence we must have

$$\sigma_{p_t}\sigma_{p_s}R_{p_tp_s} = -\frac{\sigma^2_{p_t}\bar{p}_s\left(\frac{st}{n} - \frac{s+t-1}{m}\right)}{1 - \bar{p}_t\left(\frac{t^2}{n} - \frac{2t-1}{m}\right)},$$

whence it follows that

$$\sigma^2_{p_s} = \lambda\bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\},$$

and

$$\sigma^2_{p_t} = \lambda\bar{p}_t\left\{1 - \bar{p}_t\left(\frac{t^2}{n} - \frac{2t-1}{m}\right)\right\},$$

where λ is independent of s or t, because σ_{p_s} and σ_{p_t} cannot respectively involve a particular t or s; i.e. λ can only depend on m and n, or be an absolute number.

If $s = 0$, we have

$$\sigma^2_{p_0} = \lambda\bar{p}_0\left(1 - \frac{\bar{p}_0}{m}\right),$$

which is the correct value only if $\lambda = 1$.

Hence generally we have

$$\sigma^2_{p_s} = \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\} \quad \text{.....(ix).}$$

To test this value take Troup and Maynard's more general result (see p. 398):

$$\sigma^2_{p_s} = \frac{m(m-1)(m-2)^{n-2s}}{(s!)^2(n-2s)!}\frac{n!}{m^n} + \bar{p}_s - \bar{p}_s^2 \quad \text{.....(x).}$$

Call this

$$\sigma^2_{p_s} = \epsilon + \bar{p}_s - \bar{p}_s^2.$$

Then

$$\epsilon/\bar{p}_s^2 = \frac{m-1}{m} \times \frac{m^n (m-2)^{n-2s}}{(m-1)^{2n-2s}} \times \frac{(n-s)!\,(n-s)!}{n!\,(n-2s)!}.$$

We may evaluate the factors separately,

$$u_2 = \frac{(n-s)!\,(n-s)!}{n!\,(n-2s)!} \text{ and by Stirling's Theorem } = \frac{(n-s)^{2n-2s+1}}{n^{n+\frac{1}{2}}(n-2s)^{n-2s+\frac{1}{2}}},$$

$$\log u_2 = -(2n-2s+1)\left(\frac{s}{n} + \frac{s^2}{2n^2} + \ldots\right) + (n-2s+\tfrac{1}{2})\left(\frac{2s}{n} + \frac{4s^2}{2n^2} + \ldots\right)$$

$$= -\frac{s^2}{n} - \text{etc.},$$

or

$$u_2 = e^{-\frac{s^2}{n}} = 1 - \frac{s^2}{n} \text{ nearly},$$

$$u_1 = \frac{(m-1)}{m}\frac{m^n (m-2)^{n-2s}}{(m-1)^{2n-2s}} = \left(1-\frac{2}{m}\right)^{n-2s}\left(1-\frac{1}{m}\right)^{-(2n-2s+1)}$$

$$= 1 - \frac{2(n-2s)}{m} + \frac{2n-2s-1}{m} + \text{terms of order } \left(\frac{n}{m}\right)^2,$$

which we have agreed to neglect.

Hence

$$\epsilon/\bar{p}_s^2 = u_1 u_2 = 1 - \frac{s^2}{n} + \frac{2s-1}{m},$$

and accordingly

$$\sigma^2_{p_s} = \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xi)},$$

which agrees with our previous value in (ix).

Clearly in the enteric fever case, to which Troup and Maynard apply their results, the term $(2s-1)/m$ is quite negligible, and we may always write when m is large compared with n:

$$\left.\begin{aligned} \bar{p}_s &= \frac{m}{s!}\left(\frac{n}{m}\right)^s e^{-\frac{n}{m}} \\ \sigma^2_{p_s} &= \bar{p}_s\left(1 - \frac{s^2}{n}\bar{p}_s\right) \\ \sigma_{p_s}\sigma_{p_t} R_{p_s p_t} &= -\frac{st\,\bar{p}_s \bar{p}_t}{n} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xii)}.$$

To test the accuracy of (xii) the case of enteric fever has been worked out by the long formulae, (iii) and (x), and by the short formulae of (xii). We have

	$\bar{p}_1$	$\sigma^2_{p_1}$	$\bar{p}_2$	$\sigma^2_{p_2}$	$\bar{p}_3$	$\sigma^2_{p_3}$
Long Formulae ...	3398·34	214·49	55·88	52·10	·613	·612
Short Formulae ...	3398·33	214·60*	55·89	52·33	·613	·612

* Equation (xi) used to find σ_1^2, but only this first value for which (xii) is not close enough.

The values of m and n are $m = 106{,}721$, $n = 3512$; the observed values are $p_1 = 3350$, $p_2 = 78$, $p_3 = 2$; the higher p's are zero.

It will be seen that, for such an example as enteric fever (and cancer falls into the same category), the approximate formulae are as good as the complete formulae, and there is much less danger of error in using them than in dealing with high powers of large numbers, such as arise in an equation like (x), which is based upon the differences of large numbers*.

4. *Application of Goodness of Fit Formulae.*

I shall now proceed to modify the second and third equations of (xii) or in the first place (viii) and (ix). These may be written:

$$\sigma_{p_t}\sigma_{p_s}R_{p_tp_s} = -\bar{p}_s\bar{p}_t\left(\frac{st}{n} - \frac{s+t-1}{m}\right) \quad \text{.....(xiii)}$$

$$= -\bar{p}_s\bar{p}_t\left\{st\left(\frac{1}{n} - \frac{1}{m}\right) + \frac{(s-1)(t-1)}{m}\right\}$$

$$= -\bar{p}_s\bar{p}_t\left\{\frac{st}{n\left(1+\frac{n}{m}\right)} + \frac{(s-1)(t-1)}{m}\right\} \quad \text{.....(xiv),}$$

since we are neglecting terms of the order n^2/m^2 compared with unity; and

$$\sigma^2_{p_s} = \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{s^2}{m} + \frac{(s-1)^2}{m}\right)\right\}$$

$$= \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n\left(1+\frac{n}{m}\right)} + \frac{(s-1)^2}{m}\right)\right\} \quad \text{.....(xv).}$$

Now in the cases with which we are dealing s or t is merely a number of the order 1 to 3, and small, while m is very large. Hence such terms as $(s-1)(t-1)/m$ and $(s-1)^2/m$ are negligible as compared with st/N or s^2/N, where $N = n\left(1+\frac{n}{m}\right)$. We may therefore to the same degree of approximation as before write

$$\sigma_{p_t}\sigma_{p_s}R_{p_tp_s} = -\frac{st\,p_s p_t}{N} \quad \text{..... .....(xvi),}$$

$$\sigma^2_{p_s} = \bar{p}_s\left(1 - \frac{\bar{p}_s s^2}{N}\right) \quad \text{.....(xvii)†,}$$

where

$$N = n\left(1+\frac{n}{m}\right) \quad \text{.....(xviii).}$$

I now propose to find $S(s^2\bar{p}_s)$.

* In the case of $\sigma^2_{p_1}$ the results from (x) must be worked correct to 10 figures, if we wish to get the first decimal place correct in its value.

† Equation (xvii) is as accurate as (xi) and it is better to use it than (xii).

To do this we require to know the value of

$$\frac{1^2x}{1!}+\frac{2^2x^2}{2!}+\frac{3^2x^3}{3!}+\frac{4^2x^4}{4!}+\ldots.$$

Now it clearly equals

$$x\frac{d}{dx}\left(x\frac{d}{dx}e^x\right)=x(x+1)e^x.$$

Let $x=\frac{n}{m}$, then

$$\begin{aligned} & me^{-\frac{n}{m}}\left\{\frac{1^2}{1!}\frac{n}{m}+\frac{2^2}{2!}\left(\frac{n}{m}\right)^2+\frac{3^2}{3!}\left(\frac{n}{m}\right)^3+\frac{4^2}{4!}\left(\frac{n}{m}\right)^4+\ldots\right\} \\ &= S(s^2\bar{p}_s) \text{ by the first equation of (xii)} \\ &= me^{-\frac{n}{m}}\frac{n}{m}\left(\frac{n}{m}+1\right)e^{\frac{n}{m}} \\ &= n\left(1+\frac{n}{m}\right)=N, \text{ to our degree of approximation.} \end{aligned}$$

For example $n\left(1+\frac{n}{m}\right)=3627{\cdot}6$ in the enteric data cited above, $=3627{\cdot}4$ actually, when calculated as $S(s^2\bar{p}_s)$, a difference of no importance.

Now let us write $x_s=s^2p_s$; then clearly $\sigma^2_{x_s}=s^2\sigma^2_{p_s}$, and $R_{x_sx_t}=R_{p_sp_t}$, for the new variables only differ from the old by constant multipliers. Accordingly we have

$$\left.\begin{aligned} \sigma^2_{x_s} &= \bar{x}_s\left(1-\frac{\bar{x}_s}{N}\right) \\ \sigma_{x_s}\sigma_{x_t}R_{x_sx_t} &= -\frac{\bar{x}_s\bar{x}_t}{N} \\ \text{while} \qquad S(x_s) &= N \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xix).}$$

Thus $x_1, x_2, \ldots x_s \ldots$ may be looked upon as frequencies, which have a constant sum N and for which the standard deviations and correlations are precisely those defined in my original memoir on "Goodness of Fit": see *Phil. Mag.* Vol. L. Eqns (vii) and (viii), p. 161, July, 1900. We are not troubled by p_0 for it does not enter into the system (i.e. $x_0=0$), and p_0 is merely an additional quantity to be found ultimately by the relation

$$p_0=m-\overset{n}{\underset{1}{S}}\left(\frac{x_s}{s^2}\right).$$

Proceeding exactly as in the memoir just referred to, we find

$$\chi^2=\overset{n}{\underset{1}{S}}\frac{(x_s-\bar{x}_s)^2}{\bar{x}_s}=\overset{n}{\underset{1}{S}}\left\{\frac{s^2(p_s-\bar{p}_s)^2}{\bar{p}_s}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xx).}$$

It will be observed that this result differs from the χ^2 for a series of frequencies $p_0, p_1, \ldots p_s \ldots$, i.e. from

$$\chi^2=\overset{n}{\underset{0}{S}}\frac{(p_s-\bar{p}_s)^2}{\bar{p}_s},$$

by the weighting of each contribution to the contingency with the square of the number of cases occurring in houses of the given category. A little thought seems to indicate that this weighting is *a priori* probable, and it is possible that a direct proof of it may be discoverable. At any rate when discovered, it seems so reasonable, that I am inclined to think that (xx) may be safely applied even beyond the range (i.e. m and n large and m large compared to n) for which it has been demonstrated.

In actually applying the method to statistical data of disease occurrence, we must confine our attention to those frequencies which give at least a whole house. Thus we have practically for the case of enteric

$$\bar{p}_1 = 3398, \qquad \bar{p}_2 = 56, \qquad \bar{p}_3 = 1,$$

and the observed frequencies were

$$p_1 = 3350, \qquad p_2 = 78, \qquad p_3 = 2,$$

$$\chi^2 = 1^2 \frac{(3398 - 3350)^2}{3398} + 2^2 \frac{(78 - 56)^2}{56} + 3^2 \frac{(2 - 1)^2}{1}$$

$$= \cdot 678 + 34 \cdot 571 + 9 = 44 \cdot 25.$$

I think we must say that this is the χ^2 for *four* groups, although by the nature of the expression for χ^2, the houses with no cases do not enter*. Consulting Palin Elderton's Tables (*Biometrika*, Vol. I. p. 158) we conclude that such an improbability would not occur on the average once in a million random samplings, and therefore with a high degree of probability cases of enteric do tend to repeat in the same houses. If we had judged simply on the group p_2 we should have $(p_2 - \bar{p}_2)/\sigma_2 = 22/7$ roughly $= 3$, and such an excess would occur about once in a 1000 trials. This again would be extremely improbable, but not so improbable as the result based on an examination of the entire frequency. The importance of using the latter is thus indicated. At the same time the caution given by Troup and Maynard as to the influence in houses of age and sex must have full weight, and the problem of the number of inhabitants to the house requires further consideration, which I postpone for the present.

I hope shortly to publish Dr Law Webb's data for "cancer houses," which were sent to me in 1910, discussed by this method: see below, *Miscellanea*, p. 430.

* The upper limit to the number of groups may be considered disputable. Experimentally all n balls might be cast in one compartment, or there are $n+1$ frequency classes conceivable. This is impossible in the case of multiple cases of disease, for n persons have not lived in any one house during the period under observation. To fix the upper limit the average number of persons to the house and the average duration of occupation would need to be considered. The fact, however, that χ^2 for those groups which show *theoretically* at least a unit is found *experimentally* to give reasonable values of P, justifies empirically its use to test vital statistics.

MISCELLANEA.

I. A Simplified Method of Calculating Frequencies of Occurrence, from a Large Number of Unequal Probabilities.

By Captain A. G. McKENDRICK, I.M.S., Assistant Director, Pasteur Institute of Southern India, Coonoor.

In the study of epidemics in particular it is desirable to determine, in certain cases, whether the proneness of a particular community to recurring attack by disease, is regulated by the laws of chance ; or whether other factors are operative.

Such an investigation must be carried out in two steps ; firstly, from the data given, the chance distribution must be calculated, and secondly, the chance distribution must be compared with actual figures.

Problems of this sort occur in many forms, for example :

(1) If a series of epidemics of varying magnitude occurs in a group of communities, do the numbers of communities actually attacked never, once, twice, etc., agree with the numbers as calculated by the calculus of Probabilities ?

(2) If a regiment is attacked by a number of different diseases in a given time, do the numbers of men actually attacked by none, one, two, tc. of these diseases, agree with the probable numbers?

It is only with the first step of the investigation, i.e., the calculation of the probable numbers, that this paper will deal.

M. Greenwood, Jr. (*Journal of Hygiene*, Vol. x. No. 3, p. 416) in an admirable analysis of the statistics of Plague epidemics in the Punjaub, had, amongst others, the following problem to deal with :

In the Amritsar District out of 1,062 villages,

62	were	attacked	by plague	in 1901—2
506	„	„	„	1902—3
445	„	„	„	1903—4
669	„	„	„	1904—5
276	„	„	„	1905—6
604	„	„	„	1906—7

He had to find from these data what would be the probable numbers of villages one would expect to be attacked no times, once, twice, etc. He used the ordinary summation method, that is he calculated the most probable number of infections for each possible combination of epidemics. He states that "although the calculation of the probable number of villages to be infected in any combination of epidemics is merely a matter of simple arithmetic, yet the arithmetic becomes extremely laborious, when the number of epidemics under review is even moderately large. Thus in Amritsar District there were six epidemics and a complete evaluation of the various combinations of villages attacked never, once, twice, etc., requires the determination of 64 distinct products each composed of six terms. Without a mechanical calculator this is an impracticable task, and even with such help it is very tedious."

Two years ago Major Lamb of the Plague Commission (India) gave me the problem of the Amritsar District to work out. In order to reduce the labour of the work I devised the following process, which, although it contains nothing of mathematical novelty, still attains the required result with great economy of labour. In place of the determination of 64 products of six terms each, 29 products of two terms each are evaluated, the remainder of the calculation being simple addition and subtraction. A mechanical calculator is unnecessary. The time employed in the calculation is little over an hour.

As a simple case let us cohsider only two probabilities p and q, then, the total frequency being taken as unity, the frequency F is

$$\begin{aligned} &\text{for 2 times } F_2 = p.q &&= pq \\ &\text{„ 1 time } F_1 = p(1-q)+q(1-p) = &&(p+q)-2pq \\ &\text{„ 0 times } F_0 = (1-p)(1-q) &&= 1-(p+q)+pq. \end{aligned}$$

Now let P_r=sum of the products of all the probabilities taken r at a time, P_0 being unity.

Then for the n probabilities, $a, b, c, d, e, f, \ldots\ldots$

$$\begin{aligned} (1) \qquad F_0 &= P_0 - P_1 + P_2 - P_3 + P_4 - P_5 + P_6 - \ldots \\ F_1 &= P_1 - 2P_2 + 3P_3 - 4P_4 + 5P_5 - 6P_6 + \ldots \\ F_2 &= P_2 - 3P_3 + 6P_4 - 10P_5 + 15P_6 - \ldots \\ F_3 &= P_3 - 4P_4 + 10P_5 - 20P_6 + \ldots \\ F_4 &= P_4 - 5P_5 + 15P_6 - \ldots, \text{ etc.} \end{aligned}$$

and obviously $\Sigma_0^n F_r = P_0 = 1,$

where the coefficients of the P's in the value F_r are the same as those in the expansion in powers of x, of $(1+x)^{-(r+1)}$, namely,

$$(1+x)^{-(r+1)} = 1 - (r+1)x + \frac{(r+1)(r+2)}{1.2}x^2 - \frac{(r+1)(r+2)(r+3)}{1.2.3}x^3 + \ldots.$$

The general formula for F_r is thus

$$(2) \qquad F_r = P_r - (r+1)P_{r+1} + \frac{(r+1)(r+2)}{1.2}P_{r+2} - \frac{(r+1)(r+2)(r+3)}{1.2.3}P_{r+3} + \ldots$$

$$\ldots + (-)^{n-r}\frac{(r+1)(r+2)\ldots n}{1.2\ldots(n-r)}P_n.$$

A concise and economical method of calculating the values of the P's may be exhibited in the following scheme :—

P_1 P_2 P_3 P_4 P_5 P_6

$[a+b+c+d+e+f]$

$a[b+c+d+e+f]$

$+b[c+d+e+f]$ $(a)\,b[c+d+e+f]$

$+c[d+e+f]$ $+(a+b)\,c[d+e+f]$ $\overline{(ab)}\quad c[d+e+f]$

$+d[e+f]$ $+(a+b+c)\,d[e+f]$ $+(ab+\overline{a+b}.c\ )\,d[e+f]$ $\overline{(abc)}\quad d[e+f]$

$+e[f]$ $+(a+b+c+d)\,e[f]$ $+(ab+\overline{a+b}.c+\overline{a+b+c}.d)\,e[f]$ $+\{abc+(ab+\overline{a+b}.c)\,d\}.e[f]$ $abcd.e[f]$

Write this with serial coefficients, thus:

$A[a+b+c+d+e+f]$

$B_2[b+c+d+e+f]$

$+C_2[c+d+e+f]$ $C_3[c+d+e+f]$

$+D_2[d+e+f]$ $+D_3[d+e+f]$ $D_4[d+e+f]$

$+E_2[e+f]$ $+E_3[e+f]$ $+E_4[e+f]$ $E_5[e+f]$

$+F_2[f]$ $+F_3[f]$ $+F_4[f]$ $+F_5[f]$ $F_6[f]$

then

$C_3=B_2.b$

$D_3=(B_2+C_2)\,c$ $D_4=C_3.c$

$E_3=(B_2+C_2+D_2)\,d$ $E_4=(C_3+D_3)\,d$ $E_5=D_4.d$

$F_3=(B_2+C_2+D_2+E_2)\,e$ $F_4=(C_3+D_3+E_3)\,e$ $F_5=(D_4+E_4)\,e$ $F_6=E_5.e$

A numerical example will bring out the points of the algorithm: take $a\ b\ c\ d\ e\ f$ to be 2 3 5 7 11 13

then the calculation follows through these steps,—

Step (I)

$P_1=[2+3+5+7+11+13]$

$P_2=\ 2[\ 3+\ 5+\ 7+11+13]$
$+\ 3[\ 5+\ 7+11+13]$
$+\ 5[\ 7+11+13]$
$+\ 7[11+13]$
$+11[13]$

$P_3=\ 6[\ 5+\ 7+11+13]$
$+\ 25[\ 7+11+13]$
$+\ 70[11+13]$
$+187[13]$

$P_4=\ 30[\ 7+11+13]$
$+\ 217[11+13]$
$+1111[13]$

$P_5=\ 210[11+13]$
$+2717[13]$

$P_6=2310[13]$

Step (II)

$P_1=41$

$P_2=\ 2\times39=78$
$+\ 3\times36=108$
$+\ 5\times31=155$
$+\ 7\times24=168$
$+11\times13=143$

41 652

$P_3=\ 6\times36=216$
$+\ 25\times31=775$
$+\ 70\times24=1680$
$+187\times13=2431$

5102

$P_4=\ 30\times31=930$
$+\ 217\times24=5208$
$+1111\times13=14443$

20581

$P_5=\ 210\times24=5040$
$+2717\times13=35321$

40361

$P_6=2310\times13=30030$

30030

The whole arithmetic is shown here.

Example. In P_2: The first coefficient is $C_2 = B_2 . b$; look in P_2, we see "2 [3...," hence the required coefficient is 6. Place this in front of its [ ] in P_2.

The second coefficient is $D_2 = (B_2 + C_2)\, c$. Look in P_2 we see "$\begin{matrix} 2 \\ 3 \end{matrix}$ [5..." and get $(2+3)\,5 = 25$. Place this in front of its [ ] in P_2.

The third coefficient is $E_2 = (B_2 + C_2 + D_2)\, d$. Look in P_2, we see "$\begin{matrix} 2 \\ 3 \\ 5 \end{matrix}$ [7..." and get $(2+3+5)\,7 = 70$. Place this in front of its [ ] in P_2.

The fourth coefficient is $F_2 = (B_2 + C_2 + D_2 + E_2)\, e$. Look in P_2 we see "$\begin{matrix} 2 \\ 3 \\ 5 \\ 7 \end{matrix}$ [11..." and get $(2+3+5+7)\,11 = 187$. Place this in front of its [ ] in P_2.

Similarly 217 in P_4 comes from "$\begin{matrix} 6 \end{matrix}$ [7...," which gives $31 \times 7 = 217$: and so on.

Now note that *as placed* the [ ] factor continues always the same in the *same horizontal line*, and therefore needs only to be calculated *once for all* in step (II).

Perform each calculation in step (II) directly under its place in step (I),

First, do all the parts in ordinary type,

Second, insert the parts in Italics,

Third, make the final additions.

Note. The [ ]s are all calculated in P_2, and keeping the calculation *lined* as above, the value of each [ ] has merely to be repeated for each successive P, until it runs out. Thus 31 is the third [ ] in P_2, and it reappears automatically in P_3 and P_4 and then runs out.

The process as detailed above, may be shortly described thus:—

Of the series of letters $a, b, c, d, e, f, g, h, i, k, l$... select any one, say f, and place it so as to have free positions in front of and after it: then place after it as a multiplier, the [ ] containing the sum of all the letters that follow it (that is, formally, the sum of the products of all the letters that follow it *taken one at a time*). This letter f and the square bracket $[g+h+...]$ constitute a sum of products of *two* letters each, hence if we are calculating say P_7, we need each multiplier of $f[g+h+...]$ to be the product of 5 letters: and this multiplier is "the sum of the *products* 5 *at a time*" of all the letters which precede f. Thus one term in P_7 is

$$\left(\begin{matrix} abcde + abcdf + abcdg + abcef + abceg + ... \\ + bcdef + bcdeg + ... \\ + cdefg + ... \text{ 21 terms in all} \end{matrix}\right) h\,[i+k+...].$$

The algorithm presents a systematic method of calculating the whole series of values of $P_1, P_2, P_3 \ldots P_n$, up to any required value of n, so that each calculation not only serves for its own place, but is a step in the calculation of future terms. In short the process is an economical method for the calculation of P_r, where r has all the values $r = 1, 2, \ldots n$, and where P_r is the sum of the products r at a time of a series of n quantities.

The calculation of the problem of the Amritsar District is exhibited on the opposite page.

It will be noted in this calculation, in which the whole arithmetic is shown, that in step (I) 10 products, and in step (II) 14 products of two terms each were evaluated by logarithms, and that to obtain the final values of the P's, 5 quotients of two terms each were evaluated. In all 29 logarithmic calculations of two terms each were made. The remainder of the calculation is simple addition. A complete check may be made by comparing the value of F_0 derived as above from *all* the P's with the value of F_0 obtained from $F_0 = (1-a)\,(1-b) \ldots (1-n)$.

$$a=\frac{62}{1062} \quad b=\frac{506}{1062} \quad c=\frac{445}{1062} \quad d=\frac{669}{1062} \quad e=\frac{276}{1062} \quad f=\frac{604}{1062}$$

$P_0=1$

$P_1=[62+506+445+669+276+604]\div 1062$

$$P_2=\left.\begin{array}{l} 62\,[506+445+669+276+604] \\ +506\,[445+669+276+604] \\ +445\,[669+276+604] \\ +669\,[276+604] \\ +276\,[604] \end{array}\right\}\div 1062^2$$

$$P_3=\left.\begin{array}{l} 31,372\,[445+669+276+604] \\ +252,760\,[669+276+604] \\ +677,690\,[276+604] \\ +464,230\,[604] \end{array}\right\}\div 1062^3$$

$$P_4=\left.\begin{array}{l} 13,960\times 10^3\,[669+276\times 604] \\ +190,080\times 10^3\,[276+604] \\ +265,460\times 10^3\,[604] \end{array}\right\}\div 1064^4$$

$$P_5=\left.\begin{array}{l} 9,3392\times 10^5\,[276+604] \\ +56,3150\times 10^5\,[604] \end{array}\right\}\div 1062^5$$

$P_6=25776\times 10^{10}\,[604]\div 1062^6$

$P_0=1$

$P_1=2562\div 1062$

$$P_2=\left.\begin{array}{l} 62\times 2500 \\ +506\times 1994 \\ +445\times 1549 \\ +669\times\ 880 \\ +276\times\ 604 \end{array}\right\}\div 1062^2$$

$$P_3=\left.\begin{array}{l} 31,372\times 1994 \\ +252,760\times 1549 \\ +677,690\times\ 880 \\ +464,230\times\ 604 \end{array}\right\}\div 1062^3$$

$$P_4=\left.\begin{array}{l} 13,960\times 10^3\times 1549 \\ +190,080\times 10^3\times\ 880 \\ +265,460\times 10^3\times\ 604 \end{array}\right\}\div 1062^4$$

$$P_5=\left.\begin{array}{l} 93392\times 10^5\times 880 \\ +563150\times 10^5\times 604 \end{array}\right\}\div 1062^5$$

$P_6=25776\times 10^{10}\times 604\div 1062^6$

Multiply throughout by 1062

$P_0=1062$

$P_1=2562$

$$P_2=\begin{array}{r} 155,000 \\ +1,008,960 \\ +\ 689,300 \\ +\ 588,720 \\ +\ 166,700 \\ \hline 2,608,680 \\ \hline 1062 \end{array}$$

$$P_3=\begin{array}{r} 62,555\times 10^3 \\ +\ 391,520\times 10^3 \\ +\ 596,370\times 10^3 \\ +\ 280,390\times 10^3 \\ \hline 1,330,835\times 10^3 \\ \hline 1062^2 \end{array}$$

$$P_4=\begin{array}{r} 21,624\times 10^6 \\ +167,270\times 10^6 \\ +160,330\times 10^6 \\ \hline 349,224\times 10^6 \\ \hline 1062^3 \end{array}$$

$$P_5=\begin{array}{r} 82185\times 10^8 \\ +340140\times 10^8 \\ \hline 422325\times 10^8 \\ \hline 1062^4 \end{array}$$

$$P_6=\frac{15568\times 10^{11}}{1062^5}$$

$P_0=1062 \quad P_1=2562 \quad P_2=2456{\cdot}38 \quad P_3=1179{\cdot}95 \quad P_4=291{\cdot}56 \quad P_5=33{\cdot}2008 \quad P_6=1{\cdot}1524$

$$\begin{array}{lrl} F_0=P_0-P_1+P_2-P_3+P_4-P_5+P_6= & 35{\cdot}9416 & (\ 36\ ) \\ F_1=P_1-2P_2+3P_3-4P_4+5P_5-6P_6= & 181{\cdot}9396 & (181{\cdot}9) \\ F_2=P_2-3P_3+6P_4-10P_5+15P_6= & 351{\cdot}168 & (351{\cdot}1) \\ F_3=P_3-4P_4+10P_5-20P_6= & 322{\cdot}670 & (322{\cdot}6) \\ F_4=P_4-5P_5+15P_6= & 142{\cdot}842 & (142{\cdot}6) \\ F_5=P_5-6P_6= & 26{\cdot}2864 & (\ 26{\cdot}4) \\ F_6=P_6= & 1{\cdot}1524 & (\ 1{\cdot}2) \\ \hline \Sigma_0^6 F_r= & 1062{\cdot}0000 & (1061{\cdot}8) \end{array}$$

The bracketed figures are those calculated by Greenwood.

A comparison of the amount of labour required in the two modes of calculation is shown in the following table (where $\Sigma_0^n F_r = 1$) :—

	Simplified method. Number of products + quotients	Summation method. Number of products
$n = 2$	2 of two terms	4 of 2 terms
$= 3$	6 „	8 „ 3 „
$= 4$	12 „	16 „ 4 „
$= 5$	20 „	32 „ 5 „
$= 6$	30 „	64 „ 6 „
$= 7$	42 „	128 „ 7 „
$= 8$	56 „	256 „ 8 „
$= 9$	72 „	512 „ 9 „
$= 10$	90 „	1024 „ 10 „
$= n$	$n(n-1)$ of two terms	2^n of n terms

In the case of the Amritsar district the actual frequencies were

$$F_0 = 155$$
$$F_1 = 183$$
$$F_2 = 211$$
$$F_3 = 230$$
$$F_4 = 169$$
$$F_5 = 93$$
$$F_6 = 21$$
$$\Sigma_0^6 F_r = 1062$$

obviously the distribution was not a random one.

As examples of distributions in which accordance with the laws of chance was more marked, I quote two instances from the statistics collected by the Plague Commission.

Name of Village	Total Number of Houses	Number of houses attacked in			
		First Epidemic	Second Epidemic	Third Epidemic	Fourth Epidemic
Dhand	418	101	198	40	26
Kasel	806	308	252	230	67

The comparative agreement between calculated and observed frequencies is shown in the following table :—

Number of Houses Attacked	Dhand		Kasel	
	Calculated	Observed	Calculated	Observed
Never	141·495	136	224·30	209
Once	196·77	208	350·58	383
Twice	71·26	65	189·22	169
Thrice	8·19	9	39·62	43
Four times ...	0·285	0	2·28	2
Totals	418	418	806	806

Formulæ (1) may be written, in reverse order,

$$(3)\qquad \begin{aligned} F_n &= P_n, \\ F_{n-1} &= P_{n-1} - nP_n, \\ F_{n-2} &= P_{n-2} - (n-1)P_{n-1} + \frac{n(n-1)}{1.2}P_n, \\ &\vdots \\ F_{n-r} &= P_{n-r} - (n-r+1)P_{n-r+1} + \frac{(n-r+2)(n-r+1)}{1.2}P_{n-r+2} - \dots \\ &\qquad \dots + (-)^r \frac{n(n-1)\dots(n-r+1)}{1.2\dots r}P_n \end{aligned}$$

In the special case where $a=b=c=\dots=p$, we have

$$\begin{aligned} P_n &= p^n, \\ P_{n-1} &= {}_nC_{n-1}p^{n-1} = np^{n-1}, \end{aligned}$$

and generally

$$P_{n-r} = {}_nC_{n-r}p^{n-r} = \frac{n!}{(n-r)!\,r!}p^{n-r}.$$

Thus by (3)

$$\begin{aligned} F_n &= p^n, \\ F_{n-1} &= np^{n-1} - np^n = np^{n-1}(1-p), \\ F_{n-2} &= \frac{n(n-1)}{1.2}p^{n-2}(1-p)^2, \end{aligned}$$

and so on, giving again the well-known result

$$F_r = \frac{n!}{(n-r)!\,r!}p^{n-r}(1-p)^r.$$

II. The Application of the Correlation Coefficient to Mendelian Distributions.

By E. C. SNOW, M.A.

In a paper* published in the Proceedings of the Royal Society of Edinburgh, Dr John Brownlee has employed various methods to determine theoretical values of the parental and fraternal correlations under special conditions, on the basis of the Mendelian formulæ. We do not propose to deal with Dr Brownlee's conclusions, but only to draw attention to some of his methods. The importance of criticising them is not diminished by the fact that they may in some cases give correct results. It is, indeed, for this reason the more essential that they should be scrutinised, as their employment in other circumstances, where they may not give correct results, is rendered the more likely.

The methods to be employed in determining correlation depend entirely on the nature of the frequency distributions dealt with. The following general types of distribution can be recognised: (*a*) continuous and quantitative, e.g. head length and many other anthropometric measurements, (*b*) continuous and yet for convenience of classification treated as qualitative, e.g. health and intelligence, (*c*) discontinuous and quantitative proceeding by equal steps, e.g. the number of veins on a leaf, the position of an individual within a family, (*d*) discontinuous and quantitative proceeding by unequal steps, e.g. various botanical distributions, and the frequency within any grade of various salaries in government departments, (*e*) discontinuous and qualitative, e.g. various types of occupations.

The chief methods which have been discovered for the determination of correlation are:

(i) The four-fold table method, which applies only when the table consists of two rows and two columns, and can only legitimately be employed when the distributions are perfectly continuous and at least approximately Gaussian.

(ii) The method of contingency (giving $C_2=\sqrt{\frac{\phi^2}{1+\phi^2}}$, where ϕ^2 is the mean square contingency). This can be applied whatever the number of cells and the nature of the distributions, but C_2 is only equal to r—the value of the correlation coefficient for the table—when the distribution is Gaussian and the number of cells is large.

(iii) The 'product moment' method. As all the cases with which we are concerned in the present paper are of tables of two rows and two columns or of three rows and three columns we need only discuss this method in relation to those cases. In both of them it is necessary that the observations should be supposed concentrated at points for each row and each column, and in the second of the cases it is necessary that the distances between these points for consecutive rows, and also for consecutive columns, should be equal, and also that the regression of each variable on the other should be linear. In the case of a two by two table, the value of r given by this method is

$$r^2=\frac{(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)},$$

and this, for the same case, is also the value of ϕ^2†. But this is not the same as the value of C_2 which is usually taken as the measure of relationship when the method of contingency is

* *The Significance of the Correlation Coefficient when applied to Mendelian Distributions, Proc. Roy. Soc. Edin.* Vol. xxx, Part vi. (No. 34).

† See *Drapers' Research Memoirs*, Biometric Series i. p. 21. r is also the correlation between random deviations in the means of the two variates, when these deviations are expressed in terms of the standard deviations of the variates as units. See Pearson, *Phil. Trans.* Vol. 195, A, pp. 12 and 14.

employed. C_2 is approximately the same as ϕ only when ϕ is small. If $\phi = r = \cdot 5$, its value is $\cdot 45$. Thus in such tables, and for values of r not greater than $\cdot 5$, those values will not greatly differ from the values of C_2.

Now in Mendelism we are dealing with discrete entities known as gametes. Referring first to the problem of gametic correlation and taking for an example that between parent and offspring, each individual in the population considered has a gamete which possesses 0, 1 or 2 protogenic elements of a particular character. Thus this case falls under the head (*c*). Taking parents with, say, 0 protogenic elements we have an array of offspring some with 0, some with 1, and some with 2 protogenic elements, and we can find the mean number of protogenic elements in members of the array. Now the parent's gametic character is supposed to be perfectly correlated with the number of protogenic elements for the character which he possesses, and this is likewise true for the offspring. Assuming then, that if, of four variables u, v, x and y, u is perfectly correlated with x and v perfectly correlated with y, then the correlation between x and y is the same as that between u and v, we find that the gametic correlation between parent and offspring is the same as that between the number of protogenic elements possessed by the parent and the number possessed by the offspring. Thus such a table as

Number of Protogenic elements in Gamete of Parent.

Number of Protogenic elements in Gamete of Offspring.	(2)	(1)	(0)
(2)	*a*	*d*	*g*
(1)	*b*	*e*	*h*
(0)	*c*	*f*	*k*

is equivalent to a table between parent and offspring for the particular gametic character. Since the observations are concentrated at points within each group, these points in any row or column of the above table are equidistant. Thus the legitimate method to employ on this case is (iii). A value of C_2 as by (ii) can be found to give a measure of relationship, but as the conditions referred to in (ii) are not satisfied it will not be the true value of r. We can determine 'regression' lines from a knowledge of the means of the three rows and columns. Here we must note the fundamental disagreement between continuous variation and Mendelism. For a number of characters regression lines have now been determined of offspring on parent. Many of these are perfectly continuous regressions, that is, for any small increase or decrease in the value of the parental character we can observe a corresponding small increase or decrease in the mean value of the character for the offspring. In other cases, e.g. the petals of a flower, the regressions must proceed by steps. But a number of cases which have been shown to give perfectly continuous regressions, the Mendelians claim to depend upon a single unit character, e.g. piebaldism. All that the term 'regression' can denote in a Mendelian instance is that as we jump from 2 to 1 protogenic elements in the parent, the effect on the mean number of protogenic elements in the offspring is the same as when we jump from 1 to 0, for we cannot split up the protogenic element. Thus although we use the term 'regression' in the Mendelian cases we only do so in the signification of the last sentence, viz., that an increase or decrease of one protogenic element in the parent is accompanied by an increase or decrease of, say, m protogenic elements in the mean value for the offspring. This is fundamentally different from the continuous case, in which we can observe an increase or decrease of $1°/_{\circ}$ of the parent's character followed by an increase or decrease of, say, $n°/_{\circ}$ in the mean of the value for the offspring. The comparisons throughout this paper are to characters which appear to be continuous, but which Mendelians claim to depend upon 'unit characters.'

If for the columns in the above table,

Mean of column (2) − Mean of (1) = Mean of column (1) − Mean of (0),

this Mendelian 'regression' is linear, and can be written in the form $r\sigma_y/\sigma_x$. If, in the same way, the other 'regression' is linear, it can be written in the form $r\sigma_x/\sigma_y$. Hence r can be obtained.

In the case of somatic characters, the individuals with 1 and 2 protogenic elements are considered to be somatically the same. Thus we have individuals with 0 protogenic elements, and others with a definite number, say, m. The correlation table (two rows and two columns) formed for this case will refer to individuals some concentrated at a point representing 0 protogenic elements, and others concentrated at a point representing m protogenic elements. The correct method to apply in this case is that of (iii). As before, a value of C_2 can be found, to give a measure of relationship, but this will not be the true coefficient of correlation. The two observations in each column have a mean, and these means can be joined by a line which may be called a 'regression' line. But this is less comparable to the actual regression lines found in practice for continuous distributions than in the case of gametic correlations, and it is doubtful if a correlation determined from it is of the same significance as one ascertained from a table with many rows and columns. But whether the comparison be legitimate or not, no other method of ascertaining correlation for a two-by-two table in which the observations are concentrated at points seems to be as sound as that indicated in (iii).

Dr Brownlee in his paper applies all methods to all cases *indiscriminately* and reaches very divergent results. In a particular case (p. 477) in which the product moment method gave $r = \cdot 33$, the four-fold table method gave $\cdot 53$, and the mean square contingency (C_2) $\cdot 37$, the first and third of these not greatly differing. Dr Brownlee believed that "in a Mendelian instance such as this, the four-fold table seems specially applicable," but we can imagine no single case in which that method (assuming as it does a particular form of continuous distribution) is less applicable. The fact that its use leads to a value more in accordance with results found for continuous characters could surely be used as an argument against the use of the hypothesis of the 'unit-character,' but this hypothesis is the essential foundation of Mendelism. In any problem on the determination of correlation it is very necessary to understand clearly the nature of the variation dealt with. When Dr Brownlee applies all methods to a single case, he is assuming that the distributions are at the same time both continuous and discontinuous, and we can get little information of value from his results.

We need not point out in detail the many cases in which Dr Brownlee uses methods which strike at the very root of Mendelism, and which would be rejected with scorn, if they were understood, by the supporters of Mendelian principles. He has throughout stated the method by which he has obtained his many results, so that these results can easily be assessed at their true value.

We may, however, point to one or two other instances of the looseness of thought which permeates the paper. On p. 476 Dr Brownlee uses the gametic distribution $2(AA)+4(Aa)+2(aa)$. He then clubs together the (AA) and (Aa) groups and obtains $6\,(A)+2\,(\text{not } A)$, and states "in this last case, however, the distribution is markedly skew." He then proceeds to use the four-fold table method, which should never be used if there is a suspicion that the distribution is more than slightly skew. But, in fact, Dr Brownlee knows nothing whatever about the distribution. Assuming that he was dealing with continuous variation, all he has done is to divide a frequency curve which may or may not be symmetrical into two parts, and this process of division does not in the slightest alter the distribution. If the one distribution be symmetrical, the other is also. Even such extreme numbers as $99\,(A)$ to $1\,(\text{not } A)$ may well represent a symmetrical distribution; the actual numbers depend solely upon the point at which the division is made.

Further evidence that Dr Brownlee appears to consider Mendelian distributions as perfectly continuous is given in his § 8, in which he states "if for the moment the distinguishing character of the hybrid and the dominant be assumed somewhat indefinite, we can make several

tentative divisions, either bisecting the hybrid or dividing it into such divisions that one-fourth resembles the recessive." He then applies the four-fold table method to these cases and finds the corresponding correlations are ·44 and ·50 and concludes "again, Mendelian principles do not lead to low correlations but to figures approximately equal to those found by observation." But these "tentative divisions," by assuming continuity of distribution, throw over the whole Mendelian case.

Later on Dr Brownlee deals with the cases of two and three pairs of zygotes, looking at the problem solely as one in continuous variation. For he states "it is evident that when two and three pairs of zygotes are condensed* we do not go straight back to the normal distribution. The reason of this is that the normal surface obtained when the elements are considered separately, represents something different from the surface which is condensed into the last tables." It is difficult to find the meaning of this paragraph, but at any rate it shows that Dr Brownlee has overlooked the fact that Mendelism was rediscovered by biologists who were seeking for a theory to explain *discontinuous* variation.

Dr Brownlee devotes part of his paper to an investigation, on the Mendelian basis, into the effects of selection and of assortative mating. His confusion between continuous and discontinuous variation is evident in this part as elsewhere. As regards selection, he concluded that on a Mendelian mechanism it does not follow that the higher the parental selection the lower the correlation coefficients, the result which had been reached by Prof. Pearson by biometric methods. The question is an important one and can by no means be got over so simply as Dr Brownlee appears to have done, and we have attempted to investigate it elsewhere. We may state here, however, that our results do not agree with those of Dr Brownlee, and that we find that the assumption of Mendelian discontinuous variation leads to qualitative (and often quantitative) conclusions similar to those discovered by Prof. Pearson on the assumption of continuity of distribution. There is one point, however, in Dr Brownlee's work on this subject to which we may refer. In certain cases he obtains an approximate value of the correlation coefficient for three-by-three tables and by the product moment method when the regressions are not linear. We take, for example, the table from his p. 487, which we give below.

Parent.

Offspring.	(*AA*)	(*Aa*)	(*aa*)	Totals
(*AA*)	$m+1$	$m+1$	—	$2m+2$
(*Aa*)	2	$m+3$	$m+1$	$2m+6$
(*aa*)	—	2	2	4
Totals	$m+3$	$2m+6$	$m+3$	$4m+12$

Proceeding by the ordinary product moment method the expression found for r is

$$r=\frac{m+3}{\sqrt{2}\,(m^2+14m+17)}.$$

If both regressions were linear this expression should be the square root of the product of those regressions. Now the regression of offspring on parent in the above table is linear, and equal to ·5. But the regression of parent on offspring is not linear unless $m=1$.

* This phrase is a typical example of Dr Brownlee's looseness of language. The definition of a zygote as "the cell formed by the fusion of a male with a female gamete" seems to render impossible the operation of condensing 'pairs of zygotes.' So far as we can ascertain, Dr Brownlee merely means that the character considered depends upon a number of Mendelian couplets, and not on a single one.

If x_2, x_1 and x_0 denote the positions of the means of the (AA), (Aa) and (aa) rows we find

$$x_2-x_1=\frac{m+1}{m+3} \text{ and } x_1-x_0=\frac{2}{m+3}.$$

The above value of r will be reasonably accurate only as long as these two expressions do not greatly differ, i.e. in the neighbourhood of $m=1$. Actual values of the expressions are:

m	0	·5	1	2	10	∞
x_2-x_1	·33	·43	·50	·60	·85	1·0
x_1-x_0	·67	·57	·50	·40	·15	0

Thus, except when m is very close to unity, the expressions differ considerably, and the above formula for r cannot be taken as giving a true value of the correlation for such a table. If we approximate by taking the mean of the values of x_2-x_1 and x_1-x_0 weighted according to the frequency on which each is based, we reach results which in the present case are not far different from the values given by Dr Brownlee's formula. In another case, however, which we have worked out the difference is greater. If we do not weight, the difference may be very great.

We have by no means exhausted the possible criticisms of Dr Brownlee's work. Sufficient has been done, however, to show that his results must be interpreted with very great caution. Finally, in all cases of the determination of correlation, we would emphasize the necessity for a clear understanding of the nature of the distributions dealt with.

III. A Mendelian View of racial Heredity.

By H. J. LASKI, New College, Oxford.

The application of Mendelian principles to the problems of inter-racial heredity has not thus far met with any very conspicuous success. The work of Davenport on the mulatto is quite inconclusive in character. The incomplete pedigrees of Mudge merely defeat the end which doubtless they were intended to serve. It is not, indeed, too much to say that the endeavour to make man a complex of sharply-defined unit characters has failed, and failed completely. Even the researches of Hurst, which were received with an almost lyrical enthusiasm by the adherents of Mendelism, are not above suspicion; for post-mortem microscopic examination has revealed the presence of anterior pigment in eyes or parts of eyes which, after most careful examination, were judged free from it in life. Undeterred, however, by previous failures Dr Redcliffe Salaman* has endeavoured to prove the applicability of Mendelian theory to the problems of Jewish heredity. His paper is so characteristic an example of much recent Mendelian work that it is not entirely purposeless to examine it in some detail.

Its conclusions may, at the outset, be briefly summarised. Dr Redcliffe Salaman investigated the transmission of a Jewish facial characteristic which, somewhat mysteriously, he calls Jewishness. His method was to collect cases of intermarriage between Jew and Gentile, and thus to discover whether the resulting offspring were Jewish, non-Jewish, or intermediate in appearance. He was assisted throughout by Jewish observers who, he states, seemed desirous whenever possible of claiming the child as Jewish†. This is, perhaps, a little curious in the light of our knowledge of the Jewish aversion for intermarriage and their known desire to dissociate its consequences from anything Jewish. It is remarkable that the bias of the observers influenced their judgment so little if the personal equation is so high as Dr Salaman

* *Journal of Genetics*, Vol. I. pp. 273—290.
† *Loc. cit.* p. 280.

believes. The data, it should be said, relate entirely to the Jews of Northern and Central Europe and the Gentile is either English or North European. This limitation is noteworthy because there does not seem to be a specifically Jewish type among the Sephardim, or Jews of Eastern Europe and Africa. They are rarely to be distinguished from the natives of the country to which they belong, and to distinguish between the Jew and the Armenian is almost impossible.

Dr Salaman provides the following table:

Number of families	Father	Mother	Children's appearance		
			Gentile	Jewish	Intermediate
50	Gentile	Jewish	88	15	4
86	Jewish	Gentile	240	11	4
Total 136	—	—	328	26	8

His conclusion may be briefly summarised as follows:

(1) The Gentile features, or rather, facial expression act as practically a complete dominant to the Jewish.

(2) In the matings of the hybrids the following results were reached:

(*a*) Where the Jew is father and the mother the hybrid: in nine families there were "thirteen children undoubtedly Gentile and twelve unequivocally Jewish";

(*b*) Where the mother is Jewish and the father is the hybrid: in four families there are seven children of whom two are Gentile and five are Jewish. Dr Salaman adds (*a*) and (*b*) together and obtains a very close approximation to the Mendelian expectation of equality.

(3) He has five cases where the hybrid married a Gentile and in all of these the resulting eight children are Gentile in appearance. This is not perhaps unexpected on any theory of heredity, since three-quarters of the offsprings' blood would be Gentile and the difference between the results of (3) and (2) (*b*) is not a little curious.

(4) When the Jew who lacks the characteristically "Jewish" expression marries the markedly "Jewish" type the latter is dominant. Pedigree 6 is quoted by Dr Salaman in support of this important result.

(5) Finally, "The conclusion to which these results inevitably lead is that the Jewish facial type, whether it be considered to rest on a gross anatomical basis, or whether it be regarded as the reflection in the facial musculature of a peculiar psychical state is a character which is subject to the Mendelian law of Heredity*."

These results seem open in the very highest degree to criticism. Objection must, in the first place, be made to the categories used. I do not think it is possible to understand the nature of the character of which the transmission and behaviour were investigated. One would, I think not unfairly, expect Dr Salaman to provide us with a definition or at least a more or less exact description of the character with which he deals. It is surely a lamentable lack of scientific precision not to do so. Dr Salaman himself is not at all certain what is meant by this "Jewishness." In the earlier part of his paper he is at some pains to demonstrate the fact that the Jews cannot be considered racially pure. He quotes, indeed, with approval the statement of Weissenberg† "the Jew in an anthropological sense forms no specific type, but the facial expression is absolutely characteristic"; but immediately above the opinion of Ripley‡ who favours the existence of a Jewish type is quoted as bringing comfort to the observer. But

* *Loc. cit.* p. 285.

† *Archiv f. Anthropol.* Bd. XXIII. S. 347—423 and 531—579.

‡ Ripley, *Races of Europe*, pp. 393—400.

Dr Salaman's conclusion (5) makes it a matter of indifference whether we accept the opinion (it is hardly more) of Ripley or of Weissenberg. We have nothing definite or tangible upon which to seize. The Jewish nose has a prominent place in the essay, but as to its value as a possible index of racial character Dr Salaman is perfectly agnostic. We are told of a face with "rounded features, long sloping jaw, fairly developed chin which is round and not square, a good sized forehead devoid of that angularity in the temples not uncommon among Teutonic peoples," but these highly complex anatomical peculiarities, each of which from its mere description seems to demand quantitative expression, do not constitute "Jewishness" but merely accompany it. For Dr Salaman, in fact, the "Jew is a Jew because he looks like one." This, indeed, seems to be his most characteristic argument. He deals with a character which does not admit even of qualitative expression. While it is necessarily a function of the various organs of the face, and therefore an allelomorph so highly compound as to be scarcely resoluble into its component parts, it does not admit of expression in terms of them. Now, if the facial features are each one and separately a Mendelian unit, is *each* unit in the Jewish face recessive to the corresponding unit in the Gentile face so that the whole Jewish face is recessive to the Gentile? A blend would be produced even if some units were dominant over others in the general appreciation of Jewishness; but if Dr Salaman's view be justified, then every Jewish unit must be recessive to every other Gentile unit; but since this has not been tested it is purely arbitrary to assume its truth. It is not easy to see why the shape of a Gentile nose should be dominant to the shape of a Jewish nose in precisely the same way as a Jewish chin should be recessive to a Gentile one. When we ask what "Jewishness" is, we are told that it is the quality of looking Jewish, and to the question of what it means to look Jewish Dr Salaman would doubtless reply that it is to possess the quality of Jewishness. So vicious a circle of reasoning seems, scientifically, almost ludicrous, and it can hardly have value from the standpoint of anthropology. Dr Salaman appears in some degree troubled by this. "In determining the nature of so complex a character," he writes*, "the personal equation of the observer must play an important part. I have in some cases found that observers not specially acquainted with the subject, although agreeing that a given individual of the first generation is of Gentile appearance have yet felt that there was somewhere lurking in the face an expression which suggested 'Jewishness,' and there is very little doubt that such an opinion may be well founded. I have myself come across a few cases where without doubt the recessive Jewish facial expression has come to the surface as the individual grew older." Stated in other words, the first sentence would seem to imply that individuals classed as dominant may in reality constitute a blend. This has some importance in view of the suggestion made above that not all the Jewish characters may be recessive to the Gentile, and, if this be once conceded, what becomes of the correlated inheritance which is necessary for the establishment of Mendelian theory? The second sentence indicates a very real absence of gametic purity. These facts as stated by Dr Salaman throw the gravest doubt on the value of the categories used. When a definition of the character investigated is lacking because, seemingly, it can only be "felt" by the observer; when his observers are classifying as Gentile offspring with a tinge of Jewishness—surely nothing more than a Mendelian gloss for an intermediate; when Dr Salaman observes the appearance of the supposed recessive with increase of age; the nature of Gentile dominance is very far indeed from being so satisfactory as his table would seem to show. Professor Pearson tells me of a typical case of intermarriage where the Gentile relatives say the children are Jewish, the Jewish relatives Gentile, in appearance. Here we have the dominance of Jewish features when judged by a Gentile, of Gentile when judged by a Jew. Dr Salaman's observers, as he himself tacitly admits, do not feel that his categories are sufficiently precise. The personal equation plays, I think, an even more important part than Dr Salaman imagines. In Weissenberg's experiment the Jew and the Gentile differed in their estimate by 20 per cent. and to the Jew 30 per cent. and to the Gentile 50 per cent. did not seem Jewish,—a highly

* *Loc. cit.* p. 282.

significant series and entirely unfavourable to Dr Salaman's view. It seems to show that the Jewish conception of "Jewishness" and that of the Gentile are something very different indeed, and that an independent classification of Dr Salaman's data by a Gentile observer would not lead to the same result. The fact is,—and in research of this kind it is an absolutely fatal objection to such loose and worthless categories,—the Jew catches the Gentile, and the Gentile the Jewish, features in what is probably a blend, and is guided in his judgment by the degree in which he perceives them. It is not until the nose, the eyes, pigmentation and other Gentile-Jewish differentia are reduced to measured features that any real solution of the problem can be reached.

I think therefore, that Dr Salaman's categories are open to grave and even fatal objection. Let us turn now to the results of the investigation itself. Dr Salaman has himself admitted throughout his paper the existence of a large number of Jews who do not possess the character "Jewishness." We do not find that he has given us any facts as to the appearance of those Jews who married Gentiles and whose offspring form Table I. To the ordinary observer it would appear obvious that when a Gentile marries a Jew whose features do not possess "Jewishness" the resulting offspring would be Gentile in appearance. We require a fairly large sample of the Jewish population showing the distribution of its component elements before Dr Salaman's conclusions can be accepted. Now in Weissenberg's* experiment a proportion of his samples consisting either of 30 or 50 per cent. of his population, according to the personal equation of the observer did not appear distinguishably Jewish. It seems therefore reasonable to assume that the non-Jewish looking Jew forms a fairly large proportion of the population, larger, as I shall show, than it is possible for Dr Salaman to admit. Again, it must surely have struck him that the distribution of types in his Table I is not without significance. In the first case, ♂ Gentile × ♀ Jewish, the distribution is 88 Gentile, 15 Jewish and 4 intermediate, i.e. in the sample, classing Jewish and intermediate as Jewish, 18 per cent. were Jewish in appearance. In the second case, ♀ Gentile × ♂ Jewish, gave 240 Gentile, 11 Jewish, and 4 intermediate, i.e. with the same classification as before, 5 per cent. are Jewish. This is surely very anomalous, since on Mendelian theory there is no distinction between the two cases. Dr Salaman ascribes the appearance of the recessive Jewish types and the intermediates to (1) the Jewish bias in the observers and (2) a Jewish permeation of the English people in particular districts. In reference to (1) it is only necessary to say that the bias in favour of distinguishing the offspring as Jewish must be very great when the observer makes an incorrect distinction between two types which Dr Salaman tells us cannot be mistaken. His inconsistency here is destructive of his entire position. If this incorrectness be once admitted, it is sufficient to cast doubt upon the entire investigation. In (2) Dr Salaman, I take it, means that there is a Jewish ancestry even among the Gentile inhabitants. The Gentile inhabitant would therefore be an imperfect dominant on this theory; but if there be this ancestry it is difficult to see why a Jewish looking Jew should always be dominant over a non-Jewish looking Jew, who would be often only a Jew in name and not a gametic Jew. The complete data should, I think, be tabulated before this exceedingly hypothetical explanation can be accepted as satisfactory. The mating of the hybrids with pure types seems to Dr Salaman entirely Mendelian in result, the approximate equality being the Mendelian expectation. I think the evidence I shall now adduce in criticism of this conclusion is sufficient to render it untenable.

When the father is a Jew and the mother the hybrid and conversely, the distribution of types in the resulting offspring is 15 Gentile to 17 Jewish, the Mendelian expectation being, as we have said, equality. But Dr Salaman's analysis is very far from sufficient. What was the appearance of the hybrid parent? This, the really crucial point, is left a mystery. The following scheme indicates the possibilities which might have occurred.

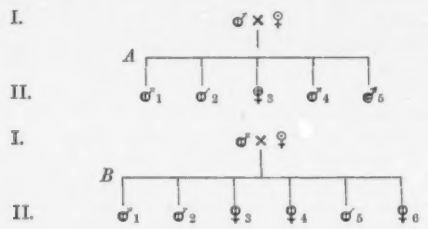
* *Globus*, Vol. xcvii. 6, 9, 10, 1910.

Hybrid Jewish in appearance × Jew Jewish in appearance,
" " " × Jew intermediate in appearance,
" " " × Jew non-Jewish in appearance.

Hybrid intermediate in appearance × Jew Jewish in appearance,
" " " × Jew intermediate in appearance,
" " " × Jew non-Jewish in appearance.

Hybrid Gentile in appearance × Jew Jewish in appearance,
" " " × Jew intermediate in appearance,
" " " × Jew non-Jewish in appearance.

It is evident from this scheme that the problem is immensely more complex than is indicated by Dr Salaman. His belief that the Jew who appears recessive carries an inhibiting factor which prevents the appearance of "Jewishness" works in any case against his view, for when crossed with pure Gentile the offspring could not possibly be Jewish on any theory. But we can go further than this. Is the non-Jewish type so markedly recessive to the Jewish type? While I thoroughly distrust any such vague category as "Jewishness," I have yet endeavoured to the best of my ability to apply it to certain data of my own and see in how far the results are consonant with Dr Salaman's. The pedigrees here given refer in every case to families where intermarriage has not, within knowledge, occurred.



⊙ = Marked Jewish type.
◑ = Non-Jewish type.
⊕ = Intermediate type.

Fig. I.

These pedigrees indicate, I think, a dominance of the non-Jewish type. I give the results of the marriage of some of the above offspring on the opposite page.

These show absolute divergence from Dr Salaman's conclusions. In every case the cross between the distinctly non-Jewish type and the distinctly Jewish type gives dominance to the former. In the case of the intermediate type no definite conclusion seems possible. The result indeed is what the biometrician is beginning almost to expect in Mendelian work on man, that while one pedigree may admit of a Mendelian interpretation others no less certainly contradict it. If we use Dr Salaman's classification, it is obvious that the non-Jewish looking Jew cannot possibly be regarded as a recessive. If it be regarded as *DR*, then in the crosses *DR* × *DR* the absence of *RR* is inexplicable. Now it does not seem to me that this divergence disproves the applicability of Mendelism to the facial expression in man, but it does demonstrate that it is purely idle to use vague categories of this kind, and then claim that any theory can be

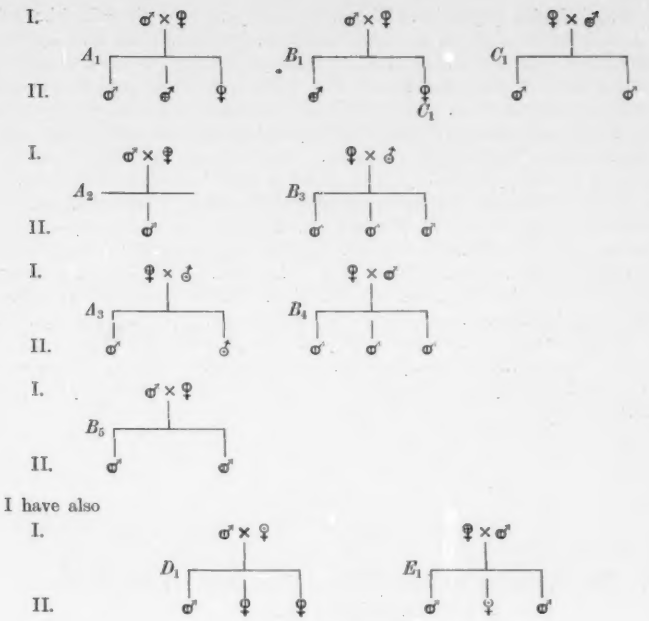
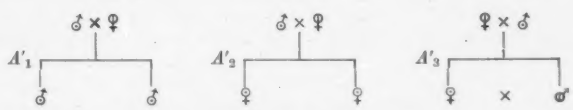


Fig. II.

demonstrated by such use. I have in my possession several pedigrees illustrating the result of crossing pure Jew with pure Gentile. These give



⊙ = Distinctly Jewish.
⊕ = Pure Gentile birth.
⊕ = Intermediate appearance.

Fig. III.

In each case the cross of a distinctly Jewish type with a no less distinctly Gentile type produces offspring either of Jewish or of intermediate appearance. It is, in fact, characteristic of the application of Mendelism to problems such as these, that imperfect categories and incomplete analysis should superficially render soluble a problem which, on a more intimate dissection, does not yield its secret to the Mendelian enthusiast.

In fact, this problem, as every genetic problem, is to be solved only by the method of quantitative analysis, that is, by the methods of biometry. We must know the numerical distribution of the various types of Jew in a population sufficiently random in character to

represent the general Jewish population. We must actually measure the "Jewishness" by analysing it into its component factors and measuring them by accepted and specially devised anthropometric methods—such as the nasal index, the cephalic index, the profile angle, etc., together with any number of new facial measurements which it would be quite easy to devise, and also pigmentation determinations. Where, as in man, we cannot be certain of the gametic constitution of an individual, we can only guess at it by an inquiry as to purity of the ancestry and the somatic characters both in Gentile and Jew. This, especially in regard to non-Jewish looking Jews, has been wholly omitted by Dr Salaman in each individual case. From such knowledge we can, by well-known biometric methods, calculate the probable character of the offspring. The result of such an investigation may be to confirm Mendelian theory—as to that I make no assertion. But we stand now, and until this fact has penetrated into the Mendelian consciousness we shall continue to stand, in the position indicated by Weldon ten years ago—"the fundamental mistake which vitiates all work based on Mendel's method is the neglect of ancestry and the attempt to regard the whole effect upon the offspring produced by a particular parent as due to the existence in the parents of particular structural characters; while the contradictory results obtained...show clearly enough that not only the offspring themselves but their race, that is, their ancestry, must be taken into account before the result of pairing them can be predicted."

IV. On "Cancer Houses," from the Data of the late Th. Law Webb, M.D.

By KARL PEARSON, F.R.S.

The data dealt with in the present paper contain a record of the house distribution of 377 cases of cancer occurring between 1837 and 1910 in the Madeley registration subdistrict. They include both cases of sarcoma and carcinoma. It has not been possible to separate the two, for in many cases the entry is merely cancer. The data include the exact position of the house, its water supply; the age, occupation, and date of death of subject, and the organ affected by the growth. The data consist, besides the record of cases, of large scale maps of the district upon which every inhabited house is marked in with blue wash and every house in which a case of cancer followed by death has occurred in the given period with red wash. A red cross marks existing cases of cancer in the living. The collection was formed by Th. Law Webb, Esq., M.D.

The deaths from cancer are as follows:

1831—1840: [20?]*	1841—1850: 14	1851—1860: 20
1861—1870: 48	1871—1880: 41	1881—1890: 77
1891—1900: 81	1901—1910: 88.	

The number of cancer deaths has thus increased very considerably, but the population has increased† and its average age has no doubt very considerably increased, so that it is not possible to say whether this increase of cancer deaths marks an actual increase of cancer.

* Based on four years of the decade only.

† Not very markedly, and lately it has fallen again.

There appear to be very few large houses in the district, most being by their descriptions of the cottage type. Certainly those in which multiple cases of cancer have occurred are often quite small. It is not very easy to determine from the maps the exact number of inhabited houses, but it appears to be about 2865*. Twelve cases of cancer appeared to be living in the district at the date of the return; these are not included in our number of cases. Taking 377 death cases in all and supposing these to be distributed at random among the 2865 houses we should anticipate according to the formulæ of my article (*Biometrika*, Vol. VIII. p. 410).

330·6 houses with one case,	in round numbers	331
21·7 houses with two cases,	„ „	22
·95 houses with three cases,	„ „	1
·03 houses with four cases,	„ „	0

A total is thus obtained of 354 houses with 377 cases and there will be 2511 houses with no cases.

The actually recorded numbers appear to be as follows: 2523 houses with no cases, 315 houses with one case; 20 houses with two cases; six houses with three cases; and one house with four cases. The identification of the houses is not always quite clear, but I think I have erred, if at all, on the side of reducing the multiple houses, e.g. I have supposed nine cases in which the house was not known to have occurred in non-multiple houses. On the other hand, when the sufferer lived in a house up to death, going perhaps to a general hospital or the workhouse just before death, I have reckoned the cancer as developed in that house.

If we now substitute in the general formula

$$\chi^2 = S\left\{s^2\frac{(p_s-\bar{p}_s)^2}{\bar{p}_s}\right\},$$

we find

$$\chi^2 = \frac{(16)^2}{331}+\frac{4\times(2)^2}{22}+\frac{9\times(5)^2}{1}+\frac{16\times(\cdot97)^2}{\cdot03}$$

$$= \text{about } 727.$$

The probability P is therefore infinitesimal, being widely outside any existing tables for χ^2 and P. We may therefore say that if these numbers be correct the distribution cannot possibly be a random one. Six houses with three cases each and one with four are wholly beyond the bounds of the possible, assuming cancer to be distributed at random among the houses.

I have made a second estimate of this improbability. I have assumed that Dr Law Webb's district actually coincides with the Madeley subregistration district and that a count of houses on the Ordnance Map is likely to be defective †.

The 1841 Census gives the number of houses as 1802 and the population as 8732. The 1851 Census gives 2006 houses and a population of 9848; the 1861 Census provides 2154 houses and a population of 10,733. In 1871 we have 2291 houses and a population of 10,535. In 1881 the numbers are 2359 houses and 10,026 persons. In 1891 we have 2228 houses and 8825 persons. In 1901, 9129 persons and 2196 houses. But it is not quite certain that the boundaries remained absolutely the same. In 1911 the population was 8859 and the *inhabited* houses 2037. I take 2000 as an average number of houses, and if 9000 to 10,000 be the average population then 4·5 to 5·0 are the average number of inhabitants per house.

* Taking about eight cancer deaths a year for the last three decades, this suggests an *average* recognised duration of the disease of about eighteen months.

† The estimate of houses must be very elastic, many houses in the period have come into being, large numbers have ceased to be: the 2865 is a maximum limit.

For $n=377$ $m=2000$, we find:

$$\left.\begin{aligned} p_1&=312{\cdot}4\\ p_2&=29{\cdot}4\\ p_3&=1{\cdot}84\\ p_4&=0{\cdot}086 \end{aligned}\right\}\text{ giving a total number of cases}=377{\cdot}06,$$

$$\chi^2=\frac{(2{\cdot}6)^2}{312{\cdot}4}+\frac{4\,(9{\cdot}4)^2}{20}+\frac{9\,(4{\cdot}16)^2}{1{\cdot}84}+\frac{16\,(\cdot914)^2}{\cdot086}$$

$$=\cdot01+17{\cdot}67+84{\cdot}65+155{\cdot}42$$

$$=257{\cdot}75.$$

P lies therefore outside any table, i.e. the probability that such a distribution could arise from random sampling is only one in many many millions. It seems therefore clear that the reduction of our number of houses by 800 makes no substantial difference in the improbability of our result. The houses with three cases are quite sufficient in themselves—even if we neglected the four-case house—to make the distribution indefinitely improbable.

We have next to consider the sources of this improbability.

The possibilities are:

(*a*) that these "cancer houses" are larger and contain more inhabitants than the others, thus they would be more likely to have multiple cases.

(*b*) that constitutions liable to cancer are hereditary and so the "cancer house" marks merely the presence of a "cancer family."

(*c*) that certain houses have been inhabited by persons following the same occupation, and that "cancer houses" are those inhabited by persons with a bad occupational mortality for cancer.

(*d*) that some houses by their environment, or by the presence of some organism render their occupiers more liable to cancer.

I propose to examine the seven instances of three or more cases from the problem of these possibilities.

(A.) No. 1. G—— V——. The first case that occurred in this house was that of a station-master, aged 28, cancer of rectum. He died in 1855. The second case was that of a school-mistress, who died, aged 64, in 1889 from cancer of the uterus. The third case was that of the servant to a nonconformist minister and his wife who were living next door. This servant attended the second case, that of the schoolmistress, in her last illness, and died in 1890, aged 57, of malignant ulceration of the intestines with perforation. The fourth case lived to her 78th year in this house in association with the third, but removed before her death elsewhere. She died at 88 years in 1899 of cancer of the liver.

(B.) No. 2. G—— V——. This is the *next* house to A above. The first recorded case in this house was that of the nonconformist minister referred to in A. He died about 1870, aged about 60, of cancer of the stomach. The second case was that of his wife who died in 1880, aged 71, of cancer of the rectum. The third case was the wife of the man who followed the widow of the nonconformist minister. She died of cancer of the breast in 1881, barely a year after the minister's wife.

We have thus in 26 years seven cases associated with two small houses, the servant to the occupants of one living in the other. It is clear that neither the size of these houses, the blood relationship of their occupants, nor any similarity of their occupations will account for these multiple cancer cases.

(C.) S—— R——. In this house the servant died in 1862, aged 69, of cancer of the breast. The second case was that of a bricklayer who died in 1885, aged 73, of carcinoma of first part of sigmoid flexure of colon. The third case was that of a woman, whose husband had left her, who died here aged 55, in 1906, of secondary carcinoma.

The adjacent house to this was also a multiple cancer house, being credited with two cases, one of carcinoma of the stomach and one of cancer of the womb.

(D.) C—— C——. The first case is that of a man aged 56, who died in 1869 of cancer of the breast. His brother had also died, but not in this house, of cancer of the rectum. On his death he was succeeded in his official position by his clerk, and the latter also retained the same housekeeper. She died in 1885, aged 57, of cancer of the stomach, and the clerk just mentioned died in the same house in 1894, aged 52, of epithelioma of the tongue.

(E.) I—— B——, F—— M——. The first case in this house was that of a pattern maker who died of cancer of rectum in 1875, aged 53. His father had died of cancer before him. The second case was that of his widow who died in 1885, aged 69, of cancer of the rectum. The third case was that of their daughter in 1888, aged 39, after three years' suffering from cancer of the left breast.

This case may show an hereditary constitutional tendency, but this does not account for *both* husband and wife dying of it. That must either be looked upon as fortuitous, or else it is evidence in favour of (*d*).

(F.) G—— B——, T—— O——. The first case was that of a woman occupant who died in 1877, aged 52, of carcinoma of the large intestine. The second case was that of a man of superior social class who died in 1895, aged 73, of carcinoma of the stomach. The third case was that of the mother-in-law of the second case, who lived in the house with her daughter; she died in 1902, aged 84, of carcinoma of the liver*.

(G.) M—— W——, B—— B——. There is a group of five cases here, all in very close proximity. It is not clear how far they are in separate parts of the same original building, but, I think, we may assume three to have been. The first case is one of cancer of the breast in a woman aged 44, in 1862; the second of malignant disease of the stomach in the wife of a nail-maker, she died aged 77 in 1894, and the last one in a woman, who had suffered from epithelioma of hip in 1887, but died of bronchitis. The other two cases were those of a woman who died in 1897, aged 62, of malignant disease of the intestines, and of a blacking-maker, who died, aged 52, of cancer not further defined, in 1865. Undoubtedly this group (G.) is less definite than the others, because the information is less easy to interpret. It did not seem desirable to omit it, for we have clearly a considerable number of cases within a very circumscribed area, but the exact division of the tenements is not clear. If it be omitted, it will not substantially alter the extreme improbability of the general distribution.

From the above specification of the individual "cancer houses," it seems improbable that occupation or heredity constitution had anything to do with the multiple cases; in only one instance do we find parents and offspring; and sex and occupations are most varied. Nor do we believe that the size of house can be concerned in the matter, as it is quite clear that we are dealing with small tenements. The cancer maps of the district show roughly a fairly uniform distribution of the cancer cases, in a district which is largely industrial. But the fact remains that the frequency of multiple case-houses is exceedingly improbable on the basis of a random

* A case of cancer of the breast occurred in the next house in 1876, a woman aged 74. Dr Law Webb writes of the three-case house: "Water, *pump*, often flooded with surface water, when stormy weather; soon dry in summer, drains in close proximity."

distribution of cases. Dr Law Webb's data provide sufficient evidence to justify a demand for a thorough investigation of the subject, such as is not feasible in the case of the individual medical man. They do not finally demonstrate that cancer is more frequent in one house than a second, but they do justify a complete inquiry into the possibility that "cancer-houses" are not wholly a myth, in other words, that immediate environment is in the long run a factor of the frequency of cancer. What is needed is a record of the houses in which cancer has occurred, say for the last 50 or 60 years in (i) a practically fully developed urban district, (ii) a completely agricultural district, and (iii) an industrial area such as occurs frequently in Lancashire or Yorkshire, etc., with relatively small factories, mines or works spread out over a rural district. The examination of the certificates of death of such districts, the careful preparation of "spot maps," and the record of occupations and relationships ought to be a perfectly straight-forward matter, and if it be carried out,—whether it justifies the inference to be drawn from the present data, or does not,—I think Dr Law Webb must be considered as a pioneer in the inquiry.

Experiment. It occurred to me on reading this paper through after completion, that the improbability of multiple cases as measured by the P derived from χ^2, however much it may appeal to the statistician, might not sufficiently impress the medical mind as demonstrating the non-random character of this cancer distribution. Above all a medical man thinking of cancer as a common disease might fail to appreciate, on reading the individual details of the multiple cases, their extreme improbability taken as a whole.

Accordingly, at the suggestion of my colleague, Dr David Heron, I arranged for the drawing at random of 377 cases out of 2000 possibilities. What we want is something equivalent to drawing 377 times at random a ball out of 2000 balls numbered 1 to 2000 in a bag, each ball being replaced after drawing.

The experiment was arranged in the following manner. A series of numbers of four figures having no exact square root, cube root or reciprocal was taken, and the figures in the seventh decimal place of the first and second and in the tenth decimal place of the third were written down. These formed the last three figures of the numbers. To obtain the first figure in the number, in one case the last figure of the cube, i.e. the twelfth was taken, and in three other cases, the tenth figure of the cube. The whole series of 377 numbers were thus taken directly from tables of cubes, square roots, cube roots and reciprocals (Barlow's). Out of the numbers thus obtained, those beginning with 0, 1, 2, 3, 4 were reckoned as belonging to the first thousand, and those beginning with 5, 6, 7, 8 and 9 to the second thousand; 0000 counted however as 2000. Thus: 3004=0004=4, but 5004=1004. We thus had equal chances for every number from 1 to 2000, provided there be no bias in taking numbers consecutively * out of such tables.

The results were as follows:

	Theory	1st Experiment	2nd Experiment	3rd Experiment	4th Experiment
p_1 ...	313	315	321	333	321
p_2 ...	29	31	28	22	28
p_3 ...	2	0	0	0	0
p_4 ...	0	0	0	0	0

The absence of triplets led me to suppose some bias in the tables tending in favour of more uniform distribution than a mere random drawing provides. Accordingly a fifth experiment

* Subject to the omission of numbers with perfect square or cube roots etc., as stated above.

was made in the more lengthy manner of drawing 1508 times a card out of a pack containing 20 cards, numbered in duplicate 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The card was returned, and the pack shuffled and the 1508 draws recorded. Each successive four gave a number which was treated exactly as in the previous four experiments. The results were:

	Theory	Experiment	Cancer
p_1 ...	313	316	315
p_2 ...	29	26	20
p_3 ...	2	3	6
p_4 ...	0	0	1
χ^2	—	approx. 6	approx. 258

It will be seen that the χ^2 of experiment is essentially within the bounds of probability ($P=\cdot 2$, say), while the cancer-data give a value of χ^2 which is 43 times as large. For the previous set of experiments the χ^2 for the third is about 25, and for the other three about 18—larger values than occur in the shuffling experiment—but of a quite different order to the cancer value. Many arguments—all houses not being of same age, some houses pulled down, and so forth—might be used to account for the multiple cancer cases, but I think these data certainly justify a fuller inquiry into the whole question. They provide some evidence, of more value than mere impression, that the hypothesis of "cancer" houses is worthy of a fuller consideration.

V. Hybridisation of Canaries.

Note on the Communication by C. L. W. NOORDUYN *to the Members of the Genetics Congress held in Paris from September* 18—23, 1911.

BY A. RUDOLF GALLOWAY, M.B., C.M., M.A.

At the Genetics Congress held at Paris in September, 1911, Mr C. L. W. Noorduyn, of Groningen, quoting *Biometrika*, Vol. VII., Nos. 1 and 2, gave from my paper on "Canary breeding" the "Table of Canary Hybrids bred since 1891, arranged to show Plumage Colour."

This table indicates the rarity of lightly variegated and "clear" hybrids, and Mr Noorduyn regretted that I did not specify the "five or six hybrids" out of the total of 526 that were not bred from ♂ wild bird × ♀ canary, but from the reverse mating of ♂ canary × ♀ wild bird.

He believed the proportion of lightly variegated hybrids to be greater from ♂ canary × ♀ wild bird than from the reciprocal mating.

In view of the prominence so kindly given to this table by Mr Noorduyn, it is necessary for me to say that the information desired is really included in the paper, but refers to five or six exceptional *matings*, not to five or six individual birds as stated by Mr Noorduyn.

The exceptional matings will be found at the following pages of my paper:

Mating	(1),	page 36, I.	1904.	1.
„	(2),	„ 39, IV 3.	1905.	
„	(3),	„ 39, IV 4.	1904–07.	
„	(4),	„ 39, IV 4.	1906.	
„	(5),	„ 39, IV 4.	1908.	1.
„	(6),	„ 39, IV 4.	1908.	2.

Mr Noorduyn will also find the plumage-colour of the canaries and, in all known cases, of their parents mentioned—information the absence of which he also regretted. The eye-colour of the canaries is also mentioned in every case, and of their parents in every case except one which was unknown.

The matings cited above were for the purpose of testing the inheritance of "pink" as against "dark" eye, a difference which can be seen as soon as the egg hatches. A large number of chicks die before the plumage appears, so that only those that survived and became fully fledged birds are mentioned in the table of hybrids quoted at the Genetics Congress by Mr Noorduyn. Of these none showed any appreciable amount of variegation, two only being *slightly* variegated.

As a matter of fact all my light hybrids, including the "clear" siskin-canary, figured in my paper have been bred from ♂ wild bird × ♀ canary.

Mr Noorduyn also reported at the Genetics Congress the breeding of a cinnamon hybrid by me, and evidently considers it to be a "reversion" to the cinnamon grandmother. With this idea, he bred canary hens from cinnamon × yellow parents, supposing that these hens, on being mated to wild birds would produce "clear" hybrids. In this he was bitterly disappointed, for he bred 27 hybrids all quite dark.

This experience is fully in accordance with the results given in my table of canary hybrids. I am afraid the production of clear hybrids is not quite such a simple matter as a reversion of this sort would explain. Otherwise clear hybrids would not be the great and valuable rarities that they are at the present day, after the labours of generations of the keenest light hybrid breeders. Indeed Mr Noorduyn's belief, that the mating of ♂ yellow canary to ♀ goldfinch results in most of the hybrids being variegated or lightly variegated, is an old one, as may be seen by reference to the earliest—1709—edition of Hervieux's "Nouveau Traité des Serins de Canarie," p. 241:—

"On met ordinairement la femelle de Serin, et le mâle Chardonnet, Linote, ou autres; mais pour moy je suis pour qu'on mette le contraire; sçavoir, le mâle Serin, et la femelle Chardonnet, Linote, ou autres, parce que le mâle ordinairement race plus que la femelle; ainsi les mulets qui sortiront d'un mâle Serin seront plus beaux, et chanteront mieux, que s'ils sortoient seulement de la femelle."

Also at page 246 we read:—

"Vous voyez par cette maniere, que vous avez de beaux mulets à esperer; car le mâle Serin, qui race pour l'ordinaire plus que la femelle, donera beaucoup de Blancs, et la femelle Chardonn donant un peu de ces differentes couleurs feront des mulets d'un prix inestimable."

VI. An attempt to ascertain the Prevalence of Syphilis in a large Urban Population.

Notice of FRITZ LENZ: Über die Verbreitung der Lues, speziell in Berlin, und ihre Bedeutung als Faktor des Rassentodes. *Archiv für Rassen- und Gesellschafts-Biologie.* May and June, 1910. Leipzig. B. G. Teubner. pp. 306 *et seq.*

The underlying ideas of the memoir are to use (1) the relative statistics in two places, and (2) the death rates from certain causes in order to estimate the number of persons attacked by the diseases which end in death from those causes. These ideas are excellent, and ultimately many valuable results may be reached, but they are extremely difficult to apply without making assumptions so wide that the conclusions become too rough to afford definite information. The particular case dealt with by Lenz is that of syphilis, which results in many cases in general paralysis of the insane, locomotor ataxia, etc., and the statistical problem involved in his work may be set out as follows: Given that syphilis is notified in Copenhagen, and that the deaths from general paralysis in Copenhagen and Berlin are known, find the proportion of males in Berlin who have at one time or another had syphilis. To solve such a problem one requires to know the age incidence of the deaths from paralysis, the age incidence of the syphilis notifications, the total populations in age groups (and the births) in both cities for several years, and some information as to the average after lifetime of syphilitics. Lenz neglects these preliminaries and boldly takes a short cut which assumes that if syphilis were notified in Berlin the notifications would bear the same proportion to the deaths from paralysis as the notifications in Copenhagen bear to the deaths from paralysis there. The syphilitic population is found by multiplying this number of notifications by the expectation of life at age 15. We have cut down Lenz's problem and have merely tried to indicate his method; he adjusts some details on the way, but the errors in the method we have just indicated exist, we think, in his work, though at times they are obscured. The weakness is that the proportionate method will not hold because the populations vary and the age incidence in the two cities can hardly be the same, while the use of the expectation at age 15 is incorrect, because this would be the youngest age at attack, and if expectation is used at all it should be for the average age of attack. Besides this the expectation of life of a syphilitic is probably less than that of the population as a whole.

These criticisms appear to us to dispose of his applications, but although the problem is an actuarial one of great difficulty it is certainly worth examination, and even though we do not agree with all his work we feel that much credit is due to Lenz for calling attention to the possibility of solving the problem of the extent of syphilis in this manner.

W. P. E.

VII. On the General Theory of the Influence of Selection on Correlation and Variation.

By KARL PEARSON, F.R.S.

(1) In 1901 a paper of mine was read before the Royal Society and shortly afterwards issued in the *Philosophical Transactions** dealing with this matter. Very shortly afterwards I found out that the formulae therein developed did not depend for their accuracy on the frequencies being Gaussian in character. All the main conclusions were deducible without this limitation,

* Vol. 200 A, pp. 1—66.

and proofs of the formulae concerned have been given by me for many years in college lectures without making this assumption. I have, however, stated the fact once or twice in print that the formulae are general, and it seems desirable to reproduce my proofs (from lecture notes) at the present time as some doubt seems to have been cast on the generality of the formulae.

(2) I begin with a preliminary proposition, which is fairly familiar. Let $x_1, x_2 \dots x_n$ be n variables, continuous or discrete but quantitatively measurable; $\bar{x}_1, \bar{x}_2 \dots \bar{x}_n$ their means, $\sigma_1, \sigma_2, \dots \sigma_n$ their standard deviations, r_{pq} the correlation coefficient of the pth and qth variables supposed found by the product moment formula. Then: What is the best linear function u of $x_1, x_2 \dots x_n$ such that an $(n+1)$th variable x_{n+1} will have a maximum correlation $r_{ux_{n+1}} = \rho$, say, with u?

Let
$$u = c_1x_1 + c_2x_2 + \dots + c_nx_n$$
$$= S(cx).$$

Clearly
$$\bar{u} = S(c\bar{x}),$$
$$N\rho\sigma_u\sigma_{n+1} = \Sigma(u-\bar{u})(x_{n+1}-\bar{x}_{n+1}) = S\Sigma c_p(x_p-\bar{x}_p)(x_{n+1}-\bar{x}_{n+1})$$
$$= NS\Sigma(c_p\sigma_p\sigma_{n+1}r_{p,n+1})$$
$$\rho = \frac{1}{\sigma_u}S(c_p\sigma_p r_{p,n+1}) \quad \dots\dots\dots\

Substitute also in (i):

$$\rho=\frac{1}{\rho}\sum_1^n (b_p r_{p,n+1}),$$

or:
$$\rho^2=-\sum_1^n\left(\frac{R_{p,n+1}}{R_{n+1,n+1}}\,r_{p,n+1}\right),$$

but:
$$R=r_{1,n+1}R_{1,n+1}+r_{2,n+1}R_{2,n+1}+\ldots+R_{n+1,n+1}.$$

Hence:
$$\rho^2=1-\frac{R}{R_{n+1,n+1}} \qquad \text{.....(iv)},$$

$$1-\rho^2=1+\sum_1^n\frac{(R_{p,n+1}r_{p,n+1})}{R_{n+1,n+1}}=\frac{R}{R_{n+1,n+1}}.$$

Hence we have the following results:

$$\rho=\pm\sqrt{1-\frac{R}{R_{n+1,n+1}}} \qquad \text{.....(v)},$$

$$u=-\frac{\sigma_u}{\rho}\left\{\sum_1^n\frac{R_{p,n+1}}{R_{n+1,n+1}}\,\frac{x_p}{\sigma_p}\right),$$

and
$$\sigma_{n+1}\sqrt{1-\rho^2}=\sigma_{n+1}\sqrt{\frac{R}{R_{n+1,n+1}}} \qquad \text{.....(vi)},$$

and is the reduced average variability of x_{n+1} for given values of x_1, x_2, ... x_n.

The probable value of x_{n+1} is given by

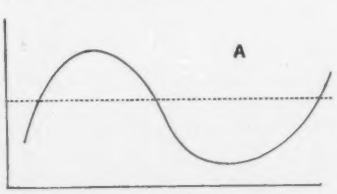
$$x_{n+1}-\bar{x}_{n+1}=\frac{\sigma_{n+1}}{\sigma_u}\,\rho\,(u-\bar{u})$$

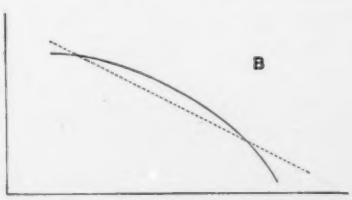
$$=-\sum_1^n\frac{R_{p,n+1}}{R_{n+1,n+1}}\,\frac{\sigma_{n+1}}{\sigma_p}\,(x_p-\bar{x}_p) \qquad \text{.....(vii)},$$

i.e. the ordinary multiple regression formula. It is the "best value," i.e. the mean value of x_{n+1}, for given $x_1 \ldots x_n$, on the assumption that we correlate x_{n+1} with that *linear* function of the n variables, which gives the highest degree of relationship as measured by the correlation coefficient. The method is absolutely independent (i) of Gaussian theory, (ii) of the continuity or discreteness of the variables, but it does assume that linearity applies within the degree of useful approximation*.

Another point deserves re-emphasising here. Equation (iv) gives ρ^2, hence whether ρ be plus or minus, the errors of random sampling will always give a positive ρ^2. It follows therefore that even if ρ be zero, we should find on making a number of trials in each case a positive value of ρ^2; let the mean value of this be $\bar{\rho}^2$, then unless the actual value of ρ^2 is significant not as compared with zero, but with $\bar{\rho}^2$, no value ought to be laid on the actual value of ρ^2. The

* The general linearity ought to be tested in all such cases. Nothing can be learnt of association by assuming linearity in a case with a regression line (plane, etc.) like *A*, much in a case like *B*. To *A* we must apply multiple correlation-ratios, the theory of which is being developed at the present time and will shortly be published.





probable error of ρ is $\cdot 67449\,(1-\rho^2)/\sqrt{N} = \cdot 67449/\sqrt{N}$ if ρ be really zero; then if $\bar{\rho}$ be the mean value of ρ we should expect ρ to be

$$\bar{\rho} \pm \cdot 67449/\sqrt{N}$$

if ρ be truly zero. In other words we must consider the question of whether the observed ρ is significant compared with this.

I have found the value of $\overline{\delta\rho^2}$, i.e. the mean increment of ρ^2 due to errors of random sampling, but I postpone its consideration in the hope of still further reducing its determinantal expression in the general case.

Let us now apply these results to the general theory of selection. Suppose we have m variates $x_1, x_2, \ldots x_m$, with means $\bar{x}_1, \ldots \bar{x}_m$, standard deviations $\sigma_1, \sigma_2, \ldots \sigma_m$ and correlations given by R the determinant

$$\begin{vmatrix} 1 , & r_{12} , & r_{13} , \ldots & r_{1m} \\ r_{21} , & 1 , & r_{23} , \ldots & r_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} , & r_{m2} , & r_{m3} , \ldots & 1 \end{vmatrix}$$

in the usual way.

Now we may suppose a selection to be made out of this variate complex of a subpopulation $x_1, x_2 \ldots x_n$, $n < m$, given by the means:

$$h_1, h_2, \ldots h_n,$$

the standard deviations

$$s_1, s_2, \ldots s_n$$

and the correlations:

$$\begin{vmatrix} 1 , & \rho_{12} , & \rho_{13} , \ldots & \rho_{1n} \\ \rho_{21} , & 1 , & \rho_{23} , \ldots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} , & \rho_{n2} , & \rho_{n3} , \ldots & 1 \end{vmatrix},$$

the selected population having values consistent with those of the unselected population.

We can then ask:

(i) How will this modify the mean and standard deviation of a non-selected variate x_p, $p > n <= m$?

(ii) How will this modify the correlation r_{pq} between two non-selected variates x_p and x_q, p and $q > n$ and $< = m$?

(iii) How will this modify the correlation r_{pt} of a non-selected and a selected variate $p > n$ and $< = m$, while $t < = n$? These are the fundamental problems of the influence of selection on variation and correlation.

(i) Let us take x_{n+1} as the non-selected organ and let the characters of one of the selected group be given by $x_t = \bar{h}_t + \xi_t$.

Then x_{n+1} will differ from its probable mean value by some quantity η_{n+1} and by (vii) we have

$$x_{n+1} = \bar{x}_{n+1} + \eta_{n+1} - \sum_1^n \left\{ \frac{R_{t,n+1}}{R_{n+1,n+1}} \frac{\sigma_{n+1}}{\sigma_t} (\bar{h}_t + \xi_t - \bar{x}_p) \right\}.$$

Or taking the mean value, $\tilde{x}_{n+1}$, $S(\eta_{n+1}) = 0$ and $S(\xi_t) = 0$, and

$$\tilde{x}_{n+1} = \bar{x}_{n+1} - \sum_1^n \left\{ \frac{R_{t,n+1}}{R_{n+1,n+1}} \frac{\sigma_{n+1}}{\sigma_t} (\bar{h}_t - \bar{x}_p) \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(viii)}.$$

This establishes the first proposition* of my *Phil. Trans.* memoir, namely: that selection about the means with any variabilities gives the same mean value for a non-selected but correlated variate as if all the selected variates had been taken at their mean selected values.

We have clearly:

$$x_{n+1}-\bar{x}_{n+1}=\eta_{n+1}-\sum_1^n\left(\frac{R_{t,n+1}}{R_{n+1,n+1}}\frac{\sigma_{t+1}}{\sigma_t}\,\xi_t\right) \quad \text{.....(ix).}$$

Now if we are dealing with N manifolds of variates:

$S(\eta^2_{n+1})/N$ = a standard deviation indicated by $\bar{\sigma}_{n+1}$,

$S(\xi_t^2)/N$ = standard deviation of selected tth organ $=s_t^2$,

$S(\xi_t\,\xi_{t'})/N = s_t s_{t'}\,\rho_{tt'}$,

$S\{\eta_{n+1}\,\xi_t\}=0$ because the tth variate is not selected in reference to the $(n+1)$th variate. Hence if we square (ix) and call Σ_{n+1} the resulting variability of x_{n+1} due to the selection of the n-variates, we have

$$\Sigma^2_{n+1}=\sigma^2_{n+1}\left\{\frac{\bar{\sigma}^2_{n+1}}{\sigma^2_{n+1}}+\sum_1^n\left(\frac{R^2_{t,n+1}}{R^2_{n+1,n+1}}\frac{s_t^2}{\sigma_t^2}\right)+2S\left(\frac{R_{t,n+1}\,R_{t',n+1}}{R^2_{n+1,n+1}}\frac{s_t s_{t'}}{\sigma_t^2}\,\rho_{tt'}\right)\right\}.$$

But as we have already seen η_{n+1} is not correlated with ξ_t. Hence we shall find the value of $\bar{\sigma}^2_{n+1}$ by putting all the s_t's zero, or by concentrating the selection at a single value of a manifold. It is therefore the value of Σ^2_{n+1} for an array of x_{n+1} for definite values of $x_1, x_2 \ldots x_n$, i.e. by (vi) $\bar{\sigma}_{n+1}$ equals $\sigma_{n+1}\sqrt{\dfrac{R_{(n+1)}}{R_{n+1,n+1}}}$, where $R_{(n+1)}$ is the determinant of $n+1$ rows and columns. Thus finally:

$$\Sigma^2_{n+1}=\sigma^2_{n+1}\left\{\frac{R_{(n+1)}}{R_{n+1,n+1}}+\sum_1^n\left(\frac{R^2_{t,n+1}}{R^2_{n+1,n+1}}\frac{s_t^2}{\sigma_t^2}\right)+2S\left(\frac{R_{t,n+1}\,R_{t',n+1}}{R^2_{n+1,n+1}}\frac{s_t s_{t'}}{\sigma_t^2}\,\rho_{tt'}\right)\right\} \quad \text{.....(x).}$$

This is in complete agreement with the value given as Equation (xlv) of my *Phil. Trans.* memoir†, and deduced there on the assumption of a Gaussian frequency distribution.

(ii) I now turn to the second of my problems the correlation between the $(n+1)$th and $(n+2)$th variables. In this work $R'_{n+2,n+2}$ denotes the determinant of n rows and columns bordered by the $(n+2)$th variate correlations, those of the $(n+1)$th being omitted. Clearly as in (ix)

$$x_{n+2}-\bar{x}_{n+2}=\eta_{n+2}-\sum_1^n\left(\frac{R'_{t,n+2}}{R'_{n+2,n+2}}\frac{\sigma_{n+2}}{\sigma_t}\,\xi_t\right) \quad \text{.....(xi).}$$

Multiply (ix) and (xi) sum and divide by the number of the manifolds, N; then if $\rho_{n+1,n+2}$ be the correlation after selection of the $(n+1)$th and $(n+2)$th variates, we have:

$$\Sigma_{n+1}\Sigma_{n+2}\rho_{n+1,n+2}=\frac{S(\eta_{n+1}\eta_{n+2})}{N}+\sigma_{n+1}\sigma_{n+2}\left\{\sum_1^n\frac{R_{t,n+1}\,R'_{t,n+2}}{R_{n+1,n+1},\,R'_{n+2,n+2}}\frac{s_t^2}{\sigma_t^2}\right.$$

$$\left.+S\left(\frac{R_{t,n+1}}{R_{n+1,n+1}}\frac{R'_{t'n+2}}{R'_{n+2,n+2}}\frac{s_t s_{t'}}{\sigma_t\sigma_{t'}}\rho_{tt'}\right)+S\left(\frac{R_{t',n+1}}{R_{n+1,n+1}}\frac{R'_{t,n+2}}{R'_{n+2,n+2}}\frac{s_t s_{t'}}{\sigma_t\sigma_{t'}}\rho_{tt'}\right)\right\} \quad \text{.....(xii).}$$

As before $\dfrac{S(\eta_{n+1}\eta_{n+2})}{N}$ will be given by the mean partial product moment of the $(n+1)$th and $(n+2)$th variates for *constant* values of the n variates concentrated at their selected means. This can be found without appeal to the Gaussian frequency surface by extending the formula (vii) to $n+1$ variates.

* Vol. 200, A, p. 13.

† *Phil. Trans.* Vol. 200, A, p. 17.

Let Δ be the determinant of $(n+2)$ rows and columns, Δ_{pq} the minor corresponding to the pth column and qth row component. Then the regression equations for x_{n+1} and x_{n+2} on the remaining variates of the $(n+2)$ group are:

$$x_{n+1}-\bar{x}_{n+1}=-\frac{\Delta_{n+2,n+1}}{\Delta_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_{n+2}}(x_{n+2}-\bar{x}_{n+2})-\sum_1^n\left\{\frac{\Delta_{t,n+1}}{\Delta_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_t}(x_t-\bar{x}_t)\right\}$$

and

$$x_{n+2}-\bar{x}_{n+2}=-\frac{\Delta_{n+1,n+2}}{\Delta_{n+2,n+2}}\frac{\sigma_{n+2}}{\sigma_{n+1}}(x_{n+1}-\bar{x}_{n+1})-\sum_1^n\left\{\frac{\Delta_{t,n+2}}{\Delta_{n+2,n+2}}\frac{\sigma_{n+2}}{\sigma_t}(x_t-\bar{x}_t)\right\}.$$

Now, when we put $x_1 \ldots x_n$ constant, the coefficients of $x_{n+2}-\bar{x}_{n+2}$ and $x_{n+1}-\bar{x}_{n+1}$ and the partial regression coefficients of x_{n+1} on x_{n+2} and x_{n+2} on x_{n+1} for constant 1 to n variates, or the square root of their product is the partial correlation coefficient, i.e.

$$_{1,2,3\ldots n}\rho_{n+1,n+2}=\bar{\rho}_{n+1,n+2},$$

say for brevity; therefore

$$\bar{\rho}_{n+1,n+2}=\sqrt{\frac{\Delta^2_{n+1,n+2}}{\Delta_{n+1,n+1}\Delta_{n+2,n+2}}}=-\frac{\Delta_{n+1,n+2}}{\sqrt{\Delta_{n+1,n+1}\,\Delta_{n+2,n+2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xiv)},$$

a well-known and familiar form*.

Now let us look at the standard deviations of the arrays of the $(n+1)$th and $(n+2)$th variates for absolutely selected values of the n variates.

The variability of the array of the $(n+1)$ variate is given by (vi), i.e.

$$\bar{\sigma}_{n+1}=\sigma_{n+1}\sqrt{\frac{R}{R_{n+1,n+1}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xv)},$$

and of the $\overline{n+2}$th variate

$$\bar{\sigma}_{n+2}=\sigma_{n+2}\sqrt{\frac{R'}{R'_{n+2,n+2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xvi)}.$$

But

$$R=\Delta_{n+2,n+2},$$

$$R'=\Delta_{n+1,n+1}$$

while clearly $R_{n+1,n+1}=R'_{n+2,n+2}=$ the second minor of Δ obtained by leaving out both $(n+1)$th and $(n+2)$th rows and columns. Hence we have:

$$R_{n+1,n+1}=R'_{n+2,n+2}=\Delta_{n+1,n+1,n+2,n+2},$$

and

$$\frac{\Sigma(\eta_{n+1}\eta_{n+2})}{N}=\bar{\sigma}_{n+1}\bar{\sigma}_{n+2}\bar{\rho}_{n+1,n+2}=-\sigma_{n+1}\sigma_{n+2}\frac{\Delta_{n+1,n+2}}{\Delta_{n+1,n+1,n+2,n+2}} \quad \ldots\ldots\ldots\text{(xvii)}.$$

Thus finally we have from (xii):

$$\Sigma_{n+1}\Sigma_{n+2}\rho_{n+1,n+2}=\sigma_{n+1}\sigma_{n+2}\left\{\frac{-\Delta_{n+1,n+2}}{\Delta_{n+1,n+1,n+2,n+2}}+\sum_1^n\left(\frac{R_{t,n+1}R'_{t,n+2}}{R_{n+1,n+1}R'_{n+2,n+2}}\frac{s_t^2}{\sigma_t^2}\right)\right.$$
$$\left.+S\left(\frac{R_{t,n+1}R'_{t',n+2}}{R_{n+1,n+1}R'_{n+2,n+2}}+\frac{R_{t',n+1}R'_{t,n+2}}{R_{n+1,n+1}R'_{n+2,n+2}}\right)\frac{s_ts_{t'}}{\sigma_t\sigma_{t'}}\rho_{tt'}\right\} \quad \ldots\ldots\ldots\ldots\text{(xviii)},$$

which is in complete agreement with the value found from the Gaussian hypothesis†.

(iii) Lastly we require the correlation $\rho_{t,n+1}$ between a selected and a non-selected variate, $t<=n$. Turning back to (ix) multiply by ξ_t, sum and divide by N, then:

$$s_t\Sigma_{n+1}\rho_{t,n+1}=\frac{S(\eta_{n+1}\xi_t)}{N}-\left(\frac{R_{t,n+1}}{R_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_t}s_t\right)-S\left(\frac{R_{t',n+1}}{R_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_{t'}}s_ts_{t'}\rho_{tt'}\right).$$

* Pearson, *Phil. Trans.* Vol. 200, A, p. 10, Equation (xxvii).

† See *Phil. Trans.* Vol. 200, A, p. 17, Equation (xlvi).

The first summation on the right is zero; hence

$$\Sigma_{n+1}\rho_{t,n+1} = -\sigma_{n+1}\left\{\frac{R_{t,n+1}}{R_{n+1,n+1}}\frac{s_t}{\sigma_t} + S\left(\frac{R_{t'n+1}}{R_{n+1,n+1}}\right)\frac{s_{t'}}{\sigma_{t'}}\rho_{tt'}\right\} \quad \text{.....(xix).}$$

This with a slight difference of notation is the result obtained on the Gaussian hypothesis*.

The above proofs justify the statement that the general selection formulae given by me are independent of any Gaussian assumption. They are really peculiar to the general idea of the manifold linear variate u which gives the maximum correlation coefficient of an $(n+1)$th variate with n other variates. They do not involve any idea of continuity or any hypothesis as to the nature of the selected means, standard deviations and correlations beyond the fundamental assumption that the selected population really exists inside the unselected population. There need be no hesitation therefore in applying these formulae to any cases whatever in which the correlation coefficients have valid application at all.

* *Phil. Trans.* Vol. 200, A, p. 17, Equation (xlvii). S in our present notation is a summation in (xix) of every value, but t, of t'. In the *Phil. Trans.* paper S_1 is a summation for *all* values of t': see p. 18.

VIII. On a Fallacious Proof of Sheppard's Correction.

The ordinary proofs of Sheppard's corrections for the moments are somewhat lengthy and depend entirely on the principle of high contact at the terminals. Mr G. U. Yule in his recent *Theory of Statistics*, p. 208, has given a proof in a few lines which is absolutely independent of this principle, and which from its very simplicity is likely, if not criticised, to be generally adopted. Unfortunately it is wholly fallacious. The error lies in the words "the correlation between X and δ is zero, for the mean value of δ is zero for every interval." What Mr Yule should have said is that the correlation between his Z and δ is zero, and he should have reached the conclusion

$$\sigma_1^2 = \sigma^2 + \tfrac{1}{12}c^2,$$

and not

$$\sigma_1^2 = \sigma^2 - \tfrac{1}{12}c^2,$$

for he is really working out the mean square for the histogram and not the true figure. He would thus have failed to obtain the correct value, which he does not appear to recognise arises solely from the fact that the 'trapezettes' *cannot* be treated as rectangles. In the case of curves of frequency without terminal contact, Sheppard's corrections are *not* the proper ones, and their general adoption without regard to their limitations is to be deprecated. Such adoption is directly encouraged by a fallacious proof of the above character. K. P.

GROWTH, VARIABILITY AND CORRELATION IN YOUNG TROUT.

By J. W. JENKINSON, M.A., D.Sc.

The growth of the organism has been made the subject of numerous quantitative investigations, and amongst those who have dealt with the growth of animals, including the human being, the names of Quetelet, Roberts, Bowditch, Minot and Boas are well known.

From their investigations it appears that there are certain features which are characteristic of growth in general.

The first of these is the fact that the growth-rate is not uniform but suffers a gradual diminution during the life of the organism. This is well established by many researches [Minot (Rabbit), Quetelet, Bowditch, Roberts, Boas (the human being), Potts and Minot (Chickens), Semper (*Limnaea*), Vernon (*Strongylocentrotus*)]. This is also known to be true of the various parts separately, though in the parts the rate of growth and the change of that rate differ a good deal. Moreover there may be more than one period of rapid growth followed by first a rapid, then a slow decline in the growth-rate, during the lifetime of one and the same organism. In the human being for example there is one such period or cycle before birth, another cycle during the first few years after birth, while the growth-rate rises once more, only once more to diminish, about the time of puberty. Robertson has recently suggested that, on the assumption that some autocatalytic chemical process underlies the phenomenon of growth, the maximum growth-rate in each such cycle should occur when the total growth occurring in that cycle is half accomplished. It may be added that results calculated on this theory are in fairly close agreement with those obtained by actual measurement.

A second feature which characterises growth is the relation between change in growth-rate and change in the variability of the organism or of its parts, the latter rising and falling with the former. While it is true that the observed decrease of variability has been in some cases ascribed to a selective death-rate, yet Boas has urged that the relation referred to must necessarily exist, however much it may be obscured by changes in variability due to other causes.

And thirdly Boas has urged and brought forward evidence to support his contention that the value of the coefficient of correlation between various organs diminishes with the diminution of the growth-rate. The data are taken from human growth and it appears that the correlation coefficient, like the variability, falls after birth, but rises again at about the time of puberty, thereafter decreasing once more.

All these points are of so much general interest and importance that it appeared to me to be worth while to endeavour to secure further data for an enquiry into the same three problems, and the animal that soon suggested itself for the purpose was the trout.

The trout is eminently suitable since a large number of parts can be readily and fairly accurately measured. Moreover there is fortunately, at Bibury in the Cotswolds, not more than thirty miles from Oxford, an excellent hatchery owned and superintended by Mr A. Severn. Here the young trout live under very favourable conditions. The temperature, a most important matter of course, is practically constant, for the hatchery is supplied directly from a spring that rises on the spot, and the temperature of this spring does not vary more than a degree the whole year round. The animals are kept free from disease, are abundantly supplied with food and their death-rate is very low; 75°/₀, often more, come to maturity; 80°/₀ of the eggs of the American Rainbow trout hatch out, 90°/₀ of those of the English brown trout. While of those Rainbow trout that hatch out less than 10°/₀ as a rule die within three months after hatching. There is however far more loss at this period amongst the English fish.

I accordingly requested Mr Severn to be good enough to supply me with some young trout at different stages after hatching, and he has very kindly provided me with the necessary material for my work. I should like to take this opportunity of thanking him for the trouble he has taken in the matter. I have used the American Rainbow trout. All the young whose measurements are given in this paper were the offspring of one pair of parents. The fish were spawned and the eggs fertilized on February 14th, 1910. In the hatchery they were kept in a separate box. The young alevins, with the yolk-sac still large, hatched out about the 19th of March; all did not hatch on the same day. The young were sent in to Oxford as follows:

I.	April 4th . .	200
	(These still had the yolk-sac.)	
II.	April 18th . .	209
III.	May 2nd . .	198
IV.	May 17th . .	200
V.	May 31st . .	161

Immediately on their arrival the fish were killed in formol.

I have thus had at my disposal nearly one thousand young trout, all born of the same parents, and killed at five successive fortnightly stages after hatching, the period of growth examined thus comprising ten weeks. Mr Severn has been kind enough to furnish me with the following particulars of the mortality that occurred in this particular lot. Five per cent. died after hatching during the first fortnight, that is before any were sent to me. When the fish were about three weeks old (that is, between Stages I and II) a trifling number died of dropsy and at the same time about 5% died which were deformed. Otherwise there was scarcely any loss at all.

The dead individuals were not sent to me and I only measured the survivors. Whether the death-rate is high enough of itself to bring about the diminution of variability which, as we shall see, actually occurs, is a question to be discussed later. But whether that is so or not I think it may be in any case worth while to describe briefly the growth and the change that occurs in the growth-rate, and in the variability both of the whole body and of different parts, both absolute and relative, as well as the correlations that exist between these parts.

I have measured the following—total length from the tip of the nose to the extremity of the tail, diameter of the eye, length of head as determined by the posterior margin of the operculum, length of the bases (properly speaking the projections of these bases on the longitudinal axis of the body) of the median fins —anterior dorsal, posterior dorsal, ventral and caudal—breadth of the body at the level of the anterior end of the anterior dorsal fin, breadth of the tail, and position of the pectoral, pelvic, anterior dorsal, posterior dorsal and ventral fins as determined by the anterior end of the base of attachment in each case. The ventral fin lies just behind the anus as a rule, so that the position of the former can be taken usually as marking the position of the latter, but there may be a gap between the two. In these cases the posterior border of the anus has been separately measured. Lastly the number of bars or spots in the middle row on the animal's side has been counted. In all cases the measurements were made on the right-hand side of the animal.

I begin with the *absolute measurements*. The mean values are given in millimetres in Table I, and in the same table the growth-rate at each successive stage, as expressed by the percentage increment of growth.

The general increase of all parts needs of course no comment. The only exception is the number of spots which is established at once in Stage II and undergoes practically no increase. What is of interest is the diminution of the growth-rate. This is evident everywhere. Generally speaking the decrease is more rapid in the early stages than in the later—which is in harmony with the results obtained by other observers—but in some cases there is irregularity. In the eye the growth-rate sinks at Stage III, to rise again at Stage IV and sink once more in the final stage. In the posterior dorsal fin there is a marked increase at Stage III, followed by a great decrease, and this by a slight rise. A slight, perhaps hardly significant rise, can be detected in the final stage of the total length.

TABLE I. *Giving in tenths of millimetres mean values (M) of total length and of the absolute dimensions of various parts in the five Stages (I—V), the differences (d) between the successive mean values and these differences expressed as percentages (%) of the mean value at the beginning of each interval. The intervals are all of the same magnitude (14 days) with the exception of that between Stages III and IV which was 15 days. The corresponding percentage differences have been reduced accordingly and the corrected values inserted in brackets.*

	Total Length			Diameter of Eye			Length of Head			Length of Anterior Dorsal Fin			Length of Ventral Fin			Length of Posterior Dorsal Fin		
	M	*d*	%	*M*	*d*	%	*M*	*d*	%	*M*	*d*	%	*M*	*d*	%	*M*	*d*	%
I	184·03 ± 1·15	—	—	14·58 ± ·12	—	—	36·05 ± ·34	—	—	22·04 ± ·24	—	—	17·32 ± ·28	—	—	14·27 ± ·31	—	—
II	238·75 ± ·85	54·72	29·73	18·95 ± ·06	4·37	29·97	53 38 ± ·23	17·33	38·07	32·72 ± ·25	10·68	48·46	25·70 ± ·31	8·38	48·38	15·50 ± ·18	1 23	8·62
III	284·65 ± ·70	45·90	19·23	21·51 ± ·06	2·56	13·51	68·14 ± ·21	14·76	27·65	*44·84 ± ·34	12·12	37·04	*34·10 ± ·32	8·40	32·68	19·88 ± ·14	4·38	28·26
IV	319·23 ± ·77	34·58	12·15 (11·34)	26·21 ± ·05	4·70	21·85 (20·39)	79·25 ± ·20	11·11	16·30 (15·21)	*42·45 ± ·43	− 2·39	− 5·33	*32·51 ± ·37	− 1·59	− 4·66	22·06 ± ·16	2·18	10·97 (10·24)
V	358 00 ± 1·47	38·77	12·14	28·08 ± ·08	1·87	7·13	87·34 ± ·30	8·09	10·21	33·14 ± ·13	− 4·31	− 10·15	29·58 ± ·23	− 2·93	− 9·01	24·78 ± ·23	2·72	12·33

	Length of Caudal Fin			Breadth of Caudal Fin			Breadth of Body			Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin	Number of Spots
	M	*d*	%	*M*	*d*	%	*M*	*d*	%	*M*	*M*	*M*	*M*	*M*	*M*	*M*
I	14·52 ± ·27	—	—	21·98 ± ·31	—	—	†	—	—	37·16 ± ·30	95·96 ± ·41	86 42 ± ·44	126·84 ± ·58	126·00 ± ·60	145·01 ± ·60	‡
II	25·16 ± ·30	10·64	73·28	34·31 ± ·43	12·33	56·10	43·08 ± ·16	—	—	52·13 ± ·20	119·54 ± ·33	109·76 ± ·34	158·47 ± ·45	157·76 ± ·47	172·34 ± ·45	9·12 ± ·06
III	37·44 ± ·21	12·28	48·81	48·50 ± ·36	14·19	41·36	51·62 ± ·22	8·54	19·82	62·18 ± ·19	139·60 ± ·33	127·01 ± ·33	183·31 ± ·43	182·62 ± ·43	196·55 ± ·45	9·75 ± ·06
IV	44·56 ± ·19	7·12	19·02 (17·75)	59·90 ± ·36	11·40	23·51 (21·94)	63·03 ± ·23	11·41	22·10 (20·62)	70·02 ± ·19	152·28 ± ·40	144·29 ± ·31	203·30 ± ·49	203·20 ± ·48	220·14 ± ·53	9·53 ± ·05
V	48·85 ± ·38	4·29	9·63	63·25 ± 58	3·35	5·59	67·20 ± ·35	4·17	6·62	76·18 ± ·28	172·85 ± ·64	161·60 ± ·59	231·24 ± ·81	230·49 ± ·85	250·40 ± ·92	9·55 ± ·05

* In Stages III and IV an error was made in many cases in the determination of the posterior end of the anterior dorsal and of the ventral fin bases. The mean values are consequently too large and hence the differences negative. The error was discovered and avoided in measuring Stage V.

† The presence of the yolk sac prevents the accurate measurement of the breadth of the body in Stage I.

‡ The spots do not appear till Stage II.

TABLE II.

Variabilities of Absolute Values.

σ (in tenths of Millimetres).

	Total Length	Eye	Length of Head	Length of Anterior Dorsal Fin	Length of Ventral Fin	Length of Posterior Dorsal Fin	Length of Caudal Fin	Breadth of Caudal Fin	Breadth of Body	Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin	Number of Spots
I	24·13 ± ·81	2·60 ± ·08	7·09 ± ·24	4·90 ± ·17	5·56 ± ·20	4·38 ± ·22	5·73 ± ·19	6·40 ± ·22	—	6·26 ± ·21	8·58 ± ·29	9·21 ± ·31	11·74 ± ·41	12·32 ± ·42	8·54 ± ·42	—
II	18·26 ± ·60	1·32 ± ·04	5·03 ± ·16	5·33 ± ·18	6·58 ± ·22	3·65 ± ·13	6·49 ± ·21	9·25 ± ·30	3·42 ± ·11	4·37 ± ·14	6·98 ± ·23	7·22 ± ·24	9·46 ± ·32	10·07 ± ·33	9·08 ± ·32	1·15 ± ·04
III	14·55 ± ·49	1·21 ± ·04	4·48 ± ·15	*7·10 ± ·24	*6·64 ± ·23	2·95 ± ·10	4·46 ± ·15	7·45 ± ·25	4·56 ± ·16	3·92 ± ·13	6·89 ± ·23	6·80 ± ·23	8·80 ± ·30	9·00 ± ·30	9·26 ± ·32	1 12 ± ·04
IV	16·13 ± 54	1·10 ± ·04	4·09 ± ·14	*8·97 ± ·30	*7·76 ± ·26	3·44 ± ·12	3·88 ± ·13	7·60 ± ·25	4·81 ± ·16	4·01 ± ·13	8·30 ± ·28	6·60 ± ·22	10·03 ± ·35	10·01 ± ·34	11·13 ± ·37	1·03 ± ·04
V	27·70 ± 1·04	1·45 ± ·06	5·72 ± ·21	2·54 ± ·09	4·36 ± ·16	4·18 ± ·16	7·05 ± ·27	11·00 ± ·41	6·55 ± ·25	5·30 ± ·20	12·15 ± ·45	11·19 ± ·42	15 23 ± ·57	16·06 ± ·60	17·02 ± ·65	1·02 ± ·04

C. of V.

I	13·11 ± ·44	17·80 ± ·56	19·68 ± ·69	22·22 ± ·81	32·08 ± 1·27	30·69 ± 1·68	39·44 ± 1·49	29·10 ± 1·08	—	16·86 ± ·59	8·95 ± ·30	10·66 ± ·36	9·25 ± ·32	9·78 ± ·33	5·89 ± ·29	—
II	7·65 ± ·25	6·96 ± ·21	9·43 ± ·30	16·30 ± ·56	25·62 ± ·92	23·52 ± ·89	25·81 ± ·89	26·96 ± ·93	7·94 ± ·26	8·23 ± ·27	5·86 ± ·19	6·58 ± ·22	5·97 ± ·20	6·38 ± ·21	5·27 ± ·19	12·56 ± ·43
III	5·11 ± ·17	5·63 ± ·19	6·57 ± ·22	*15·83 ± ·55	*19·47 ± ·70	14·79 ± ·51	11·96 ± ·40	15·36 ± ·53	8·83 ± ·31	6·30 ± ·21	4·94 ± ·16	5·35 ± ·18	4·80 ± ·16	4·91 ± ·16	4·71 ± ·16	11·51 ± ·41
IV	5·05 ± ·17	4·21 ± ·15	5·16 ± ·18	*21·12 ± ·74	*23·88 ± ·85	15·59 ± ·55	8·71 ± ·29	12·69 ± ·42	7·63 ± ·25	5·73 ± ·19	5·45 ± ·18	4·58 ± ·15	4·93 ± ·17	4·93 ± ·17	5·05 ± ·17	10·87 ± ·42
V	7·74 ± ·29	5·16 ± ·21	6·55 ± ·24	6·65 ± ·23	14·74 ± ·55	16·87 ± ·67	14·43 ± ·56	17·39 ± ·67	9·75 ± ·37	6·96 ± ·26	7·03 ± ·26	6·92 ± ·26	6·58 ± ·25	6·97 ± ·26	6·80 ± ·25	10·68 ± ·42

* For the reasons stated in the note to Table I. these values are untrustworthy.

In the second place the growth-rate differs considerably—both initially and finally—in the various parts. The maximum observed is 73·28 for the second stage of the length of the caudal fin, the minimum 5·59 for the last stage of the breadth of the caudal fin (the negative values obtained for the two last stages of the anterior dorsal, and of the ventral fins, are due to the excessive value given to the means, owing to an error in measurement, in the two previous stages). And lastly, the rate of decline of the growth-rate differs. Thus between Stages II and III there may be a decrease of 10 (total length), 11 (head, anterior dorsal fin, ventral fin), 15 (breadth of caudal fin) or 25 (length of caudal fin). Similarly the decrease in the final stage varies from 5 to 16.

Apart from the irregularities exhibited in the cases alluded to there is no evidence that in the period under observation more than one growth cycle is involved.

We turn next to the *variabilities* of these various dimensions (Table II, in which both standard deviations and coefficients of variability are given). The coefficient of variability (C. V.) may be taken as giving the best measure. Two things are at once evident. First there is a regular decrease during the first four stages (length of anterior dorsal fin and length of ventral fin must be ruled out for the reasons already given). Like the decrease in the growth-rate this decrease of variability is also more rapid at first than later on, at least this is generally so. There are however exceptions. In the posterior dorsal fin (length), caudal fin (length) and caudal fin (breadth) there is nearly as much or even more difference between the variabilities in the third and second stages as there is between the variabilities in the second and first. It is not easy to associate these anomalies with anomalies in the change of the growth-rate of the corresponding parts. It is true that the growth-rate changes irregularly in the posterior dorsal fin: but in the caudal fin length and breadth the difference between the growth-rate from the first to the second stages and that from the second to the third stages is very great. Again, the growth-rate in the eye increases temporarily between Stages III and IV, but there is no corresponding increase in the variability.

Secondly, in respect of all characters except the number of spots there is a slight increase of variability in the final stage, an increase which is not, as a rule, associated with any increase in the growth-rate. This slightly increased variability never however becomes as large as the variability in the first stage and is usually a good deal smaller. The question is whether this general decrease of variability in the *whole* period is to be ascribed in whole or in part to the decrease in the growth-rate, for, since there was a certain mortality, it may be reasonably contended that this was selective and caused the diminished variability.

The 5% that, as stated, died in the first fortnight, need not be considered, but some attempt must be made to make an allowance for the 5% that died between Stages I and II.

In Table III are given the corrected values for the mean, standard deviation and coefficient of variability of the total length in the last four stages on the assumption that this 5°/₀ (say 40 individuals) should have been uniformly spread over these stages. The extra individuals have been placed (α) in the lowest class only of the distribution, and (β) equally in the lowest and the highest class. The

TABLE III.

Total Lengths.

	Values as given in the tables			Corrected Values					
				(α) By the inclusion of 10 extra individuals in the lowest class			(β) By the inclusion of 5 extra individuals in the lowest and 5 in the highest class		
	M	σ	*C. V.*	*M*	σ	*C. V.*	*M*	σ	*C. V.*
I	184·03	24·13	13·11	—	—	—	—	—	—
II	238·75	18·25	7·65	236·22	21·29	9·01	238·16	20·20	8·48
III	284·65	14·55	5·11	282·19	18·92	6·70	284·11	16·88	5·94
IV	319·23	16·13	5·05	316·26	19·39	6·13	319·11	18·88	5·92
V	358·00	27·70	7·74	351·99	36·21	10·29	356·66	33·55	9·41

If all 40 extra individuals are put in the lowest class in Stage II, then

$M = 228{\cdot}36, \quad \sigma = 26{\cdot}44, \quad C.\ V. = 11{\cdot}58.$

variabilities are of course increased, and more by the former than by the latter method. The former perhaps is fairer since Mr Severn says those that died were deformed. But even so, the series shows the same decrease up to Stage IV and a rise in Stage V, and, what is more important, the variability in no case becomes equal to that found in Stage I. Only by including all 40 in the lowest class of Stage II can the variability be made even to approximate to that seen in Stage I. The values are given at the foot of the tables. In this case of course the values for the coefficient of variability in Stages III and IV remain unaltered.

I admit that the method is unsatisfactory, but in the absence of any actual knowledge of the size of those that died, it is the best that can be done. The decrease of variability is then perhaps not wholly due to a selective death-rate.

Is it then to be attributed to decrease in growth-rate? This seems possible, but the absence of any marked increase in growth-rate in the last stage suggests that still some other factor may be concerned; and again there is the apparent absence of any connection between the irregularities in the change of growth-rate and anomalies in the alteration of variability.

I had thought at one time that the increased variability in Stage V might be due to imperfect sampling in the collection of the material, smaller, weaker individuals having been perhaps left at the bottom of the box in earlier stages, and

only gathered in with the last lot. But I have been assured by the keeper of the hatchery, who himself collected them, that he made no such selection; the fish were taken quite at random.

I can therefore offer no explanation. The change is certainly not to be attributed to any alteration in the environment.

We pass now to the *relative dimensions.* These have been expressed as percentages of the total length.

The means of these indices and their standard deviations are given in Table IV.

First we may notice that the proportions of many organs remain constant throughout or nearly so—the eye, the three median fins, body-breadth, position of pectoral and pelvic fins. There is, however, a slight decrease in the proportion (except in body-breadth), which means, in the case of the position of the fins, that they move a little nearer the head, and this is true also of the positions of the median fins and of the anus. The hinder end of the operculum, on the other hand, moves further away from the head end: in Stage I it is in front of the pectoral fin, but subsequently more and more behind it. But these changes in the proportions of the parts are not very serious.

It is only when we look at the length and breadth of the caudal fin that we see how great may be the changes that occur in their index value. The length increases from 4 °/₀ to nearly 14 °/₀ of the total body-length, the breadth from 12 °/₀ to 18 °/₀ (with a subsequent slight decrease). As has already been shown these two dimensions grow very rapidly indeed in the first period, twice or more than twice as rapidly as the whole body, and this excess in the growth-rate is maintained up to the fourth stage. By the time Stage V is reached the growth-rate of these parts is smaller than that of the total length, and their index value has also decreased.

The figures in Table I also show that the head grows faster than the whole body, although in this case the excess is not so great. And again in the first two stages (the figures for the others are unreliable) growth-rate is high in the anterior dorsal and ventral fins (Table I) and the indices are correspondingly increasing.

The variabilities (σ) of the indices are usually small. Generally speaking they exhibit a progressive decrease. So in the eye, head-length, length of median fins, breadth of body, position of paired and median fins, and of anus. In some of these cases (position of pelvic fin, and of median fins and of anus) there is an appreciable rise of variability in the final stage, though the value of the variability in the first stage is never reached. The two exceptions to this regular decrease of variability are seen in the length and breadth of the caudal fin. Here the variability first increases, then decreases only to rise again to more than its original value in passing from the fourth to the fifth stage.

As we know already these two dimensions are anomalous in other respects. The mean index value rises at first rapidly then more slowly, but finally decreases a little; the growth-rate is very high at first but experiences finally a sharp fall.

TABLE IV.

Mean Values (M) of Indices in per cents.

	Eye	Length of Head	Length of Anterior Dorsal Fin	Length of Ventral Fin	Length of Posterior Dorsal Fin	Length of Caudal Fin	Breadth of Caudal Fin	Breadth of Body	Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin
I	7·91 ± ·04	19·40 ± ·08	11·91 ± ·08	9·09 ± ·11	7·01 ± ·14	4·33 ± ·06	11·64 ± ·11	—	20·23 ± ·06	52·22 ± ·19	47·01 ± ·11	68·46 ± ·17	68·29 ± ·16	71·87 ± ·19
II	7·97 ± ·02	22·35 ± ·06	13·63 ± ·08	10·22 ± ·11	6·38 ± ·07	10·00 ± ·10	13·73 ± ·15	17·72 ± ·10	21·81 ± ·06	50·13 ± ·09	46·04 ± ·07	66·36 ± ·12	66·19 ± ·10	71·19 ± ·11
III	7·56 ± ·02	23·93 ± ·05	*15·76 ± ·11	*11·92 ± ·10	6·95 ± ·05	12·68 ± ·06	16·59 ± ·10	18·16 ± ·05	21·88 ± ·05	49·09 ± ·06	44·65 ± ·05	64·38 ± ·07	64·32 ± ·07	68·94 ± ·06
IV	7·43 ± ·02	24·86 ± ·05	*13·28 ± ·13	*10·15 ± ·11	6·93 ± ·05	13·93 ± ·04	18·28 ± ·09	19·39 ± ·06	21·95 ± ·05	47·64 ± ·05	45·27 ± ·05	63·63 ± ·06	63·62 ± ·06	69·07 ± ·04
V	7·85 ± ·03	24·67 ± ·05	10·58 ± ·06	8·19 ± ·05	6·83 ± ·05	13·56 ± ·07	17·01 ± ·13	18·76 ± ·05	21·24 ± ·05	48·18 ± ·07	44·98 ± ·06	64·18 ± ·09	64·19 ± ·09	69·30 ± ·07

Differences between successive Mean Index Values.

II	+ ·06	+ 2·95	+ 1·72	+ 1·13	− ·63	+ 5·67	+ 2·09	—	+ 1·58	− 2·09	− ·97	− 2·10	− 2·10	− ·68
III	− ·41	+ 1·58	—	—	+ ·57	+ 2·68	+ 2·86	+ ·44	+ ·07	− 1·54	− 1·39	− 1·98	− 1·87	− 2·25
IV	− ·13	+ ·93	—	—	− ·02	+ 1·25	+ 1·69	+ 1·23	+ ·07	− 1·45	+ ·62	− ·75	− ·70	+ ·13
V	+ ·42	− ·19	—	—	− ·10	− ·67	− 1·27	− ·63	− ·71	+ ·54	− ·25	+ ·55	+ ·57	+ ·23

Variabilities (σ) of Indices in per cents.

	Eye	Length of Head	Length of Anterior Dorsal Fin	Length of Ventral Fin	Length of Posterior Dorsal Fin	Length of Caudal Fin	Breadth of Caudal Fin	Breadth of Body	Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin
I	·92 ± ·03	1·68 ± ·06	1·77 ± ·06	2·30 ± ·08	1·97 ± ·10	1·15 ± ·04	2·27 ± ·08	—	1·25 ± ·04	3·87 ± ·13	2·32 ± ·08	3·56 ± ·12	3·33 ± ·11	2·68 ± ·13
II	·50 ± ·01	1·27 ± ·04	1·75 ± ·06	2·32 ± ·08	1·34 ± ·05	2·19 ± ·07	3·10 ± ·11	2·13 ± ·07	1·19 ± ·04	1·95 ± ·06	1·54 ± ·05	2·54 ± ·08	2·24 ± ·07	2·15 ± ·08
III	·39 ± ·01	1·11 ± 04	*2·21 ± ·08	*2·16 ± ·07	·95 ± ·04	1·25 ± ·04	2·07 ± ·07	1·07 ± ·04	1·07 ± ·04	1·34 ± ·04	1·11 ± ·04	1·53 ± ·05	1·56 ± ·05	1·20 ± ·04
IV	·43 ± ·01	·98 ± ·04	*2·66 ± ·09	*2·28 ± ·08	·95 ± ·04	·85 ± ·03	1·85 ± ·06	1·15 ± ·04	1·01 ± .04	·96 ± ·04	·98 ± ·04	1·23 ± ·04	1·20 ± ·04	·99 ± ·03
V	·45 ± ·02	·97 ± ·04	1·09 ± ·04	·99 ± ·04	·99 ± ·04	1·36 ± ·05	2·48 ± ·09	·86 ± ·04	1·01 ± ·04	1·23 ± ·05	1·20 ± ·04	1·64 ± ·06	1·62 ± ·06	1·38 ± ·05

* For the reason stated in the note to Table I these values are untrustworthy.

It might be urged therefore that where the part is growing fast relatively to the whole its variability is high, while as its relative growth-rate diminishes its variability decreases too. Thus, between Stages I and II the index value of the caudal fin length passes from 4·33 to 10·00, but between Stages II and III from 10·00 only to 12·68, and between Stages III and IV from 12·68 to 13·93, that is there are successively diminishing increases of 5·67, 2·68, and 1·25, and all this time the variability is also decreasing (2·19, 1·25 and ·85). A similar relation can be seen between rate of change of index value and variability in the breadth of the caudal fin between Stages III and IV.

That the growth-rate is not however the only factor concerned is seen from the fact that in the fifth stage of these two dimensions (length and breadth of caudal fin) and between Stages II and III of the latter, a diminution in index value is accompanied by an increase of variability.

A similar relation between the relative growth-rate—as measured by the differences between successive index values—and the variability obtains in other cases. Thus, although the index value of head-length increases it does so at a decreasing rate, and the variability decreases at the same time, so with the eye-diameter, the body-breadth, the position of the pectoral and posterior dorsal fins, and, but in the final stage only, the positions of the pelvic and ventral fins and of the anus. In others the relation does not hold good. It does however appear to be true of the majority that the variability depends more closely on the relative than we have seen it to do on the absolute growth-rate.

Correlations.

Table V gives the values of such correlation coefficients as have been worked out in Stages I, III and V.

The value of the coefficient varies, but between the same pair of correlates is of the same sort of magnitude throughout.

TABLE V. ρ.

	Total Length and Breadth of Caudal Fin	Total Length and Breadth of Body	Total Length and Length of Anterior Dorsal Fin	Total Length and Length of Ventral Fin	Total Length and Length of Posterior Dorsal Fin	Total Length and Length of Head	Length of Head and Diameter of Eye	Length of Anterior Dorsal and of Posterior Dorsal Fin	Length of Ventral and of Posterior Dorsal Fin	Position of Pectoral and of Pelvic Fin
I	·888 ± ·01	—	·764 ± ·02	·836 ± ·01	·495 ± ·05	·945 ± ·00	·766 ± ·02 † (·943 ± ·01)	·432 ± ·06	·515 ± ·05	·872 ± ·01
III	·736 ± ·02	·790 ± ·02	·493* ± ·04	·584* ± ·03	·452 ± ·04	·729 ± ·02	·646 ± ·03	·441* ± ·04	·646* ± ·03	·691 ± ·02
V	·727 ± ·03	·869 ± ·01	·602 ± ·03	·607 ± ·03	·523 ± ·04	·848 ± ·01	·647 ± ·03	·391 ± ·05	·581 ± ·04	·825 ± ·02

* Unreliable for the reason already given.

† The second and higher value is obtained by the omission of one very aberrant individual.

With two exceptions (total length correlated with length of posterior dorsal fin and length of ventral correlated with length of posterior dorsal fin) there is a sensible diminution in the value of the coefficient. This may depend on the fall in the growth-rate, as Boas has suggested.

On the other hand in passing from the third to the fifth stage there is, except in the case of total length and breadth of caudal fin, a rise in the value of the coefficient, and, as we have seen, this is not accompanied by any rise in the absolute growth-rate.

Nor does there appear to be any uniform connection between a rise or fall in the correlation coefficient and a rise or fall in the relative growth-rate.

Thus between Stages III and V the index values of head and of body-breadth increase, and the coefficient of correlation between these and the total length increases too. But between the same two stages the index value of the position of the pectoral and pelvic fins decreases, while the coefficient increases. In the case of the breadth of the caudal fin on the other hand the coefficient diminishes steadily while the index value increases.

Boas has found that the correlation is greater between axial organs—stature and height sitting, or stature and head-length—than between longitudinal and transverse parts.

In the Trout there is—so far—no indication of this. The coefficient value often is high between longitudinal parts, e.g. total length and length of anterior dorsal or ventral fins, or total length and length of head, but it may be low between such parts, e.g. total length and length of posterior dorsal fin, or length of posterior dorsal fin and that of anterior dorsal or of ventral fin. On the other hand it is high between the total length and the breadth of the body, and of the caudal fin.

Between the positions of one pair of metameric organs—the pectoral and pelvic fins—it is high: between the dimensions of other pairs of such organs (the median fins) it is lower.

Lastly, constancy through the series in the index value is not necessarily a sign of high correlation (e.g. the length of the posterior dorsal fin and total length).

The main results of the investigation may be briefly summarised as follows:

(1) During the first ten weeks after hatching there is a decline in the absolute growth-rate. This is more rapid in earlier than in later stages.

This is true not only of the whole body but of its parts. The growth-rate and the rate of its decline are not the same in the different parts.

(2) There is diminution in the absolute variability accompanying and possibly caused by this general diminution of growth-rate. At the same time there is a sensible increase in variability in the final stage which is not accompanied by any rise in the growth-rate. Also at certain other stages in certain parts the growth-rate may change while the variability does not alter correspondingly.

(3) The indices or proportions of the parts relatively to the whole may exhibit a gradual increase, or an increase at first rapid, later less so, followed eventually by a decrease, or a gradual diminution from the beginning, followed sometimes by a slight final increase or may be nearly constant.

There seems to be some reason to suppose that the changes in the variabilities of the indices, which exhibit on the whole a gradual decrease, though in some cases an increase is shown, depend on changes in the relative growth-rate.

(4) Correlations between parts are generally high. They exhibit a general decrease, with one or two exceptions. This may depend on the observed decrease in the growth-rate.

In the last, as compared with the middle stage, there is however an increase as a rule.

It is difficult to connect this last change with any corresponding change in either the absolute or the relative growth-rate.

It is hoped that these results, though somewhat meagre, may prove useful in the future, as it is proposed if possible to compare the growth-rate, variability and correlation under normal conditions and under conditions which have been made unfavourable experimentally.

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Biometric Workers and Statistical Reviewers.

By E. C. SNOW, M.A.

THE letter which follows this note was sent to the Editors of the *Journal of the Royal Statistical Society* in reply to an editorial criticism in the December number of the Journal of my recent memoir "The Intensity of Natural Selection in Man*." The remarks made in the course of that criticism seemed to me to be based on an inadequate reading of the memoir and a lack of appreciation of the work given to the reduction of the statistical data with which it deals. I thought that an immediate reply was desirable, and wrote pointing out the errors of fact, quotation and opinion into which I considered the writer had fallen.

As the Editors have seen fit to extract and publish only certain statements out of my letter without consulting me in the matter, I take this opportunity of issuing my detailed reply.

SIRS,

The writer of the notes on my recent memoir on "The Intensity of Natural Selection in Man" in the December number of the *Journal of the Royal Statistical Society* asks that discussion should be given to a certain point involved in that memoir. I gladly take this opportunity of initiating that discussion since, in addition to the specific point which it is desired to clear up, there is in those notes one flagrant misquotation from the memoir which requires correction, and also two or three other matters which it seems eminently desirable to place in their proper perspective in order that attention may be directed to the kernel of the problem.

In the first of the three paragraphs dealing with the subject, and before the memoir is mentioned, a quotation is given from Mr Yule's Appendix to the Report of the Medical Officer of the Local Government Board in which he states that the data he employed were such that his conclusions could not be put forward "as anything but a very tentative hypothesis." This coming before the title of the memoir or even any connection of mine with the subject is announced would lead the casual reader to conclude that I had given more weight to Mr Yule's work than he himself would desire. Yet the identical quotation used in the note is given in full in the memoir (p. 8) and reiterated on p. 10. As the charge of discourtesy is afterwards made against me, I think it might in common fairness have been made clear that I gave no greater emphasis to Mr Yule's work than he himself desired.

The point on which the writer of the note specifically asks for discussion admits of a very simple investigation. Before stating this I may point out that in giving an example of my method he uses the phrase "the remaining deaths under 5 years of age during 1903—7." In the original the word "remaining" is italicised (p. 15). This may seem at first sight to be

* *Drapers' Company Research Memoirs.* No. VII. Dulau and Co.

a small matter, but his failure to notice the emphasis placed on that word is, I think, the basis of the whole of the writer's attack on the method of measuring environment adopted in the memoir. He appears to think that by making those *remaining* deaths constant throughout the districts, I have also made the deaths of the particular cohorts I dealt with constant during their first five years of life. He asks, "Has Mr Snow found anything more than a somewhat obscured consequence of the obvious result that, in a group of districts with the same total mortality for the first five years of life, the districts with the higher mortality for the first three years will tend to have the lower mortality for the last two. It seems to us exceedingly doubtful whether the method of 'correction for environment' used has not led to a series of correlations which are largely fallacious and of no significance as regards selection." It is remarkable that the reviewer did not take the trouble to test the value of his suggestion, for I gave complete data in the table on p. 33 of the memoir for that to be done, and the time required was only a few moments at the outside. In fact an approximation sufficient for his purpose could be worked out mentally in the space of a few seconds. In the table referred to I gave values of ${}_{03}\sigma_1$ and ${}_{03}\sigma_2$ as well as of ${}_{03}r_{12}$ for five sets of data (three from English figures and two from Prussian—the latter, however, dealing with the first ten years of life, x_1 referring to the first two, and x_2 to the next eight years of life) distinguishing males from females. Thus there were ten distinct sets of values given, on any one of which he could have tested his suggestion. If we let x_5 denote x_1+x_2, the phrase "the same total mortality for the first five years of life" obviously requires the values of ${}_{03}\sigma_5$ to be appreciably zero in each case. Values of this can be found from a well-known elementary formula

$$ {}_{03}\sigma_5^2 = {}_{03}\sigma_1^2 + {}_{03}\sigma_2^2 + 2\,{}_{03}r_{12}\cdot{}_{03}\sigma_1\cdot{}_{03}\sigma_2, $$

a proof of which is given in Mr Yule's *Introduction to the Theory of Statistics*. The values of ${}_{03}\sigma_5$ as found from this formula are given in the following table together with such of the other figures from the memoir as are necessary for an understanding of the matter in its present bearings.

Data		$\bar{x}_0$	$\bar{x}_1$	$\bar{x}_2$	${}_{03}\sigma_1$	${}_{03}\sigma_2$	${}_{03}r_{12}$	${}_{03}\sigma_5$
Males:	1870	3227	644	51	44·582	10·634	−·4483	40·93
English Rural	1871	3226	602	60	42·247	9·589	−·3574	39·84
Districts	1872	3291	588	62	41·220	12·108	−·2271	40·24
Prussian Rural	1881	9407	2270	729	198·042	210·803	−·9278	78·68
Districts	1882	9297	2424	732	148·377	206·413	−·6050	166·02
Females:	1870	3090	531	48	34·340	11·126	−·4666	30·77
English Rural	1871	3114	491	59	28·468	12·736	−·2857	27·67
Districts	1872	3150	494	61	24·011	11·175	−·5089	20·54
Prussian Rural	1881	8917	1914	711	169·182	183·986	−·8483	98·30
Districts	1882	8793	2004	729	117·355	178·700	−·6078	142·17

The figures in the last column at once proclaim how groundless is the assertion that the negative values of ${}_{03}r_{12}$ are due to the fact that the method I used for correcting for environment was equivalent to assuming all the districts had the same total mortality for the first five years of life. Taking the first case on the table—the cohort of English males born in rural districts in 1870:—*when correction is made for environment* the mean total mortality in the first five years of life was 695, *with a standard deviation of* 41; the mean mortality in the first three years was 644 with a standard deviation of 45; and in the next two the figures were 51 and 11 respectively, the partial correlation for a constant environment between the mortality in the first three years of life and the next two being −·45. With such a range of values as is indicated by a mean

value of 695 and a standard deviation of 41 it cannot be maintained for a moment that the reviewer had any ground whatever for the opinion he stated. (For the reader who has not referred to the original memoir I may say that I have found there the magnitude of the (negative) correlation which should be expected if there were no selection; this arising solely from the fact that the districts with a high mortality in the first period would have a smaller population to be aimed at in the second, and therefore, without selection, a certain amount of negative correlation should be expected. But in every case except the Prussian one for the 1881 cohort—where the (negative) value of ${}_{03}r_{12}$ is very high—this correlation is less than the corresponding probable error of ${}_{03}r_{12}$.)

The complaint is also made of my "dangerous measure of environment." I admit that the method of measuring environment in the memoir is rather subtle, and that its use requires great care. I believe, however, that the real danger lies in the ease with which it may be misunderstood by those who have not the time or inclination to ascertain what it really means. I hope soon to complete the development of a criterion which shall give a measure of the goodness of any particular correction for environment, etc., but in the meantime I may point out that the danger is considerably less than that involved in the correction which necessitates the correlation of rates. There is often a likelihood of spurious correlation arising when this is done, and there is evidence accumulating that a number of earlier investigations in the field of correlation may be invalidated on account of the use of the correlation between rates. The measurement of environment has always been a difficulty in statistical researches requiring its use and, for the purposes of future work, I shall be grateful if the reviewer can produce any *facts* invalidating the one employed by me, or can make suggestions for a more valuable one. The only improved method I have been able to think of is fundamentally similar to the one actually adopted, but involves the introduction of more variables.

I must confess to considerable amazement that the *Journal* critic should write "It seems to us also that on reconsideration he is not likely to adhere to his present opinion that any investigation on the selectivity of the death-rate must be made on a homogeneous group of districts; heterogeneity is really of the essence of the case in the most important form of the problem." He can hardly have appreciated the facts I stated on p. 10 and the tables on pp. 19 and 21 of the memoir. Briefly, I found that when treating of a mixture of districts I obtained practically zero correlations. When I took out from these the rural ones and also the urban ones (there was a residue of areas coming completely under neither head) I found very significant negative values for the former, but zero correlations for the latter. On this problem and an earlier one (*Journal*, May 1911) I have spent many weeks in analysing heterogeneous material into more or less homogeneous groups, and no amount of verbal disquisition as apart from statistical analysis is likely to overthrow the experience I gained in those weeks. As I perceive the problem of the selectivity of the death-rate, we have statistically to get the populations in a large number of districts under uniform environmental conditions. Differences in the mortality in a given period will then arise solely from *random* causes. We then want to investigate to what extent the mortality in a district which is below the mean of all districts in that period is followed by a mortality which is above the mean of all districts in a following period. The difficulty is the statistical one of the measurement of environment. I spent considerable time in trying to devise a method of attacking the problem which could be applied to a mixture of urban and rural districts, but failed, chiefly on account of the migratory habits of urban populations. I claim, then, that the measurement of selection investigated by the method I adopted essentially requires the analysis of the material into groups of similar characteristics.

In the third of the *Current Notes* dealing with the subject there is a serious misquotation from the memoir. I quote the whole of the sentence in which it occurs. "The aims of the Report seem fairly evident to any reasonably careful reader, but Mr Snow judges that 'the investigation was specially intended to discover whether selection exists under modern

conditions,' and makes consequently some criticisms which are entirely wide of the mark and might have been couched in more courteous terms." The inference here is that I carelessly read the Report and assumed that its object was *specially* to investigate the existence or non-existence of selection. I did no such thing, for in my memoir the word written is not "specially" but "specifically" (p. 8), and the charge of careless reading must recoil on the reviewer. I never for one moment thought that the Report was *specially* concerned with that problem; its chief object was the exceedingly important one of the determination of the distribution and causes of excessive infantile mortality. That one of the *specific* objects of the Report was to determine if selection existed cannot be denied. The first of the three objects stated on p. 1 of the Report reads "to determine, on the basis of our national statistics, whether reduction of infant mortality implies any untoward influence on the health of survivors to later years." Again, on p. 9, "attempts to reduce infant mortality are regarded by many as an interference with natural selection, which must be inimical to the average health of those surviving.... The statistics in the following pages do not support this view." In the memoir I have given reasons for the opinion that the statistics referred to do nothing of the kind, though here I am only concerned in showing that the author of the Report intended part of it to refer to the problem of selection. Other quotations to the same end could be given from the Report, though it is true that on p. 17 there is another statement which contradicts them all. I need not quote from the many newspaper accounts of the Report in support of my contention that I did not carelessly assume it to deal with the problem of selection, but I will cull a statement from the review in our own *Journal* (Jan. 1911, p. 208), "Perhaps the most striking conclusion of the report—which is admittedly of a somewhat provisional character in the absence of an extended series of annual statistics of the kind given—is the proof that the alleged selective effect of infantile mortality is not supported by the data available: there may be a slight selective influence extending to the second or third year of life, but it is exceedingly slight at best."

I regret that the charge of discourtesy should be made against me, as I attempted throughout to be studiously courteous, and the *Journal* critic is the first one who in speaking of my attitude to the Report has expressed the opinion that I have erred on the side of *under*-courtesy. On re-reading my words the only sentence I can find which, I think, remotely admits of the charge made is one in which reference is made to the fact that, in the Report, those who have spent months in the laborious collection of *facts* and *figures* relating to the problem, and have come to the conclusion that selection *is* operative, are pilloried with the offensive label a "school of thought." I shall be grateful if the reviewer will point out to me the passages to which he refers, for other critics have expressed the opinion that I was too favourable in my attitude to the Report.

Finally I should like to summarise the evidence we have at present as to the existence of a selective death-rate in man. Full references to the originals are given both in the *Current Note* and in the memoir. Fuller details are given in the latter.

(*a*) The work of Miss Beeton and Prof. Pearson. This was indirect, but its results have never, so far as I am aware, been called into doubt, and have long been accepted by the scientific world. The authors showed that at least from 50 °/₀ to 70 °/₀ of all deaths were due to selective action.

(*b*) The investigation of Prof. Ploetz on German data. The result of this was to show that at least 60 °/₀ of the mortality at ages less than five was due to selective causes.

(*c*) The recent report of the Medical Officer of the Local Government Board and Mr Yule's Appendix to it. This is the only work, whose existence I am aware of, which, since the publication of (*a*), has appeared to call into doubt the operation of a selective death-rate. I have elsewhere given reasons against the validity of that work, and Mr Yule's statement as to the tentative nature of its results has often been quoted.

(*d*) The memoir frequently referred to in this letter. Not every result found there pointed to the existence of selection, but, in my opinion, those which were founded on the best data conclusively showed the prevalence of some degree of selection in the mortality of a general population. I do not believe that the criticisms brought against that work by the *Journal* reviewer in any way affect the legitimacy of its results.

I am, Sirs, etc.,

E. C. SNOW.

January 5th, 1912.

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